Ollivier-Ricci Curvature approach to Cost-Effective Power Grid Congestion Management

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***Abstract*— A new method for congestion management is developed based on the Ollivier-Ricci curvature notion applied to the power grid. This curvature notion is utilized to detect those lines that carry a significant amount of power compared to other lines (negative curvature lines/edges). Once the areas prompt to congestion are identified, a curvature maximization is achieved via load balancing using FACTS devices. Simultaneously, a set of loads/storage devices are deployed within the congested areas in a cost-effective manner, aimed at draining the power away from the negative curvature areas (congestion areas) and minimizing the generated energy throughout the grid; guaranteeing stability under phase angle and voltage constraints.**

I. Introduction

Power Grid Congestion Management has become an active research area over the last few years; many novel algorithms and strategies have been disseminated within the research community. A comprehensive literature review for the reader could be found in [8], [9], [10], [11], [12], [13] and [14]. Some of the most renowned startegies and optimization algorithms include: Generators Rescheduling (GR), load shedding, Distributed Generation (DG), Optimal Power Flow (OPF), Flexible Alternating Current Transmission System (FACTS) devices, Artificial Bee Colony algorithm (ABC), Genetic Algorithms (GA) and Strength Pareto Evolutionary Algorithm (SPEA), just to name a few.

This work proposes a congestion management (CM) method that uses the grid Ollivier-Ricci curvature (definition in Sec. III) as the pillar for a cost-effective congestion reduction technique. The very first step in the method aims at effectively detecting all the *critical (negatively curved) edges* prompt to receive most of the active power within the grid. This is achieved by applying the Ollivier-Ricci Curvature (ORC) concept, which traces its origins back to the *earth mover's distance* idea developed in the Napoleon era with the objective of effectively move earth from one point to the other to level off the landscape. This measure is known as the Wasserstein 1-metric distance within the mathematicians' community, and it is also known as the *transportation problem* (Gaspard Monge, 1781).

Once the negative curvature lines are identified using ORC (recall from [18] that negative curvature carries a congestion connotation), a simultaneous load balancing (using FACTS) and storage deployment is implemented within the grid. Both the load balancing and the storage deployment are dictated by the initial ORC analysis. Finally, once a FACTS/storage deployment is performed, an OPF algorithm is implemented to minimize the cost of energy production while maintaining stability under the usual phase angle and voltage constraints.

A. Outline of paper

We begin in Section II with an overview of various simplifications of the power flow equations, the "DC power flow equations," and derive the P and Q-graphs, all of which are resistive network models of the various power flows. The construction of such mathematical models will be our very first step to start analyzing the power grid. Let us emphasize that this work will analyze the IEEE 300 bus system, a benchmark example that starts showing how an existing connection architecture with no more than 300 nodes already has congestion implications.

Sec. III summarizes the appropriate literature to review the notions of curvature (in the ORC sense) for discrete spaces and the notion of congestion within grids/graphs.

Sec. IV starts the construction of the proposed method for congestion management, applying the ORC to the resistive network model of the IEEE 300 bus system, aiming at detecting negatively curved lines/edges. Once the congested regions are identified, Sec. IV-B executes a heuristic procedure to simultaneously balance the loads using FACTS and deploy storage devices within the grid. All these is done following up the ORC outcome of the P-graph model of the IEEE300 bus, where the FACTS and storages are deployed within the negative curvature areas. While doing the heuristic tune up/allocation mentioned above, an OPF algorithm is run behind the scene to assure a cost-effective generation, minimizing a quadratic generation cost functional under AC model assumptions and under constraints on the phase angles and bus voltages, to secure grid stability under such changes.

All of the simulations and methods presented till this point have taken into account *line rating considerations*; this allows the method to handle realistic line limits. Sec. V summarizes the main definitions and ideas embraced within the procedure to account for the capacity of the lines, usually determined by the thermal rating [29]. Finally, Sec. VI recaps what has been proposed in this work and opens up future research lines.

