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Evidence of Long-Range Dependence in Power Grid

L. Shalalfeh
Dept. of Electrical Engineering
University of Southern California
Los Angeles, California 90089
shalalfe@usc.edu

P. Bogdan
Dept. of Electrical Engineering
University of Southern California
Los Angeles, California 90089
pbogdan@usc.edu

E. Jonckheere
Dept. of Electrical Engineering
University of Southern California
Los Angeles, California 90089
jonckhee@usc.edu

***Abstract*—Analyzing the voltage stability of the smart grid requires accurate mathematical models of the load. Several static and dynamic load models were proposed, but most of them did not incorporate the frequency component. In order to build more accurate dynamic load models in the power system, we must examine the mathematical characteristics (short range versus long range dependence) of the power system variables. Toward this end, we demonstrate that the voltage magnitude, frequency, and voltage phase angle of Phasor Measurement Unit data exhibits long-range dependence. Our findings call for the development of a fractal modeling approach of the smart grid.**

I. Introduction

The smart grid term was firstly introduced in 2005 [**?**] and the objective was to make the grid stronger, smarter and more secure. Simply speaking, the aim is to design a power grid that it can cope with all new developments of renewable energy plants and to sustain future energy demand such as the increase of deployed plug-in hybrid electric vehicle (PHEV).

Aiming to design a smart grid that is efficient, reliable, resilient, stable, and secure requires both a fundamental understanding of its dynamics and an accurate mathematical modeling. For instance, in order to have a stable power grid, the system should be able to regain the state of operating equilibrium after being subjected to a disturbance with most of the system variables remaining bounded [**?**]. The system stability can be classified into: *voltage*, *frequency*, and *rotor angle stabilities*. One fundamental challenge for ensuring voltage stability of the power grid is represented by the voltage collapse phenomenon in which the voltage drops to a low level due to a system disturbance [**?**].

The voltage collapse phenomenon has been studied extensively in the 80's and 90's. It was argued that the voltage collapse is either static or dynamic in nature; several researchers studied the voltage collapse from both aspects. In the static case, the voltage collapse was studied by looking at the feasibility of the load flow [**?**], the minimum singular value of the Jacobian matrix [**?**], and static bifurcations of the load flow equations [**?**]. The static approach describes the load using the active and reactive powers, as shown in Eq. (1):

$$P(V) + jQ(V) = k_p(V)^{n_p} + jk_q(V)^{n_q} \tag{1}$$

where, k_p and k_q are the nominal active and reactive powers respectively. Voltage collapse was studied dynamically by investigating the interaction between the generator and the load [**?**] and the interaction between the load and the On-Load Tap Changer (OLTC) [**?**]. The popular dynamic load model was introduced by Hill [**?**] by representing the active power as a first-order nonlinear differential equation, as shown in Eq. (2):

$$T_p\dot{P}_d + P_d = P_s(V_L) + k_p(V_L)\dot{V}_L \tag{2}$$

The dynamic model represents the load recovery with time constant (T_p) after a step change in the load voltage. The system frequency in this model is assumed to be constant.

Even though many studies of the voltage collapse assumed no change in the system frequency during the voltage drop, there is still some debate on the need to investigate the dynamics of the frequency during this phenomenon. In [**?**], the author shows how the frequency dynamics can play a role in the voltage collapse, and explains how constant frequency assumption is not completely justifiable in case of a large time constant of the voltage. More recently, the research community and the government energy agency [**?**] advocate for the necessity of a better dynamic load modeling that can explain and describe the voltage and frequency variations over multiple time scales (short and long).

Starting from these premises and aiming to build a more accurate dynamic load model, in this paper, we perform an in-depth statistical analysis of the power grid processes and identify a few fundamental mathematical characteristics that are essential for understanding the voltage collapse phenomenon. Using real power system data, we demonstrate the existence of fractal and non-stationary behavior in the power grid that justifies the need for capturing the frequency dependence as well as hints towards constructing long-range memory (dependence) models of the power grid dynamics. The remaining of the paper is organized as follows: Sec. II summarizes the prior work and our novel contribution. Sec. III describes the source of data used in our statistical analysis. Sec. IV demonstrates that voltage magnitude (V), frequency (f), and phase angle (θ) processes in the power grid exhibit long range memory (correlations). Sec.V concludes the paper and points out some future directions.

