Finding and Characterising Robust Quantum Controls

Irtaza Khalid[1,†], Carrie Weidner[2], S. G. Schirmer[3], Edmond Jonckheere[4], Frank C. Langbein[1]

[1]Cardiff University, [2]QETLabs, University of Bristol, [3]Swansea University, [4]University of Southern California

[†]khalidmi@cardiff.ac.uk

TL;DR

1. The **average fidelity is a statistical robustness-infidelity measure (RIM_1)** as it is the first order optimal transport distance from the perfectly robust distribution δ_1.
2. Higher-order RIMs are equivalent up to scaling to lower-order RIMs. **RIM_1 is a sufficient** controller robustness measure.
3. This extends to compare quantum control algorithms: **algorithmic RIM** (**ARIM**).
4. Numerical results on the energy landscape control of **XX-Heisenberg chains** indicate that not all high-fidelity controllers are also robust (see Fig. 3).
5. There exists some benefit to incorporating **certain noise in finding low RIM controllers** (see Fig. 4) due to smoothing.

Quantum Control Problem

The state transfer optimisation problem is given by

$$t^*, \boldsymbol{\Delta}^* = \underset{t,\boldsymbol{\Delta}\in\mathbb{T}}{\arg\max} \underbrace{|\langle\psi^*| U_c(t,\boldsymbol{\Delta}) |\psi^i\rangle|^2}_{=:\mathcal{F}} \tag{1}$$

where $U_c = \exp(-iH_c(\boldsymbol{\Delta})t)$ and the **XX Heisenberg Hamiltonian** $H_c(\boldsymbol{\Delta})$ of the L-body chain with energy landscape controls $\boldsymbol{\Delta}$ in the single-excitation subspace is [1]:

$$\frac{H_c(\boldsymbol{\Delta})}{\hbar} = \sum_n^L (\Delta_n |n\rangle\langle n| + J|n\rangle\langle n\pm 1|) \tag{2}$$

We **structurally perturb** the coherent dynamics by adding Hamiltonian noise: $(H_c)_{ij} \to (1+(S_{\sigma_{\mathrm{sim}}})_{ij})(H_c)_{ij}$ using

$$(S_{\sigma_{\mathrm{sim}}})_{l,m} = J(p_1\delta_{l,m\pm1}) + p_2\Delta_{l,m}\delta_{l,m}, \quad p_1, p_2 \sim \mathcal{N}\left(0, \sigma^2_{\mathrm{sim}}\right) \tag{3}$$