II. Resistive network models of power flows

Given two buses k and m specified by their voltage magnitude and phase angle pairs (V_k, θ_k) and (V_m, θ_m), resp., connected by a transmission line with admittance $Y_{km} = G_{km} - jB_{km}$, the power flow equations are well known as

$$\begin{aligned} P_{km} &= G_{km}V_k^2 + B_{km}V_kV_m\sin(\theta_k-\theta_m) \\ &\qquad - G_{km}V_kV_m\cos(\theta_k-\theta_m), \\ Q_{km} &= B_{km}V_k^2 - B_{km}V_kV_m\cos(\theta_k-\theta_m) \\ &\qquad - G_{km}V_kV_m\sin(\theta_k-\theta_m), \end{aligned}$$

where P_{km} and Q_{km} are the active and reactive power, resp., flowing from bus k to bus m. Under the standard approximations of a nearly lossless lines ($G_{km} \approx 0$) with small phase angle differences ($\theta_k \approx \theta_m$), the power flow equations are simplified to become

$$P_{km} = B_{km}V_kV_m(\theta_k - \theta_m), \quad Q_{km} = V_kB_{km}(V_k - V_m).$$

Hence, P_{km} can be viewed as the current flowing through a resistor $\rho_{km} = 1/B_{km}V_kV_m$ driven by a voltage potential difference $\theta_k - \theta_m$. Active powers injected at some buses are then modeled as currents injected at the corresponding nodes of the resistive network. Let us call this resistive network the *P-graph.*

Similarly, but subject to a discrepancy to be explained soon, Q_{km} can be viewed as the current flowing through a resistor $\rho_{km} = 1/B_{km}V_k$ driven by a voltage potential difference $V_k - V_m$. The discrepancy relative to the active power case is that the resistor is directional, $\rho_{km} \neq \rho_{mk}$. We refer to this directed resistive network as the *Q-digraph.* Ricci curvature concepts for such digraphs are developed in the context of Finsler geometry.

III. Curvature, Congestion and the ORC

This work suggests that one of the main steps towards a succesfull congestion management method is to be able to effectively detect congestion areas. Having that in mind, this work proposes a geometric approach to the congestion management problem. Basically, we propose, as core strategy of the method, the utilization of the Ollivier-Ricci curvature notion to detect areas prompt to congestion (*negative curvature areas*). The bridge between negative curvature and congestion in power networks was established in [16] and [18]. The Ollivier-Ricci concept has recently been applied to many different fields outside mathematics itself. A clear example is the usage of ORC to differentiate biological networks corresponding to cancer cells from normal cells [6], and the detection of changes in brain structural connectivity in people with ASD (Autism Spectrum Disorders) [7]. It need not be said that ORC has been long applied in image processing too ([15]); thus, ORC as a first step in CM seems natural. The following subsection briefly defines the ORC for graphs. A better detailed review of the ORC notion can be found in [1], [2], [3], [4] and [5].

A. Wasserstein Distance, Earth Mover's Distance (EMD) and ORC

Let H be a discrete metric space equipped with a metric $d(.,.)$, and let c_{ij} be the cost of moving a unit mass from x_i to x_j; both x_i and x_j belong to H. Denote with p and q two probability distributions in H. Let $\pi_{ij} \geq 0$ be the amount of mass to be transferred from x_i to x_j. The so-called OPT (Optimal Mass Transportation) is the problem of finding an optimal transfer of mass from p to q with minimum cost. This can be formulated as ([7]):

$$\min_{\pi} \sum_{i,j} c_{i,j}\pi_{i,j}, \tag{1}$$

subject to

$$\begin{aligned} &\sum_j \pi_{i,j} = p_i, \qquad \forall i, \\ &\sum_i \pi_{i,j} = q_j, \qquad \forall j, \\ &\pi_{i,i} \geq 0, \qquad \forall i,j, \end{aligned} \tag{2}$$

where i and j are connected via an edge. If the previously formulated problem is solved with a cost $c_{i,j} = d(x_i, x_j)^r$ for any positive integer r, then it is said that the solution of the optimization problem is the r-$Wasserstein\ Distance$. Moreover, if $r = 1$, the solution is called *Earth Mover's Distance*.

Let now (X, d) be a geodesic metric space equipped with probability measures $\{p_x : x \in X\}$. Then the Ollivier-Ricci curvature $k(x, y)$ along the geodesic joining x to y is defined as

$$W_1 = (1 - k(x, y))d(x, y), \tag{3}$$

where W_1 is the EMD distance and d the geodesic distance within the space.

Recall now from Sec. II that our method will initially calculate a resistive network model called P-graph; thus, we will have a $G = (V, E, W)$ graph, where V is the set of nodes/buses, E is the set of lines/edges, and W is the set of resistances/weights. Following our previous equations, the geodesic distance $d(x, y)$ of the ORC formulation for a graph will be represented by the minimum number of steps/hops needed to go from x to y. Therefore, after this recap, a simple and short way to calculate the ORC $k(x, y)$ can be implemented through a linear programming script, and this is what has been done in the forthcoming sections.

