Information Transfer in Spintronics Networks under Worst Case Uncertain Parameter Errors

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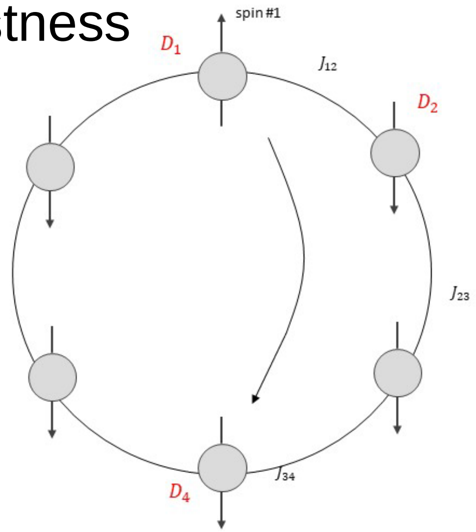
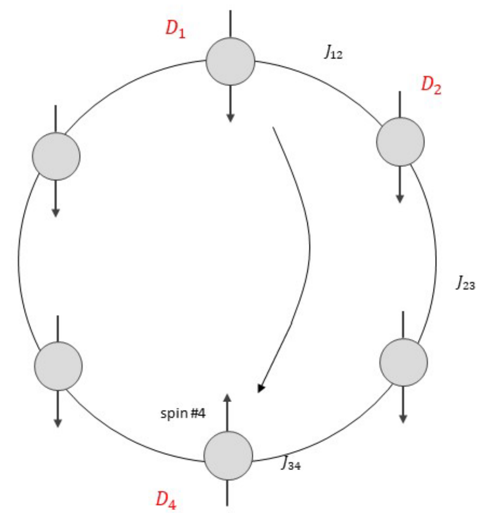
Problem Statement/Setup

Goal:

- Optimal information transfer in quantum devices
- High fidelity & high robustness



$|\Psi(0)\rangle = |1\rangle = |100000\rangle$



$|\Psi(t_f)\rangle = |4\rangle = |000100\rangle$

Network :

- N spins
- Spin- ½ particles with states $|0\rangle$ & $|1\rangle$
- Nearby spins interact with couplings J_{ij}
- Single excitation subspace

Problem Statement/Setup

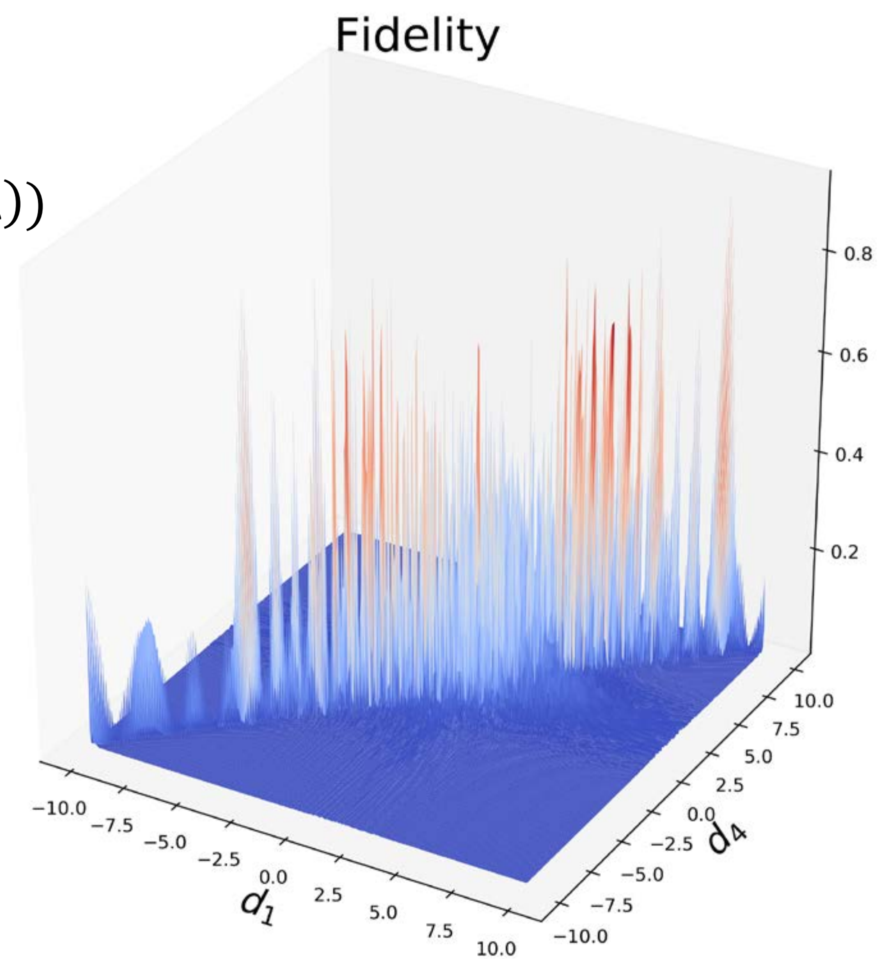
Dynamics:

- External control input on spin $\#i$ is d_i
- Hamiltonian $H_\Delta = \sum_{n=1}^{N} d_n Z_n + \sum_{n\neq m}^{N} J_{mn}(X_n X_m + Y_n Y_m)$
- $H_\Delta = \begin{bmatrix} d_1 & J_{12} & 0 & \cdots & J_{1N} \\ J_{21} & d_2 & J_{23} & & 0 \\ & \vdots & & \ddots & \vdots \\ J_{N1} & 0 & 0 & \cdots & d_N \end{bmatrix}$
- Schrodinger's equation: $i\hbar \frac{d\Psi}{dt} = -H_\Delta \Psi$
- Fidelity: $F_t(\Delta) = \left|\langle OUT \middle| e^{-iH_\Delta t} \middle| IN \rangle\right|^2$

Problem Statement/Setup

Formulation:

- Find $\Delta = \mathrm{diag}\{H_\Delta\} = [d_1, d_2 \cdots d_n]^T$
- Minimize fidelity error: $\min\limits_{\Delta,t} \ (1 - F_t(\Delta))$
- Subject to
  - bias field bounds: $d_L \leq d_i \leq d_U$
  - $t_f \leq T$



Robustness

Uncertainties:

- Bias controls d_n
- Couplings J_{ij}

Goal:

- Solution robust against uncertainties
- Precisely, low fidelity error & low sensitivity

Large-Scale Differential δ

Minimize fidelity error & worst-case large-scale sensitivity (mixed sensitivity):

- $\min_{\Delta,t}\ a(1-F_t)+(1-a)\|\delta F_t(\Delta)\|_\infty$
- $a\epsilon[0,1]$
- Subject to
  - bias field bounds: $d_L \leq d_i \leq d_U$
  - $t_f \leq T$

worst-case perturbation

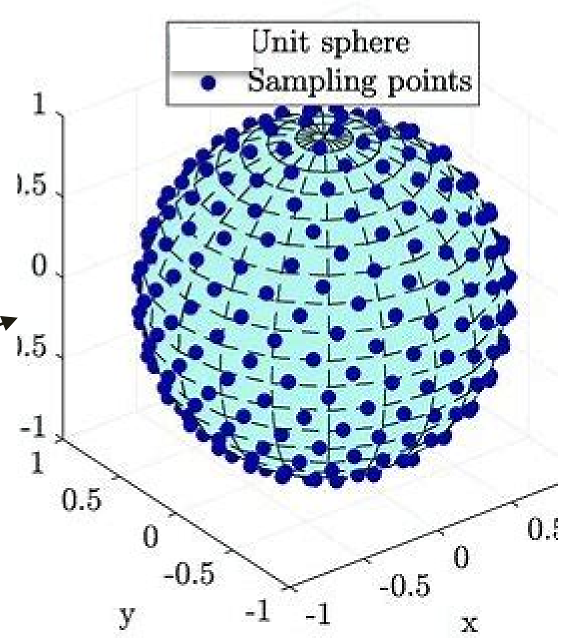
New metric:

- ***Large-scale*** differential δ: $\|\delta F_t(\Delta)\|_\infty$
- ***Large-scale*** version of $\partial F_t(\Delta) = \ max_{\|h\|=1}|\nabla F_t(\Delta)\cdot h|$
- "Flat" fidelity peaks

Fidelity Differential

Forward difference of the Fidelity:

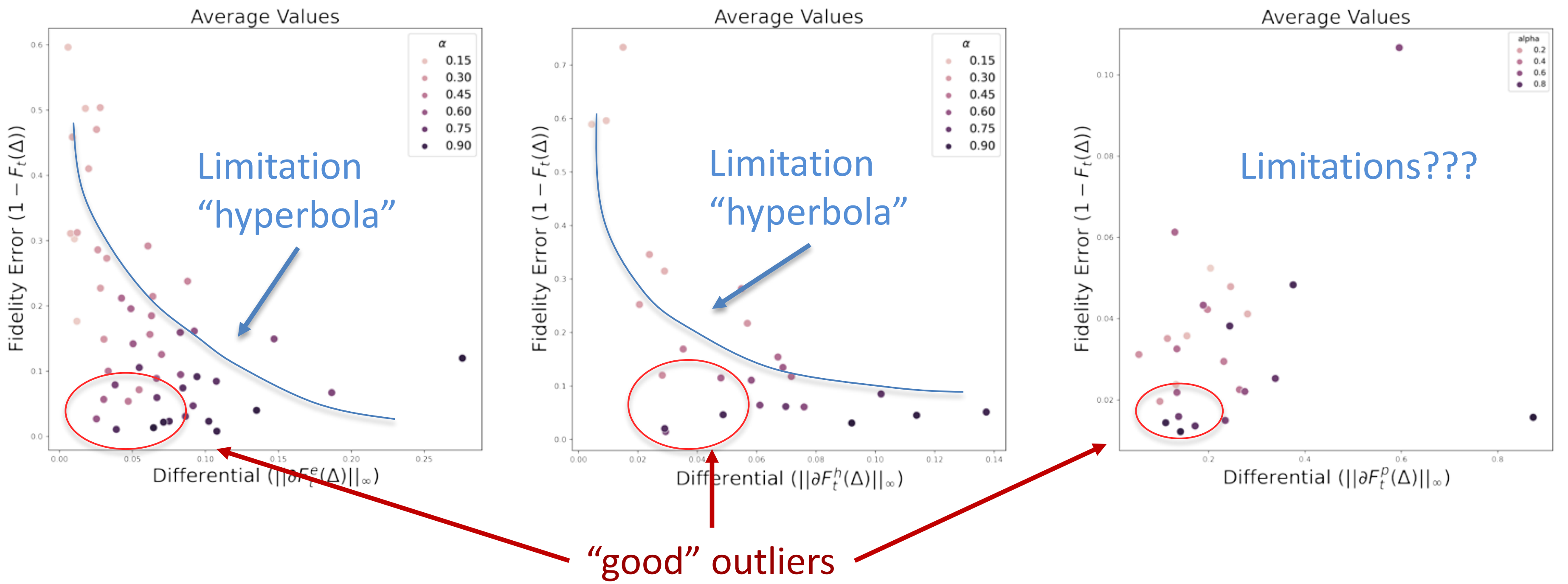
1. Perturbation I:
   - w.r.t. e_i $\delta F_t^e(\Delta) = \frac{F_t(\Delta+\varepsilon e_i)-F_t(\Delta)}{\varepsilon}$
2. Perturbation II:
   - w.r.t. h: $\delta F_t^h(\Delta) = \frac{F_t(\Delta+\varepsilon h)-F_t(\Delta)}{\varepsilon}$