II. Prior Work and Novel Contribution

The self-organized criticality has been discussed in the context of blackout size in power system [**?**],[**?**]. In [**?**], a long-range dependence in the electricity prices has been measured. In the literature, the long-range dependence has not been applied to power system times series (voltage magnitude (V), frequency (f), and voltage phase angle (θ)). There are several methods to study the long-range dependence in time series. For stationary time series, we can use methods like, R/S, Variance, Absolute Moments, and Whittle.These methods do not give accurate results in case of non-stationary time series. However, Detrended Fluctuation Analysis (DFA) is a powerful and robust method to examine the long-range dependence in non-stationary time series. Our main contributions are:

- We first investigate the statistical properties of the real power system data (voltage magnitude (V), frequency (f), and phase angle (θ)) measured by several PMUs in Texas synchrophasor network.
- Based on the observed non-stationarity, we employ the DFA method to calculate the scaling exponent α of time series (voltage magnitude (V), frequency (f), and phase angle (θ)) at different locations in the synchrophasor network. The scaling exponents indicate the type of mathematical models that should be constructed to describe the power system dynamics.

III. Statistical properties of the power system

We will first describe the power system data we use in our analysis and then summarize our statistical analysis of several Phasor Measurement Units (PMUs) time series at various locations.

A. Description of the power system measurements

We use real data from the power system obtained through several PMUs installed in the Texas synchrophasor network. Texas synchrophasor network has several PMUs distributed over several locations. The measured data of our interest are voltage magnitude (in p.u.), frequency (in Hz), and voltage phase angle (in degrees). These data are available online [?] for three PMUs at three locations: Baylor University, Harris Substation, and McDonald Observatory. There are four data sets of measurements at each location with one hour duration and at sampling rate of 30 samples/s. The data sets have been recorded on 5/25/2015 (6:00-7:00 PM and 10:00-11:00 PM), 05/27/15 (12:00-1:00 PM), and 05/30/15 (9:00-10:00 AM).

The plots of the voltage magnitude (V), frequency (f), and phase angle (θ) of data set 1 at Baylor university are shown in Figs. 1(a), 1(b), and 1(c) . We unwrapped the measurements of the phase angle, as shown in Fig. 1(d). The voltage magnitude (V) in Fig. 1(a) varies between 0.9632 and 0.9955 p.u., and the frequency (f) varies between 59.89 and 60.04 Hz, as shown in Fig. 1(b). There are noticeable events around the 40th minute of the voltage magnitude (V) plot and around 3rd minute in the frequency (f) plot. Due to the low sampling rate (30 samples/s) of the measured data, the frequency components of the data will be limited to 15 Hz and will not be useful to detect any high-frequency events.

B. Statistical analysis of PMU measurements

Examining the stationarity of the PMU measurements is very important for choosing the most appropriate mathematical modeling framework. For instance, working under the assumption that the processes are stationary and trying to construct compact mathematical models (with few parameters) can lead to misguiding conclusions (e.g., applying a stationary method for quantifying long-range memory to a non-stationary time series can lead to misleading exponents). Consequently, in what follows, we investigate the stationarity of the voltage magnitude (V), frequency (f), and phase angle (θ) by estimating their empirical cumulative distribution functions (CDFs) over moving time intervals.

A process is called second-order stationary if its mean and variance are constant over time, and the auto-covariance of the data does not depend on time. Moreover, the stationarity in the strict sense means the joint statistical distribution of any subset of the time series does not depend on time. To investigate the stationarity of the PMU data, we divide each data set into four intervals each with 900 seconds length and estimate their corresponding empirical CDFs for each data set. Identical empirical CDFs indicate that the processes are stationary; otherwise, they are non-stationary.

The empirical CDFs of the voltage magnitude (V) in data set 3 are shown in Fig. 1(e). The empirical CDFs of the voltage magnitude (V) are not identical and empirical CDF of the first interval (black) is intersecting the empirical CDF of the second interval (blue). Also, the empirical CDFs of the frequency (f), shown in Fig. 1(f), are not identical and the empirical CDF of the first interval (black) is crossing the empirical CDFs of the three other intervals (blue, green, and red). The empirical CDFs of the phase angle (θ) and unwrapped phase angle, shown in Figs. 1(g) and 1(h), are crossing each other several times. So, the voltage magnitude (V), frequency (f), and phase angle (θ) in data set 3 are non-stationary. The non-stationarity has been confirmed for all the locations in the Texas synchrophasor network.