IV. Ollivier-Ricci Curvature Driven Optimal Power Flow (ORC-OPF)

A. Negative Curvature Detection via ORC

We start this section by building up a *P-graph* of the IEEE 300 bus system; this is done following Sec. II. Once the resistive network of the power grid is acquired, we calculate the ORC of every edge in the model, aiming at detecting those edges that have negative curvature. Bear in mind that what we actually have is a pure resistive network model (P-graph) that can be seen as an undirected weighted graph; thus, an ORC calculation for every edge is straightforward using Sec. III.

Let us define a convenient terminology:

Definition 1 (Critical Edge): A critical edge is a transmission line possessing a negative curvature value.

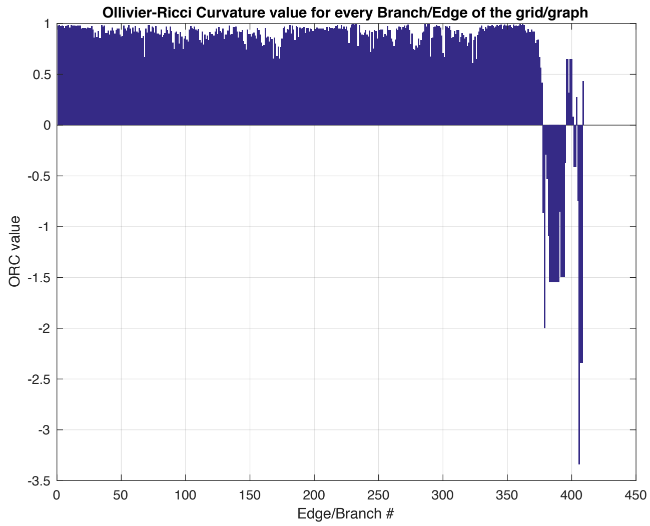

Fig. 1. Curvature Values of Critical Edges

Definition 2 (Critical Buses): A bus is critical if it defines one of the vertices of a *critical edge*.

Fig. 1 shows the ORC value for every edge in the grid. We actually have 409 edges within the grid (x-axis), and almost 25 edges show a negative ORC value (y-axis).

Observe that this brief section has identified the cause/reason to propose a congestion management procedure; it has detected the *critical edges*, which are the lines prompt to be overloaded within the grid. Therefore, if we are able to smooth over the curvature throughout the grid, we will be avoiding congestion. This is actually what is done in the next section, and it is basically a heuristic procedure that makes positively curved those areas of the grid originally negatively curved.

B. FACTS, Storage Devices and the full ORC-OPF

Recall from Sec. II that the P-graph is basically a resistive network model composed of $\rho_{km} = 1/B_{km}V_kV_m$ resistances, where k and m are the different buses of the grid ($k \neq m$), V_k and V_m are the voltages at buses k and m resp., and B_{km} is the susceptance value of the line (edge) that joins buses k and m.

Clearly, in order to change the negative curvature value of an edge, we need to change the resistance value (distance) of such edge, with the aim of obtaining a positive curvature value for it.

The only possible variables that we have available for changing the resistances are the voltages V_k and V_m, and the susceptance B_{km}, but if we leave the voltages unchanged for the grid voltage stability operations, we are only left with the susceptance value B_{km} ($\approx 1/X_{km}$).

This is actually the only choice due to the DC assumptions (mostly the $G_{km} \approx 0$ assumption), which sets the format of the ρ_{km} to be dependent only on voltages (V_k and V_m) and the susceptance (B_{km}).

This seems to be a serious limitation, because just a change in the suceptance might not be enough to do a nontrivial load balancing of the grid. Here is the approach that will be taken: for the purpose of the ORC calculation on each edge, we will enforce the DC assumptions in order to be able to have a relevant P-graph and consequently a *curvature* value for each edge. Once the curvature values are computed for all lines and the negative curvature areas are spotted, we will switch to AC conditions to smooth out the curvature (which is actually more realistic). This will allow us to have $G_{km} \neq 0$, and consequently we will have a new variable (R_{km}) to adjust, which in turn will facilitate the curvature smoothing process. Actually, while no control can be directly exercised upon the line susceptances, the apparent susceptances can be modified by, for example, FACTS series compensation to modify line impedance and static synchronous series compensator (SSSC) that connects an inductive or capacitive reactance in series with the transmission line.