3. Hamiltonian Perturbation:
   - w.r.t. H_Δ^p: $\delta F_t^p(\Delta) = \frac{F_t^p(\Delta)-F_t(\Delta)}{\sigma_\varepsilon}$
   - $H_\Delta^p = \sum_{n=1}^N d_n Z_n + \sum_{n\neq m}^N J_{mn}(1+\varepsilon_n)(X_nX_m + Y_nY_m)$
   - $F_t^p(\Delta) = \left|\left\langle OUT\left|e^{-iH_\Delta^p t}\right|IN\right\rangle\right|^2$
   - $\varepsilon_n \sim N(0,\sigma_\varepsilon)$

$\frac{h}{\|h\|}$ — h drawn from 0-mean Gauss distribution

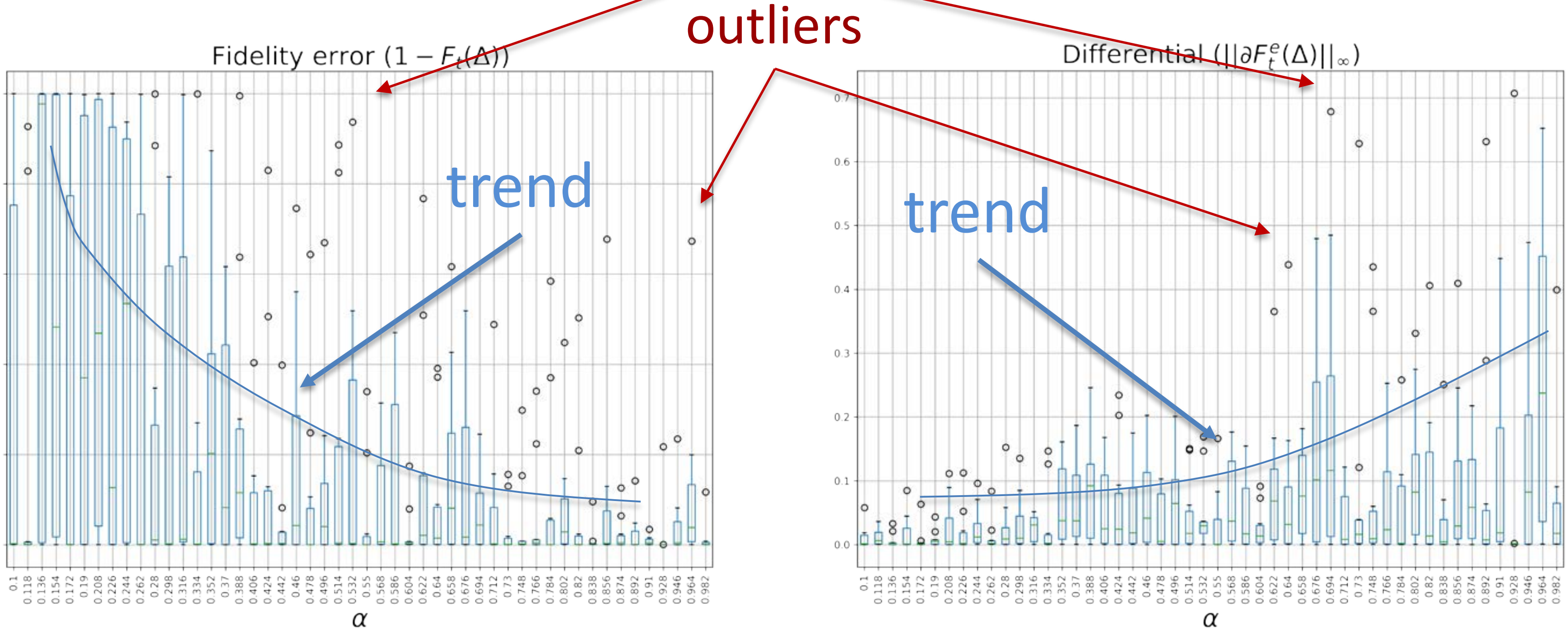
Results



- Statistically, we do see a "hyperbola" limiting the fidelity versus robustness conflicting objectives.
- In a sea of outliers, we can always find some high fidelity/high robustness controllers!

