CSCI 104
Queues and Stacks

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ARRAY-BASED LIST IMPLEMENTATIONS
BOUNDDED DYNAMIC ARRAY STRATEGY
A Bounded Dynamic Array Strategy

• Allocate an array of some user-provided size
  – Capacity is then fixed

• What data members do I need?

• Together, think through the implications of each operation when using a bounded array (what issues could be caused due to it being bounded)?

```cpp
#ifndef BALISTINT_H
#define BALISTINT_H

class BAListInt {
 public:
  BAListInt(unsigned int cap);

  bool empty() const;
  unsigned int size() const;
  void insert(int pos, const int& val);
  void remove(int pos);
  int const & get(int loc) const;
  int& get(int loc);
  void set(int loc, const int& val);
  void push_back(const int& val);

 private:
};
#endif
```

balistint.h
A Bounded Dynamic Array Strategy

• What data members do I need?
  – Pointer to Array
  – Current size
  – Capacity

• Together, think through the implications of each operation when using a static (bounded) array
  – Push_back: Run out of room?
  – Insert: Run out of room, invalid location

```cpp
#ifndef BALISTINT_H
#define BALISTINT_H

class BAListInt {
public:
  BAListInt(unsigned int cap);
  bool empty() const;
  unsigned int size() const;
  void insert(int pos, const int& val);
  void remove(int pos);
  int const & get(int loc) const;
  int& get(int loc);
  void set(int loc, const int& val);
  void push_back(const int& val);
private:
  int* data_; 
  unsigned int size_; 
  unsigned int cap_; 
};
#endif
```
Implementation

- Implement the following member functions
  - A picture to help write the code

```cpp
BAListInt::BAListInt (unsigned int cap)
{
}

void BAListInt::push_back(const int& val)
{
}

void BAListInt::insert(int loc, const int& val)
{
}
```
Implementation (cont.)

• Implement the following member functions
  – A picture to help write the code

```cpp
void BAListInt::remove(int loc) {

}
```
Array List Runtime Analysis

- What is worst-case runtime of set(i, value)?
- What is worst-case runtime of get(i)?
- What is worst-case runtime of pushback(value)?
- What is worst-case runtime of insert(i, value)?
- What is worst-case runtime of remove(i)?
Const-ness

- Notice the get() functions?
- Why do we need two versions of get?
- Because we have two use cases...
  - 1. Just read a value in the array w/o changes
  - 2. Get a value w/ intention of changing it

```cpp
#ifndef BALISTINT_H
#define BALISTINT_H

class BAListInt {
 public:
    BAListInt(unsigned int cap);

    bool empty() const;
    unsigned int size() const;
    void insert(int pos, const int& val);
    bool remove(int pos);

    int& const get(int loc) const;
    int& get(int loc);

    void set(int loc, const int& val);
    void push_back(const int& val);

 private:
};
#endif
```
// ---- Recall List Member functions ------
// const version
int& const BAListInt::get(int loc) const
{ return data_[i]; }

// non-const version
int& BAListInt::get(int loc)
{ return data_[i]; }

void BAListInt::insert(int pos, const int& val);

// ---- Now consider this code ------
void f1(const BAListInt& mylist)
{
  // This calls the const version of get.
  // W/o the const-version this would not compile
  // since mylist was passed as a const parameter
  cout << mylist.get(0) << endl;
  mylist.insert(0, 57); // won't compile..insert is non-const
}

int main()
{
  BAListInt mylist;
  f1(mylist);
}
// ----  Recall List Member functions ------
// const version
int& const BAListInt::get(int loc) const
{ return data_[i]; }

// non-const version
int& BAListInt::get(int loc)
{ return data_[i]; }

void BAListInt::insert(int pos, const int& val);

// ---- Now consider this code ------
void f1(BAListInt& mylist)
{
    // This calls the non-const version of get
    // if you only had the const-version this would not compile
    // since we are trying to modify what the
    // return value is referencing
    mylist.get(0) += 1;  // mylist.get(0) = mylist.get(0) + 1;
    mylist.insert(0, 57);
    // will compile since mylist is non-const
}

int main()
{ BAListInt mylist;
  f1(mylist);
}

Moral of the Story: We need both versions of get()
UNBOUNDED DYNAMIC ARRAY STRATEGY
Unbounded Array

- Any bounded array solution runs the risk of running out of room when we insert() or push_back()
- We can create an unbounded array solution where we allocate a whole new, larger array when we try to add a new item to a full array

```
push_back(21) => 21

Old, full array
30 51 52 53 54 10

Allocate new array

Copy over items
30 51 52 53 54 10

Add new item
30 51 52 53 54 10 21
```

We can use the strategy of allocating a new array twice the size of the old array.
Activity