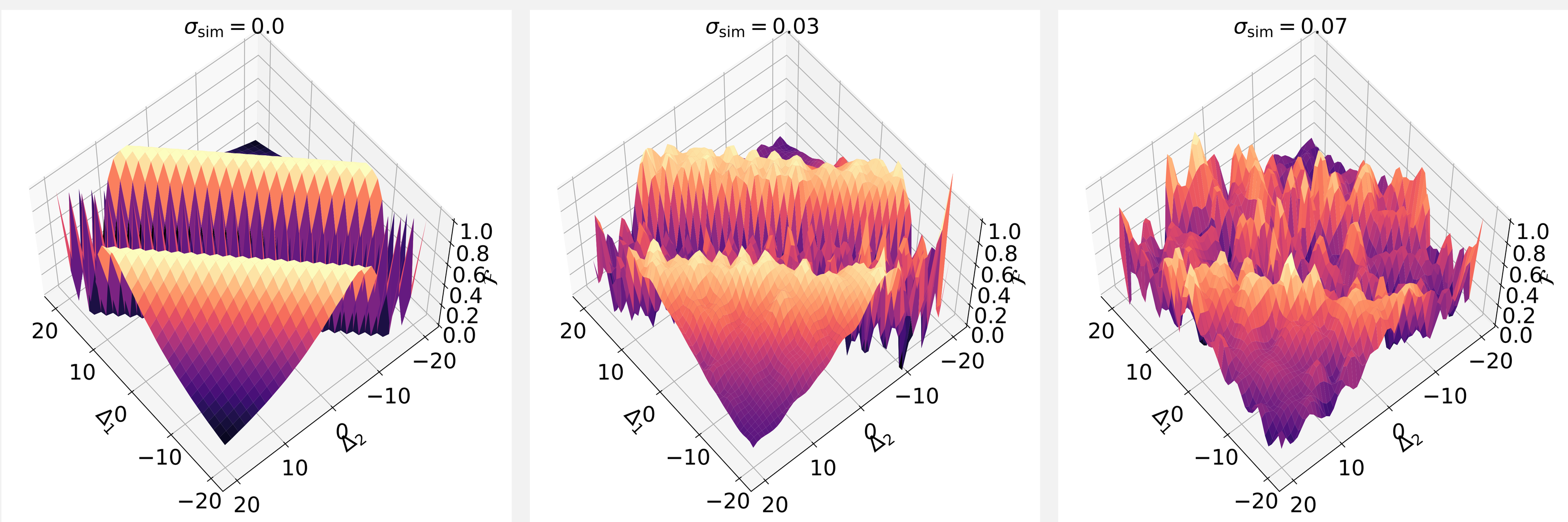

Figure 1: Fidelty $\mathcal{F}$ landscape (Eq. (1); $\Delta_1 = \Delta_3$) with one structured perturbation (Eq. (3)) for $L = 3$ and an end-to-end transition.

Robustness-Infidelity Metric

The robustness-infidelity metric RIM_p is the pth-order Wasserstein distance of a **fidelity distribution** $\mathcal{P}_{\sigma_{\mathrm{sim}}}(\mathcal{F})$ from the perfectly robust fidelity distribution $\delta(\mathcal{F}-1)$ induced by the uncertainty $S_{\sigma_{\mathrm{sim}}}$

$$\mathrm{RIM}_p := \mathcal{W}_p(\mathcal{P}_{\sigma_{\mathrm{sim}}}(\mathcal{F}), \delta(\mathcal{F}-1)) = \mathbb{E}_{f\sim\mathbf{P}(\mathcal{F})}[(1-f)^p]^{\frac{1}{p}} \tag{4}$$

1. Works for any bounded fidelity measure $\mathcal{F}$.
2. Can be used for controller post selection.
3. The pth order Wasserstein distance is a metric on the space of controllers that facilitates robust optimisation due to its structure-preserving properties.
4. It permits nested definitions such as the ARIM.
5. Using reinforcement learning, we implicitly optimise a discounted RIM_1 as the cumulative return per episode.