IV. Existence of long-range dependence

In this section, we first explain the DFA method, and then, study the long-range dependence in the time series of the voltage magnitude, frequency, and voltage phase angle.

A. Detrended Fluctuation Analysis (DFA)

The measured data of the voltage magnitude (V), frequency (f), and phase angle (θ) in the power system are non-stationary, therefore we will not be able to use methods like, Variance, Absolute Moments, and R/S to confirm the long-dependency in the data. That is because these methods were derived based on the assumption that the data are stationary. The DFA method was introduced in [?] to study the mosaic organization of DNA nucleotides. This method has been proven as a robust method to show existence of long-dependency in time series.

This method has been applied successfully in several non-stationary data like, heartbeat fluctuation [?], daily temperature [?], and wind speed [?]. For a data set y with N data points, the DFA analysis can be summarized in four steps:

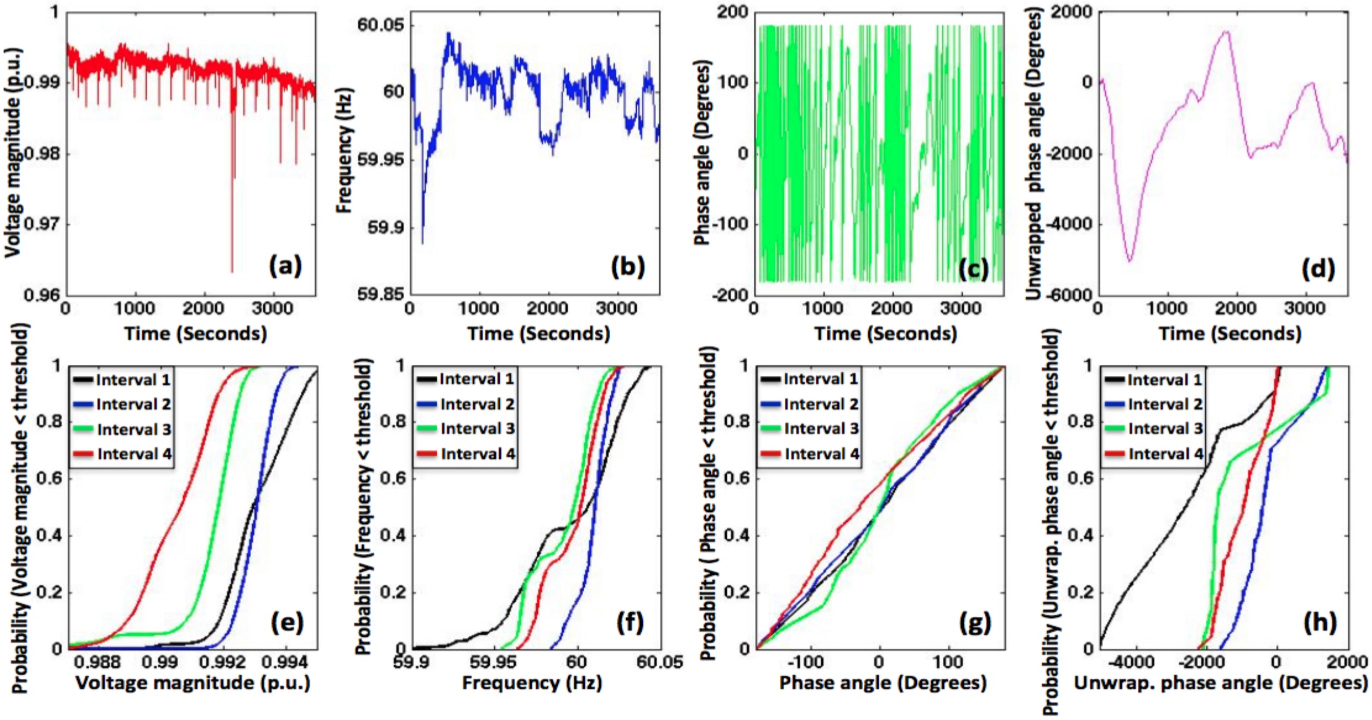

Fig. 1: (a) The voltage magnitude at Baylor University on 05/27/15 (12:00-1:00 PM) (b) The frequency at Baylor University on 05/27/15 (12:00-1:00 PM) (c) The voltage phase angle at Baylor University on 05/27/15 (12:00-1:00 PM) (d) The unwrapped phase angle (e) The empirical CDFs of the voltage magnitude at four intervals (900 seconds each) (f) The empirical CDFs of the frequency at four intervals (900 seconds each) (g) The empirical CDFs of the phase angle at four intervals (900 seconds each) (h) The empirical CDFs of the unwrapped phase angle at four intervals (900 seconds each)