• What function implementations need to change if any?

```cpp
#ifndef ALISTINT_H
#define ALISTINT_H

class AListInt {
    public:
        bool empty() const;
        unsigned int size() const;
        void insert(int loc, const int& val);
        void remove(int loc);
        int& const get(int loc) const;
        int& get(int loc);
        void set(int loc, const int& val);
        void push_back(const T& new_val);
    private:

        int* _data;
        unsigned int _size;
        unsigned int _capacity;
};

// implementations here
#endif
```
Activity

• What function implementations need to change if any?

```cpp
#ifndef ALISTINT_H
#define ALISTINT_H

class AListInt { 
public:
    bool empty() const;
    unsigned int size() const;
    void insert(int loc, const int& val);
    void remove(int loc);
    int& const get(int loc) const;
    int& get(int loc);
    void set(int loc, const int& val);
    void push_back(const T& new_val);
private:
    void resize(); // increases array size
    int* _data;
    unsigned int _size;
    unsigned int _capacity;
};

// implementations here
#endif
```
Resizing

- Implement the resize method for an unbounded dynamic array

```cpp
#include "alistint.h"

void AListInt::resize()
{
}
```
A LOOK AHEAD: AMORTIZED RUNTIME
Example

• You love going to Disneyland. You purchase an annual pass for $240. You visit Disneyland once a month for a year. Each time you go you spend $20 on food, etc.
  – What is the cost of a visit?

• Your annual pass cost is spread or "amortized" (or averaged) over the duration of its usefulness

• Often times an operation on a data structure will have similar "irregular" (i.e. if we can prove the worst case can't happen each call) costs that we can then amortize over future calls
Amortized Run-time

- Used when it is impossible for the worst case of an operation to happen on each call (i.e. we can prove after paying a high cost that we will not have to pay that cost again for some number of future operations)

- Amortized Runtime = (Total runtime over k calls) / k
  - Average runtime over k calls
  - Use a "period" of calls from when the large cost is incurred until the next time the large cost will be incurred
Amortized Array Resize Run-time

• What is the run-time of insert or push_back:
  – If we have to resize?  
    – $O(n)$
  – If we don't have to resize?  
    – $O(1)$

• Now compute the total cost of a series of insertions using resize by 1 at a time

• Each new insert costs $O(n)$... not good

`push_back(21) =>
Old, full array
0 1 2 3 4
30 51 52 53 54
Increase old array size by 1
0 1 2 3 4 5
Copy over items
30 51 52 53 54 21
push_back(33) =>
Increase old array size by 1
0 1 2 3 4 5 6
Copy over items
30 51 52 53 54 21 33

Resize by 1 strategy`
Amortized Array Resize Run-time

- What if we resize by adding 5 new locations each time
- Start analyzing when the list is full...
  - 1 call to insert will cost: n+1
  - What can I guarantee about the next 4 calls to insert?
    - They will cost 1 each because I have room
  - After those 4 calls the next insert will cost: (n+5)
  - Then 4 more at cost=1
- If the list is size n and full
  - Next insert cost = n+1
  - 4 inserts after than = 1 each = 4 total
  - Thus total cost for 5 inserts = n+5
  - Runtime = cost / inserts = (n+5)/5 = O(n)

push_back(21) =>

Old, full array

Increase old array size by 5

Copy over items

Resize by 5 strategy
Consider a Doubling Size Strategy

• Start when the list is full and at size \( n \)
• Next insertion will cost?
  – \( O(n+1) \)
• How many future insertions will be guaranteed to be cost = 1?
  – \( n-1 \) insertions
  – At a cost of 1 each, I get \( n-1 \) total cost
• So for the \( n \) insertions my total cost was
  – \( n+1 + n-1 = 2^n \)
• Amortized runtime is then:
  – Cost / insertions
  – \( O(2^n / n) = O(2) \)
    = \( O(1) = \) constant!!!
Specialized List ADTs

STACKS AND QUEUE ADTS
Lists

• Ordered collection of items, which may contain duplicate values, usually accessed based on their position (index)
  – Ordered = Each item has an index and there is a front and back (start and end)
  – Duplicates allowed (i.e. in a list of integers, the value 0 could appear multiple times)
  – Accessed based on their position (list[0], list[1], etc.)