Results

Perturbation I $\partial F_t^e(\Delta)$

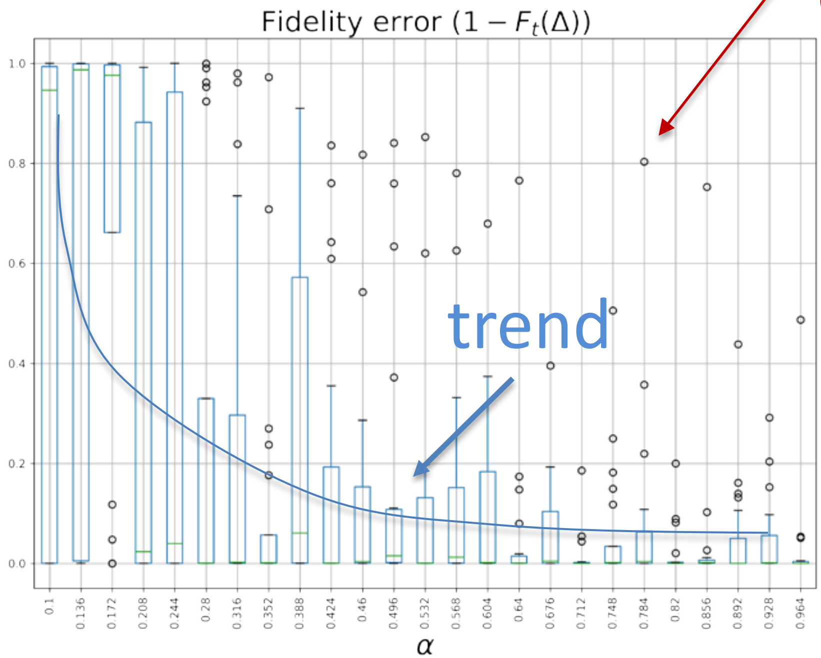


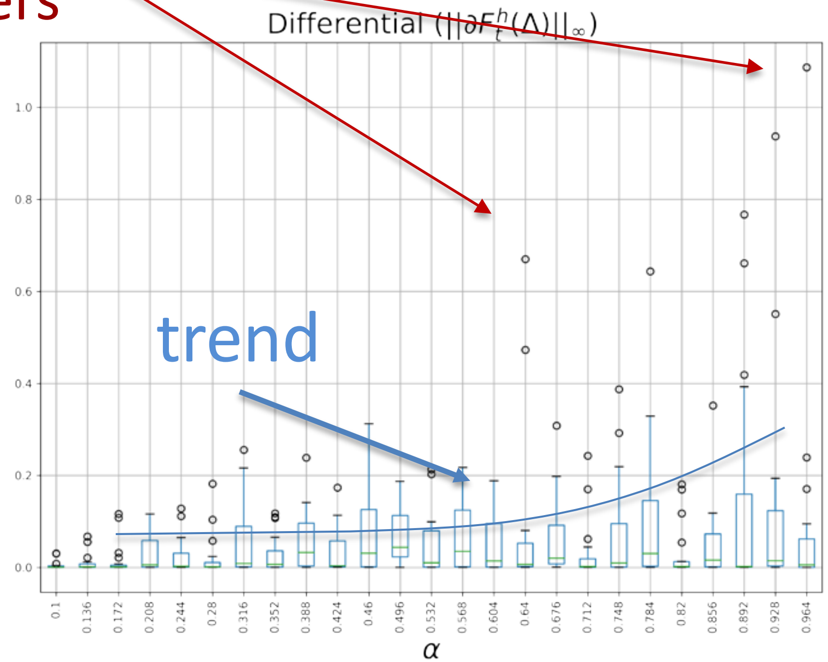
- Fidelity error ↓ as a ↑
- Differential ↑ as a ↑