1) Subtract the data set average y_{avg} from each data point $y(i)$ and integrate the data set using Eq. (3),

$$y_{int}(k) = \sum_{i=1}^{k} (y(i) - y_{avg}) \tag{3}$$

2) Divide the integrated data set into equal-sized boxes n and find the linear least square line y_n inside each box. Then, subtract the least square linear fitting $y_n(k)$ from the integrated data $y_{int}(k)$ to generate the detrended data y_d, as shown in Eq. (4),

$$y_d(k) = y_{int}(k) - y_n(k) \tag{4}$$

3) Find the root mean square (rms) fluctuation of the detrended data y_d using Eq. (5),

$$F(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N} (y_d(k))^2} \tag{5}$$

4) The second and third steps are repeated at different box sizes n, and we find the least square line of the log-log plot of $F(n)$ versus n.

The slope of the least square linear fitting is called the scaling exponent α. A linear relationship between $\log_{10}(F(n))$ and $\log_{10}(n)$ means $F(n) \propto n^{\alpha}$. Based on the value of α, the long-range dependence in the data can be determined. A scaling exponent α between 0 and 0.5 means that the data are anti-correlated. The measured data is random if α is equal to 0.5. For α between 0.5 and 1, a long-range power-law dependence exists in the data. Finally, $\alpha > 1$ indicates long-range dependence, but not in the power-law form. If the log-log plot of $F(n)$ versus n has one linear trend, then the data is called mono-fractal. Some data sets exhibit a change in the slope of the least square line at a transition point. This phenomenon is called crossover and the data are considered to possess multi-fractal characteristics.

B. Analysis of the power system data using DFA

As part of the DFA analysis, we have to find the root mean square (rms) of fluctuation at different box sizes n. We choose the box size (n) to change from 100 data points to 10,000 data points with logarithmic step size equal to $10^{\frac{1}{10}}$. A logarithmic spacing is preferred because choosing a linear spacing will make the weight of the log-log plot higher by moving from 100 to 10,000. The range of the $\log 10(n)$ is from 2 to 4 with 0.1 step size. We applied the DFA method on four data sets of voltage magnitude (V), frequency (f), and phase angle (θ) at three different locations. So, our idea is to run the DFA analysis on data sets that are distant in time and space to make sure that results are robust and comparable.