This curvature smoothing process (making the negative curvature areas positive using FACTS) has been done heuristically and it was done at the same time a collection of loads were deployed in the surroundings of the critical edges. Clearly, this second stage of the proposed method allows energy storage by converting the power consumed by the deployed loads to Gibbs free energy, while at the same time minimizing the overall cost of generating active power within the grid.

Once the load balancing is performed and the loads are deployed, a new set of power flow equations are embedded in a convex optimization algorithm, with the objective of minimizing a polynomial cost function of the active and reactive power of each generator. Clearly, this is also done under the AC assumptions. We illustrate below the structure of the nonlinear programming algorithm:

Algorithm IV.1: AC COST OPTIMIZATION($\theta, V, P_g, Q_g,$)

$$\min_{\theta,V,P_g,Q_g} \sum_{k=1}^{\texttt{gensize}} \mathcal{C}_{\mathrm{AC}}(P_{g,k}, Q_{g,k}),$$

subject to

$$\begin{cases} F_{\mathrm{AC}}(\theta, V, P_g, Q_g) = 0, & \\ \underline{\theta}_i \quad \leq \theta_i \leq \quad \bar{\theta}_i, & i = 1, ..., \texttt{bussize}, \\ \underline{v}_i \quad \leq v_i \leq \quad \bar{v}_i, & i = 1, ..., \texttt{bussize}, \\ \underline{P}_{\,g,k} \quad \leq P_{g,k} \leq \quad \bar{P}_{g,k}, & k = 1, ..., \texttt{gensize}, \\ \underline{Q}_{\,g,k} \quad \leq Q_{g,k} \leq \quad \bar{Q}_{g,k}, & k = 1, ..., \texttt{gensize}. \end{cases}$$

return (θ, V, P_g, Q_g)

In the algorithm, $P_{g,k}$ and $Q_{g,k}$ stand respectively for the active and reactive power generated by generator k, `gensize` is the number of generators in the grid, $x = [\theta, v, P_g, Q_g]$ is the optimization state variable where θ is the phase angle vector carrying the buse's phase angle, v stands for the bus voltages vector, P_g and Q_g are respectively the vectors of active and reactive powers generated by the generators; `bussize` is the number of buses in the grid; $\mathcal{C}_{\mathrm{AC}}(.)$ is a degree-2 cost function that weights the cost of generation of each generator k:

$$\mathcal{C}_{\mathrm{AC}}(P_{g,k},Q_{g,k}) = \alpha_{g,k}(P_{g,k})^2 + \beta_{g,k}(P_{g,k}) + \\ +\delta_{g,k}(Q_{g,k})^2 + \psi_{g,k}(Q_{g,k}) + \gamma_{g,k}. \tag{4}$$

$F_{\mathrm{AC}}(\cdot) = 0$ represents the dynamic of the AC power flow. Finally, $(\underline{P}_{g,k}, \bar{P}_{g,k})$, $(\underline{Q}_{g,k}, \bar{Q}_{g,k})$, $(\underline{\theta}_i, \bar{\theta}_i)$ and $(\underline{v}_i, \bar{v}_i)$ are the min and max limits for $P_{g,k}$, $Q_{g,k}$, θ_i and v_i, respectively. Observe that the cost function is composed of $'\texttt{gensize}'$ order-two polynomials that could be built up differently for each generator; thus, we can weigh (choosing $\alpha_{g,k}$, $\beta_{g,k}$, $\delta_{g,k}$, $\psi_{g,k}$ and $\gamma_{g,k}$) each generator cost differently by shaping each polynomial separately.

The combination of the different steps made till this point constitutes what we call the Ollivier-Ricci Curvature Driven OPF (ORC-OPF), and it can be summarized in the following table:

The overall procedure has been implemented in Matlab using the MATPOWER package. A modified version of the Ulas Yilmaz ORC software has been generated and implemented based on [15]. The results and a comparison with a standalone AC OPF optimization method applied to the IEEE 300 bus system are summarized in Table II. The total amount of storages arranged were 24, all of them deployed in the surroundings of critical lines. Also, a total of 30 critical lines were adjusted (load balancing) so as to maximize the curvature.

TABLE II
TOTAL COST FUNCTIONS VALUES (AC MODEL WITH CONVENTIONAL OPF AND WITH ORC-OPF IMPLEMENTATION)

(dollars/hr)	Total Cost Function Value
with conventional OPF	719730.00
with ORC-OPF	613730.00

Observe from Table II that the optimal cost of generation proposed by the ORC-OPF spend almost 15% less energy than the optimal generation proposed by the standalone OPF; thus, these numbers clearly show the effectiveness of the proposed method. Moreover, it can be claimed now that the structure of the grid is no longer so prompt to congestion as the original one, because the ORC-OPF method not only has achieved less energy usage, but it has also smoothed over the negative curvatures areas indicated in Fig. 1. This can be clearly seen in Fig. 2, where no trace of negative ORC values can be found—and this is actually the main goal of the method.