• What are some operations you perform on a list?
# List Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
<th>Input(s)</th>
<th>Output(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>insert</td>
<td>Add a new value at a particular location shifting others back</td>
<td>Index : int Value</td>
<td></td>
</tr>
<tr>
<td>remove</td>
<td>Remove value at the given location</td>
<td>Index : int</td>
<td>Value at location</td>
</tr>
<tr>
<td>get / at</td>
<td>Get value at given location</td>
<td>Index : int</td>
<td>Value at location</td>
</tr>
<tr>
<td>set</td>
<td>Changes the value at a given location</td>
<td>Index : int Value</td>
<td></td>
</tr>
<tr>
<td>empty</td>
<td>Returns true if there are no values in the list</td>
<td></td>
<td>bool</td>
</tr>
<tr>
<td>size</td>
<td>Returns the number of values in the list</td>
<td></td>
<td>int</td>
</tr>
<tr>
<td>push_back / append</td>
<td>Add a new value to the end of the list</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>find</td>
<td>Return the location of a given value</td>
<td>Value</td>
<td>Int : Index</td>
</tr>
</tbody>
</table>
Stacks & Queues

• Lists are good for storing generic sequences of items, but they can be specialized to form other useful structures.

• What if we had a List, but we restricted how insertion and removal were done?
  – **Stack** – Only ever insert/remove from one end of the list
  – **Queue** – Only ever insert at one end and remove from the other
First-In, First-Out (FIFOs)

QUEUE ADT
Queue ADT

• Queue – A list of items where insertion only occurs at the back of the list and removal only occurs at the front of the list
  – Like waiting in line for a cashier at a store
• Queues are FIFO (First In, First Out)
  – Items at the back of the queue are the newest
  – Items at the front of the queue are the oldest
  – Elements are processed in the order they arrive
A Queue Visual

Items leave from the front (pop_front)

Items enter at the back (push_back)
Queue Operations

• What member functions does a Queue have?
  – push_back(item) – Add an item to the back of the Queue
  – pop_front() - Remove the front item from the Queue
  – front() - Get a reference to the front item of the Queue (don't remove it though!)
  – size() - Number of items in the Queue
  – empty() - Check if the Queue is empty
A Queue Class

• A sample class interface for a Queue
• Queue Error Conditions
  – **Queue Underflow** – The name for the condition where you call pop on an empty Queue
  – **Queue Overflow** – The name for the condition where you call push on a full Queue (a Queue that can't grow any more)
    • This is only possible for Queues that are backed by a bounded list

```cpp
#ifndef QUEUEINT_H
#define QUEUEINT_H

class QueueInt {
  public:
    QueueInt();
    ~QueueInt();
    size_t size() const;
    // enqueue
    void push_back(const int& value);
    // dequeue
    void pop_front(); // dequeue
    int const & front() const;
    bool empty() const;

  private:
    // ???
};
#endif
```
Other Queue Details

• How should you implement a Queue?
  – Compose using an ArrayList
  – Compose using a singly-linked list w/o a tail pointer
  – Compose using a singly-linked list w/ a tail pointer
  – Which is best?

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</tr>
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<td>(Singly-linked w/o tail ptr)</td>
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Queue Applications

• Print Jobs
  – Click “Print” on the computer is much faster than actually printing (build a backlog)
  – Each job is processed in the order it's received (FIFO)
  – Why would you want a print queue rather than a print stack

• Seating customers at a restaurant
• Anything that involves "waiting in line"
• Helpful to decouple producers and consumers
Last-In, First-Out (LIFOs)

STACK ADT
Stack ADT

- Stack: A list of items where insertion and removal only occurs at one end of the list

- Examples:
  - A stack of boxes where you have to move the top one to get to ones farther down
  - A spring-loaded plate dispenser at a buffet
  - A PEZ dispenser
  - Your e-mail inbox

- Stacks are LIFO
  - Newest item at top
  - Oldest item at bottom

(push)

(pop)

Top item

Stack
Stack Operations

• What member functions does a Stack have?
  – push(item) – Add an item to the top of the Stack
  – pop() - Remove the top item from the Stack
  – top() - Get a reference to the top item on the Stack (don't remove it though!)
  – size() - Get the number of items in the Stack

• What member data does a Stack have?
  – A list of items
  – Top/Last Item Pointer/Index
Stack Axioms

- For all stacks, s:
  - s.push(item).top() = item
  - s.push(item).pop() = s
- Let’s draw the stack for these operations:
  - s.push(5).push(4).pop().top()
A Stack Class

- A sample class interface for a Stack
- How should you implement a Stack?
  - Back it with an array
  - Back it with a linked list
  - Which is best?
- Stack Error Conditions
  - Stack Underflow – The name for the condition where you call pop on an empty Stack
  - Stack Overflow – The name for the condition where you call push on a full Stack (a stack that can't grow any more)

```cpp
#ifndef STACKINT_H
#define STACKINT_H

class StackInt {
public:
  StackInt();
  ~StackInt();
  size_t size() const;
  bool empty() const;
  void push(const int& value);
  void pop();
  int const & top() const;
};
#endif
```
Array Based Stack