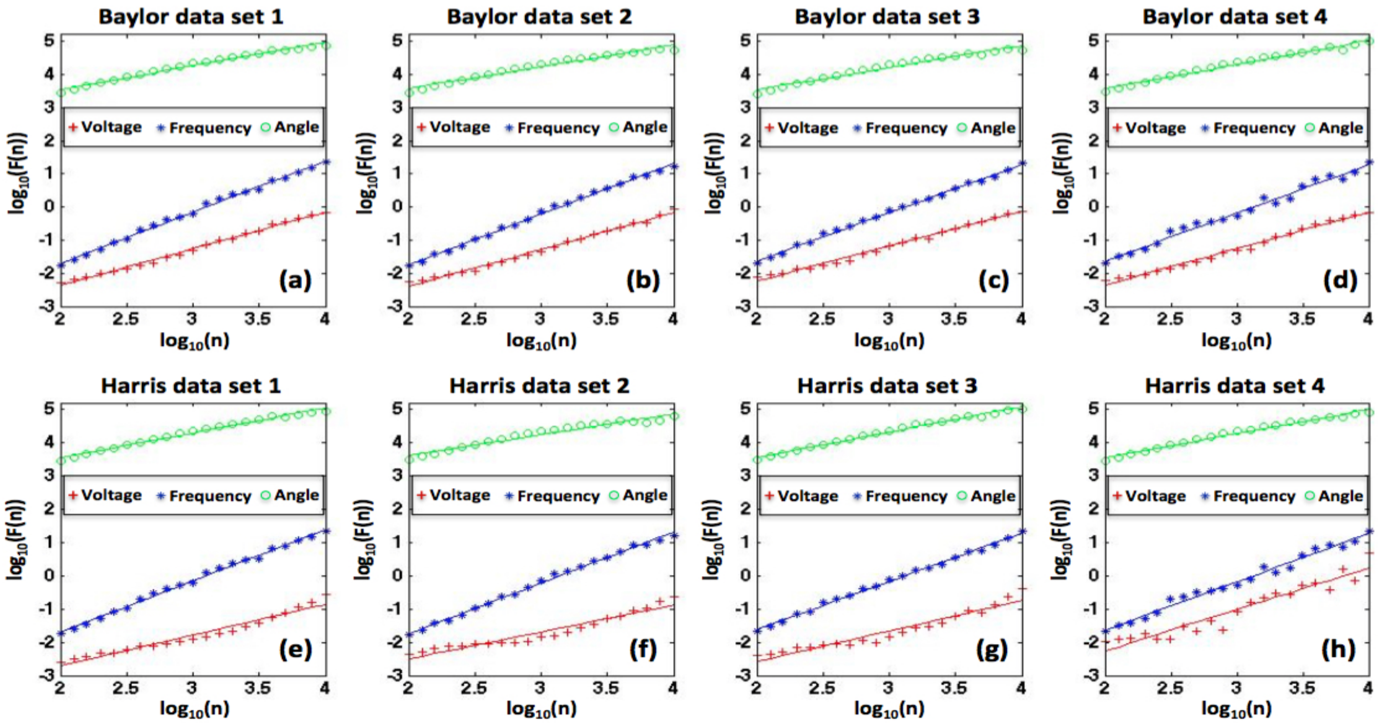

Fig. 2: Log-log plots of the rms fluctuation function $F(n)$ versus the box size n of V, f, and θ of different data sets: (a) Data set 1 at Baylor University (b) Data set 2 at Baylor University (c) Data set 3 at Baylor University (d) Data set 4 at Baylor University (e) Data set 1 at Harris Substation (f) Data set 2 at Harris Substation (g) Data set 3 at Harris Substation (h) Data set 4 at Harris Substation

1) DFA analysis of the voltage magnitude: The DFA analysis of the voltage magnitude (V) at the three different locations are shown in Table. I. The scaling exponents of the four data sets at Baylor location are between 0.91 and 1.11. These values are close to the scaling exponent of the white noise ($1/f$) which has scaling exponent equal to 1. At Harris location, the scaling exponents are in the range of [0.81, 0.92] and they are also not far from the scaling exponent 1 (white noise). The McDonald location has scaling exponents between 1.30 and 1.37 which are clearly higher than the other two locations. The McDonald PMU was installed in the Texas synchrophasor network particularly at the McDonald Observatory because it is among several wind power plants in that part of the network [?]. Our explanation of the higher scaling exponents at this location could be related to the voltage fluctuation associated with the connected wind power plants in the area [?]. Figs. 2 (a-h) show the rms fluctuation function of the voltage magnitude (V) at Baylor and Harris using red-color data points. Finally, since the scaling exponent at three locations varies between 0.81 and 1.37, long-range dependence does exist in the voltage magnitude (V).

2) DFA analysis of the frequency: The scaling exponents of the frequency (f) of the four data sets at each location are shown in Table. I. The three locations (Baylor, Harris, and McDonald) have similar scaling exponents between 1.45 and 1.54. All the data sets of the frequency (f) exhibit long-range dependence, but not in a power-low form. The frequency (f) has scaling exponents similar to Brownian noise which has 1.5 scaling exponent. The plots of the rms fluctuation function of the frequency (f) are shown in blue color in Figs. 2 (a-h) for Baylor and Harris locations.

Data Set	Baylor			Harris			McDonald		
	V	f	θ	V	f	θ	V	f	θ
#1	1.11	1.54	0.71	0.92	1.54	0.75	1.32	1.54	0.74
#2	1.11	1.53	0.66	0.81	1.53	0.63	1.30	1.53	0.64
#3	1.05	1.45	0.67	0.91	1.45	0.76	1.37	1.45	0.73
#4	0.91	1.49	0.63	0.89	1.49	0.64	1.32	1.49	0.64

TABLE I: Scaling exponents of voltage magnitude, frequency, and phase angle for four data sets at three locations: Baylor University, Harris Substation, and McDonald Observatory

3) DFA analysis of the phase angle: The scaling exponents of the phase angle (θ) data are shown in Table. I. The scaling exponents of the phase angle (θ) varies between 0.63 and 0.76. Since the scaling exponents of the phase angle (θ) are between 0.5 and 1, that means the phase angle (θ) data have long-range power-law dependence. Figs. 2 (a-h) show the rms fluctuation function of the phase angle (θ) in green color.