Notice that Fig. 2 shows more edges than Fig. 1; the reason is that the edges belonging to the deployed loads added after the ORC-OPF implementation have been included.

We also highlight the fact that in order to calculate the ORC curvature we are using a graph abstraction, the P-graph. This is basically done because the ORC needs a graph and a distance to be computed. As a consequence, we

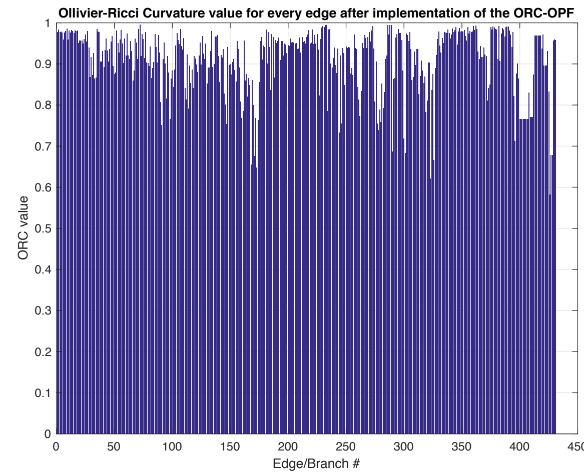

Fig. 2. Curvature Values of Critical Edges *(no negative curvature values are present)*

utilize the DC assumptions to eliminate the non-linearities and generate a simple graph/model of the grid. Although we worked with AC assumptions during the load balancing and the OPF energy optimization, we analyze part of the results (curvature maximization) under the DC assumptions, because it is the fundamental tool to generate the ORC values that guide the overall method. We however choose the AC model within the main steps of the procedure, just to be closer to the real behavior of the grid, where dissipation and non-linearities are present.

Nevertheless, if we were to run the entire approach under DC assumptions, the curvature maximization results/figure would not really change and the final costs of energy generation would be just a little smaller than in the AC case, as depicted in Table III.

TABLE III
TOTAL COST FUNCTIONS VALUES (DC MODEL WITH CONVENTIONAL OPF AND WITH ORC-OPF IMPLEMENTATION)

(dollars/hr)	Total Cost Function Value
with conventional OPF	706290.00
with ORC-OPF	599550.00

It is just worth mentioning that if we were to run the entire method under the DC scenario, the DC cost optimization algorithm would look like:

Algorithm IV.2: DC COST OPTIMIZATION (θ, P_g)

$$\min_{\theta, P_g} \sum_{k=1}^{\texttt{gensize}} \mathcal{C}_{\mathrm{DC}}(P_{g,k}),$$

$$\text{s.t.} \begin{cases} F_{\mathrm{DC}}(\theta, P_g) = 0, \\ \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i, & i = 1, ..., \texttt{bussize}, \\ \underline{P}_{g,k} \leq P_{g,k} \leq \bar{P}_{g,k}, & k = 1, ..., \texttt{gensize}. \end{cases}$$

return (θ, P_g)

where the same notation as in Algorithm IV.1 has been used.

TABLE I

OLLIVIER-RICCI CURVATURE DRIVEN OPF "MAIN STEPS" *(recall that critical buses and critical edges are defined in Def. 2 and Def. 1, resp.)*

ORC-OPF (IV-B)	identify critical buses
	adjust B's and R's (w / FACTS) of critical lines
	deploy loads around critical buses
	run AC Cost Optimization Algorithm

Notice that the process is simplified and that neither the bus voltage nor the reactive power generated are involved in this DC cost optimization.

V. LINE RATING CONSIDERATIONS

As mentioned in previous sections, all of the simulations have taken into account *line rating* considerations; this has been accomplished by restricting the active power flowing through the lines to a maximum of 700MW. This is actually a way to account for the capacity of the lines, usually determined by the thermal rating [29], [31], [32].

In particular, the following definitions have been applied:

Definition 3 (DC Utilization Factor): The utilization factor for the branch (k, m) under *DC model assumptions* is defined as

$$\mu_{\text{DC}} = P_{k,m}/LC_{k,m},$$

where LC stands for *line capacity*, the maximum active power allowed (in MW) through the branch (k, m).