• A sample class interface for a Stack
• If using an array list, which end should you use as the "top"?
  – Front or back?
• If using a linked list, which end should you use?
  – If you just use a head pointer only?
  – If you have a head and tail pointer?

```cpp
#ifndef STACKINT_H
#define STACKINT_H

class StackInt {
  public:
    StackInt();
    ~StackInt();
    size_t size() const;
    bool empty() const;
    void push(const int& value);
    void pop();
    int const& top() const;
  private:
    AListInt mylist_;  // or LListInt mylist_
};
#endif
```
Stack Examples

• Reverse a string

```cpp
#include <iostream>
#include <string>
#include "stack.h"
using namespace std;
int main()
{
    StackChar s;

    string word;
    cout << "Enter a word: ";
    getline(cin,word);

    for(int i=0; i < word.size(); i++)
        s.push(word.at(i));

    while(!s.empty()){
        cout << s.top();
        s.pop();
    }
}
```

Type in: "hello"
Output: "olleh"
Another Stack Example

- Depth First Search (See Graph Traversals later in this semester)
- Use a stack whenever you encounter a decision, just pick and push decision onto stack. If you hit a dead end pop off last decision (retrace steps) and keep trying, etc.
  - Assume we always choose S, then L, then R
  - Strait or Left
    - Choose straight...dead end
    - Pop straight and make next choice...left
    - Next decision is Straight or Right...choose Straight...

http://www.pbs.org/wgbh/nova/einstein/images/lrk-maze.gif
Stack Usage Example

• Check whether an expression is properly parenthesized with '(', '[', '{', '}', ']', ')'
  – Correct: \((7 * [8 + [9/{5-2}]]))\)
  – Incorrect: \((7*8\)
  – Incorrect: \((7*8[\)
• Note: The last parentheses started should be the first one completed
• Approach
  – Scan character by character of the expression string
  – Each time you hit an open-paren: '(', '[', '{' push it on the stack
  – When you encounter a ')', ']', '}' the top character on the stack should be the matching opening paren type, otherwise ERROR!

\((7 * [4 + 2 + 3])\)

\[
\begin{array}{c}
( \{ \} ) \\
[ ] \\
\end{array}
\]

\[
\begin{array}{c}
(7 * [4 + 2 + 3]) \\
7 * \\
7 \\
( \\
\end{array}
\]
Queue with two stacks

- To enqueue(x), push x on stack 1
- To dequeue()
  - If stack 2 empty, pop everything from stack 1 and push onto stack 2.
  - Pop stack 2

---

stack1 | stack2
Time=1

stack1 | stack2
Time=2

stack1 | stack2
Time=3
Double-ended Queues

DEQUE ADT
The Deque ADT

- **Deque** - Equally good ($\Theta(1)$) push and pop on either end
- What list implementation supports this already?
  - ________________

(push_front)  
(pop_front)  
(push_back)  
(pop_back)
STL Deque Class

- Uses an array-based approach
- Similar to vector but allows for push_front() and pop_front() options
- Useful when we want to put things in one end of the list and take them out of the other

```cpp
#include <iostream>
#include <deque>

using namespace std;

int main()
{
    deque<int> my_deq;
    for(int i=0; i < 5; i++){
        my_deq.push_back(i+50);
    }
    cout << "At index 2 is: " << my_deq[2] << endl;
    for(int i=0; i < 5; i++){
        int x = my_deq.front();
        my_deq.push_back(x+10);
        my_deq.pop_front();
    }
    while( !my_deq.empty() ){
        cout << my_deq.front() << " ";
        my_deq.pop_front();
    }
    cout << endl;
}
```

1. my_deq
   0 1 2 3 4
   50 51 52 53 54
2. my_deq
   0 1 2 3 4 after 1st iteration
   51 52 53 54 60
3. my_deq
   0 1 2 3 4 after all iterations
   60 61 62 63 64
4. my_deq

1 2 3 4
STL Vector vs. Deque

- `std::vector` is essentially a Dynamic Array List
  - Slow at removing and inserting at the front or middle
  - Fast at adding/remove from the back
  - Implies it could be used well as a (`stack / queue`)

- `std::deque` gives fast insertion and removal from front and back along with fast random access (i.e. `get(i)`)
  - Almost has "look and feel" of linked list with head and tail pointers providing fast addition/removal from either end
  - Implies it could be used well as a (`stack / queue`)
  - Practically it is likely implemented as a circular array buffer
Circular Buffers

- Take an array but imagine it wrapping into a circle to implement a deque
- Setup a head and tail pointer
  - Head points at first occupied item, tail at first free location
  - Push_front() and pop_front() update the head pointer
  - Push_back() and pop_back() update the tail pointer
- To overcome discontinuity from index 0 to MAX-1, use modulo operation
  - Cannot just use `back++;` to move back ptr
  - Instead, use `back = (back + 1) % MAX;`
- Get item at index i
  - Must be relative to the head pointer
SOLUTIONS
Other Queue Details

• How should you implement a Queue?
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