Results

Perturbation II $\partial F_t^h(\Delta)$

"bad" outliers

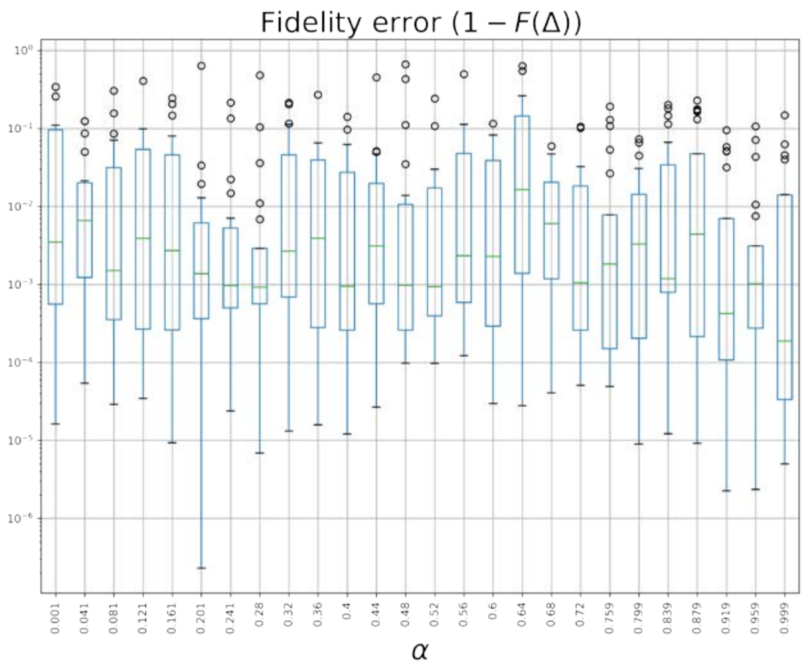


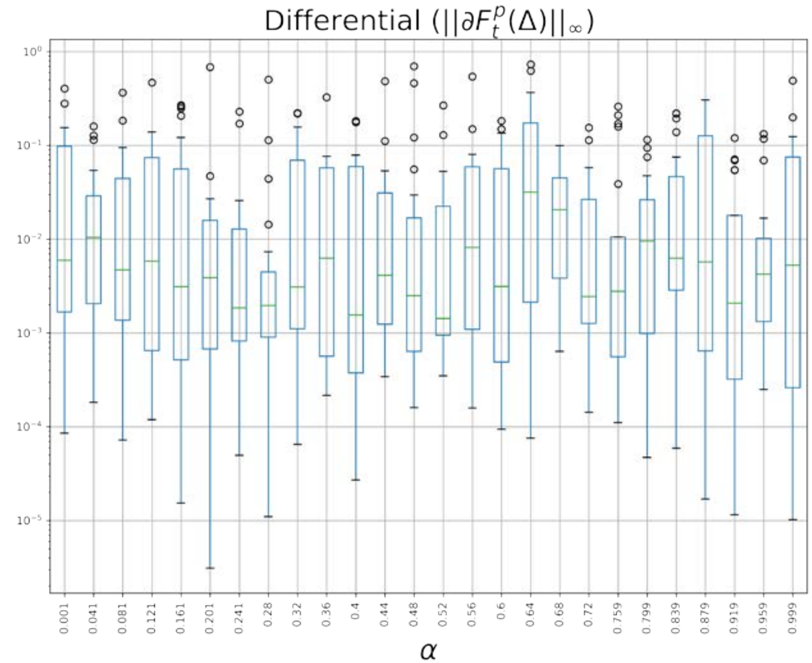


- Fidelity error ↓ as a ↑
- Differential ↑ as a ↑

Results

Hamiltonian Perturbations $\partial F_t^p(\Delta)$

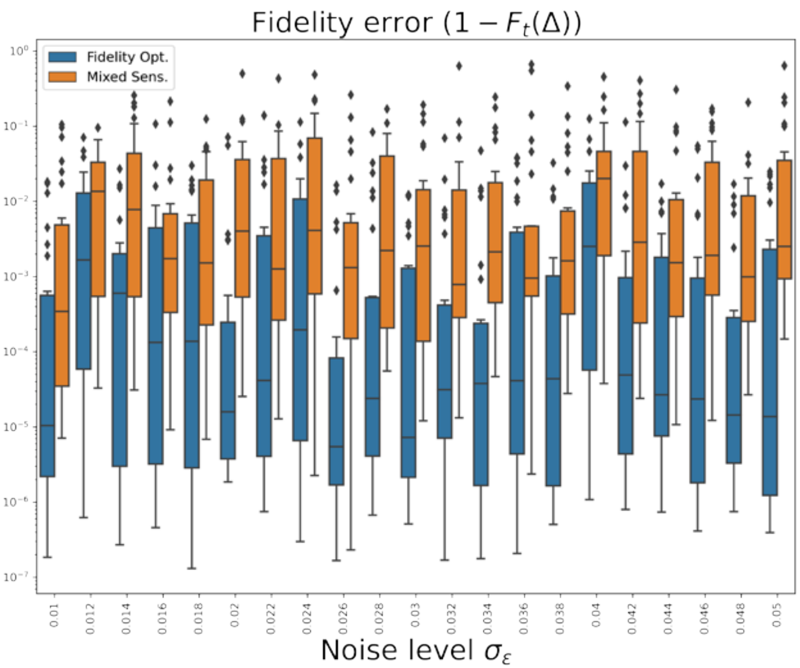


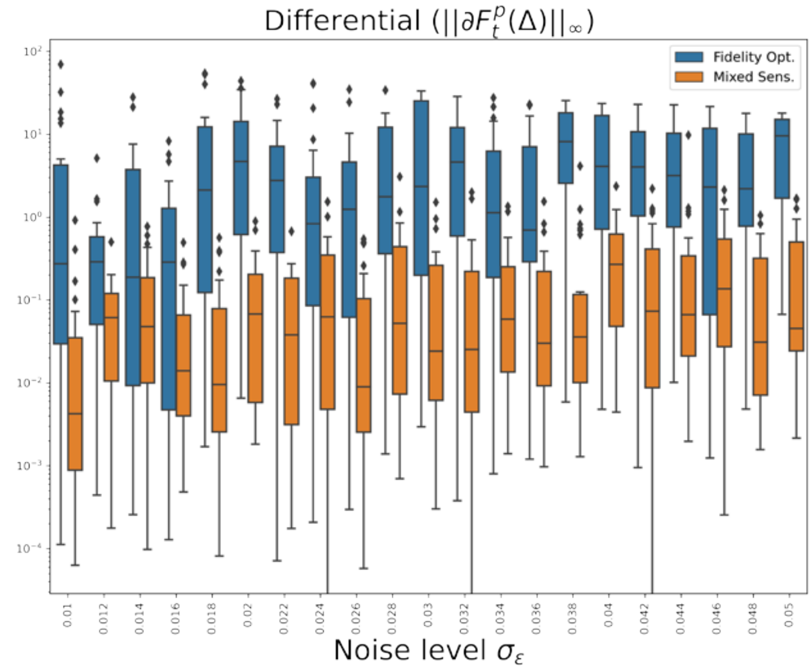


- Fidelity error ↓ as a ↑
- Differential ↑ as a ↑

Results

Hamiltonian Perturbations $\partial F_t^p(\Delta)$





- Fidelity error ↑ & differential ↑ as σ_ε ↑
- Mixed sensitivity offers robustness with slightly greater fidelity error

Concluding Remarks

- Optimization method to balance between low fidelity error and robustness
- Defined "Large-scale" differential for robustness
- Lack of error vs sensitivity conflict

Future work

- "Fair" sampling of h in the unit sphere
- Second – order differential of the fidelity (Hessian)
- Information transfer and robustness under decoherence

Thank you!
Questions?