C. DFA analysis of the surrogate power system data

To show the robustness of our result, we generated a randomly shuffled version of the PMU data (surrogate data), which has an identical CDF compared to the original data.

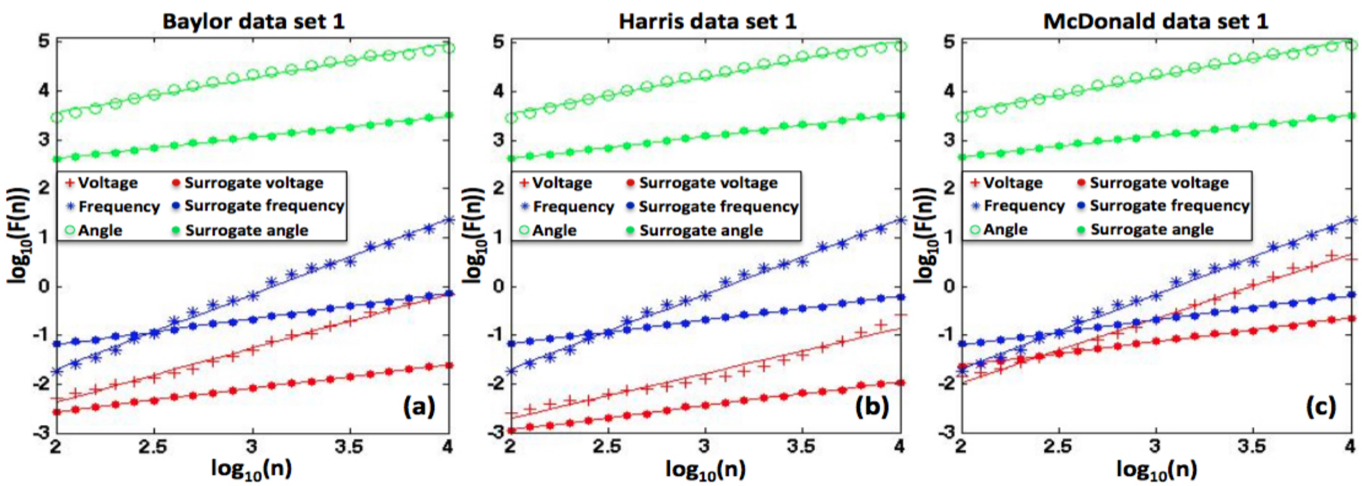

Fig. 3: Log-log plots of the rms fluctuation function $F(n)$ versus the box size n of original and surrogate data at: (a) Baylor University (b) Harris Substation (c) McDonald Observatory

Then, we run the DFA analysis on both the original and surrogate data to identify any change in the correlation. If we observe a decrease in the scaling exponent α, the fractal behavior is due to the temporal structure of the measured data. This temporal structure plays a fundamental rule in constructing an accurate mathematical model. We generated log-log plots of the rms fluctuation function $F(n)$ versus n for the original and surrogate on three data sets at three different locations, as shown in Fig. 3. The results show that the scaling exponents of voltage magnitude (V), frequency (f), and phase angle (θ) have changed to around 0.5 after the random shuffling of the data. Of note, time series characterized by 0.5 scaling exponent can be modeled through first order differential equations. However, the observed temporal structure characterized by scaling exponent higher than 0.5 implies that the dynamics of the system variables should be characterized through fractional order differential equations.

V. Conclusion

In this work, we performed a compressive statistical analysis of the PMU measurements, which demonstrates the existence of mono-fractality and non-stationarity as two main mathematical characteristics. These findings have recently been corroborated by European PMU data analysis; furthermore, the same analysis on India voltage collapse data seems to indicate that monitoring the PMU fractal dimension is able to anticipate imminent voltage collapse. The next challenge is to develop models able to reproduce the long range dependency of frequency, voltage and phase angle PMU data. Fractional dynamics models would reproduce such long range dependency. It is believed that the fractional powers of the frequency in the Berg model, properly re-interpreted in the time domain, would produce such models, as it has already been able to anticipate hitherto unknown voltage collapse scenarios [**?**].

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