Definition 4 (AC Utilization Factor): The *utilization factor* for the branch (k, m) under *AC model assumptions* is defined as

$$\mu_{\text{AC}} = \sqrt{P_{k,m}^2 + P_{m,k}^2}/LC_{k,m}.$$

The impact of adding the line limitation constraints

$$\mu_{\text{DC};\,k,m} \leq 1, \quad \mu_{\text{AC};\,k,m} \leq 1,$$

in the DC and AC ORC-OPF approach is barely noticeable in the overall final cost function values for a line limit of $LC_{k.m} = 700\text{MW}$, although it underlines an important advantage within the complete method: the ORC-OPF scheme is able to handle realistic line limits.

Fig. 3 shows a line utilization histogram (in percentage) for the IEEE 300 bus system with a line rating of 700 MW (on the active power of the branches) under AC analysis. As mentioned earlier, the *line rating* inclusion within the proposed load balancing encompasses a promising analysis tool: it would basically consists in the direct inclusion of *thermal rating* considerations for *bare overhead conductors*.

The increase of *thermal stress* due to variable weather or other conditions [27], [28], [31], [32] could easily trigger a line overloading that might end up in a blackout (e.g., 1996 Western North America blackout [29]). As another scenario, when a major line trips, power is rerouted along other lines that may not have been designed to carry such an amount of power and hence are likely to be overloaded and trip, leading to a chain reaction effect [30]. Thus, a real time power flow calculation that includes a DLR (Dynamic Line Rating [31]) could certainly help to assess power grid functionality.

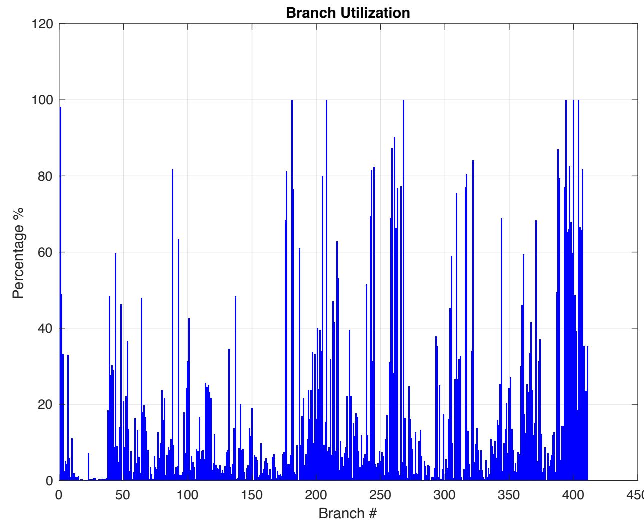

Fig. 3. *Utilization Factor* (μ_{AC}) for the AC model of the IEEE300 bus system with ORC-OPF implementations and a 700 MW active power constraint

VI. DISCUSSION AND CONCLUSION

This work has proposed an Ollivier-Ricci curvature-based electrical load balancing procedure that can take line rating into consideration. Once the Ollivier-Ricci curvature analysis has identified the $stress\ points$, line admittances are adjusted by FACTS, loads are being deployed, and finally the generation is optimally readjusted within feasibility constraints (including line ratings) in such a way as to reduce the overall cost of generation.

It is suggested that the loads that are deployed to mitigate congestion be used to store energy, even recharge electrical vehicles, although the latter would require thorough scheduling analysis, which is left for further research.

The case-study investigated in this paper is the IEEE 300 bus system, in which load balancing immediately appears to be an issue.

By choosing these nontrivial examples, the curvature analysis has revealed restrictions that the topological-combinatorial properties of the power network impose on what can be achieved in terms of load balancing.

From this latter perspective, the paper has proposed to start re-thinking what can be done, and at what cost. The best option appears to be the combined load deployment/cost reduction.

As is known, the power grid is a dynamic nonlinear system acting at different time-scales, some aspects of which, like the fractal behavior of the PMU signals, are still poorly understood [25], [26]. Since it is still very unclear how the fractional dynamics betrayed by the PMU signal analysis can be used for enhanced modeling, here we have limited ourselves to utilize the AC model for combined curvature smoothing and generation cost reduction.

The new ORC-OPF optimization procedure has reduced the overall cost of generation, necessary to sustain the power flow, relative to the standalone OPF. The line rating considerations have opened a door for the future inclusion of thermal rating calculations. Finally, as a future step of this work, an optimization procedure that includes the hitherto unknown dynamical effects revealed by the data driven approach of [25], [26] could further enhance the combined curvature smoothing/cost reduction for real grids.

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