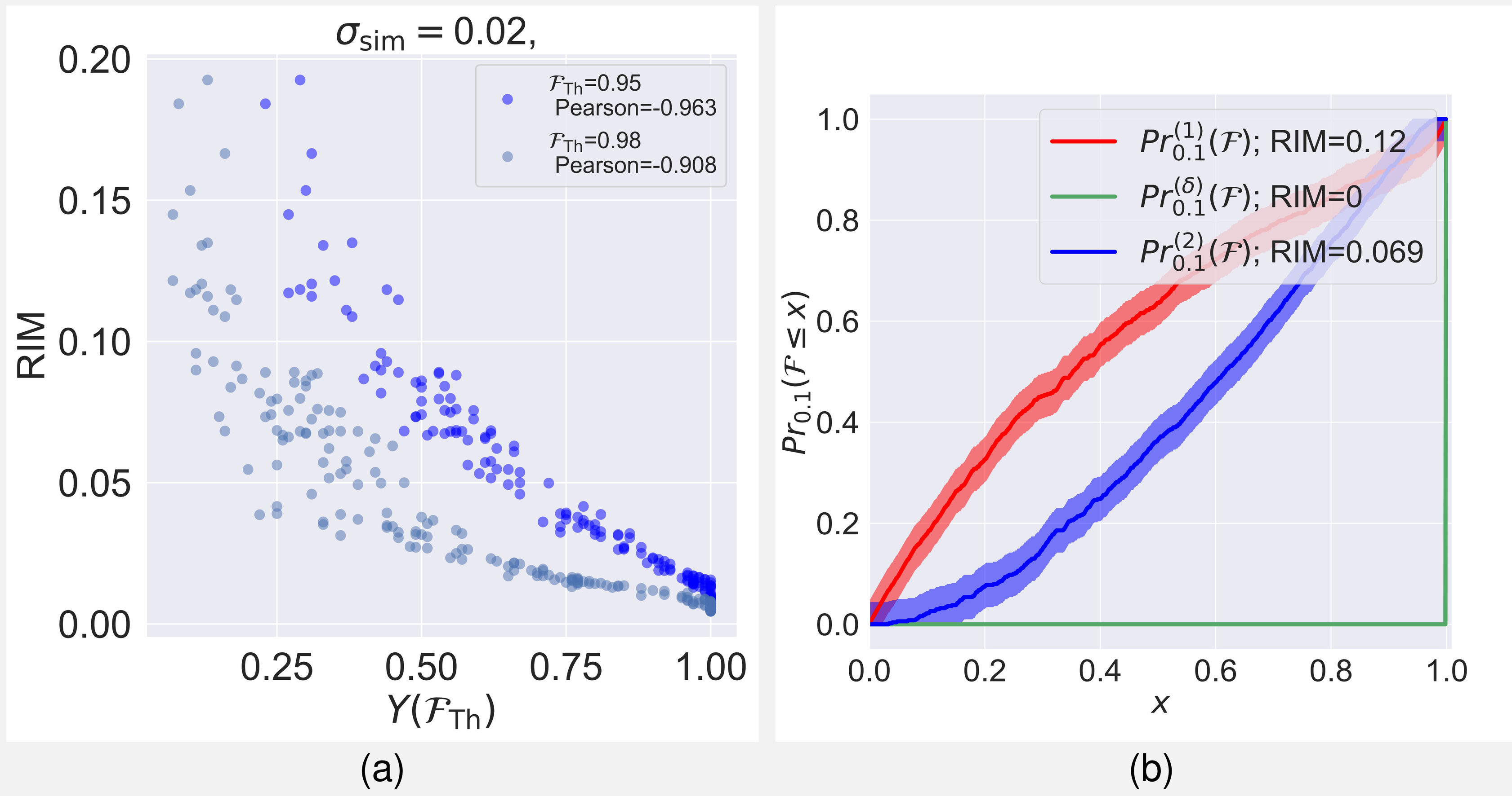

Figure 2: (a) Comparison of RIM with yield Y (fraction of fidelities greater than threshold fidelity $\mathcal{F}_{\mathrm{th}}$) at values $\mathcal{F}_{\mathrm{th}} = 0.95, 0.98$ for 200 controllers. (b) Illustration of how RIM is calculated for a single controller. Both figures are generated for a chain of length $L = 5$ and a bit transition from $|1\rangle$ to $|3\rangle$.

References

[1] Frank C Langbein, Sophie Schirmer, and Edmond Jonckheere. Time optimal information transfer in spintronics networks. In *2015 54th IEEE Conference on Decision and Control (CDC)*, pages 6454–6459. IEEE, 2015.
[2] Cédric Villani. *Optimal transport: old and new*, volume 338. Springer, 2009.
[3] Ciyou Zhu, Richard H. Byrd, Peihuang Lu, and Jorge Nocedal. Algorithm 778: L-bfgs-b: Fortran subroutines for large-scale bound-constrained optimization. *ACM Trans. Math. Softw.*, 23(4):550–560, December 1997.
[4] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms, 2017.
[5] Waltraud Huyer and Arnold Neumaier. Snobfit – stable noisy optimization by branch and fit. *ACM Trans. Math. Softw.*, 35(2), July 2008.
[6] John A Nelder and Roger Mead. A simplex method for function minimization. *The computer journal*, 7(4):308–313, 1965.

Results

Individual controller comparison using the RIM for chain of length $L = 5$ and transition from $|1\rangle$ to $|3\rangle$. Controllers are obtained numerically using L-BFGS ($\sigma_{\mathrm{train}} = 0$) [1, 3], PPO [4], SNOBFit [5] and Nelder-Mead [6] ($\sigma_{\mathrm{train}} = 0, 0.01, \ldots, 0.05$). Controller performance is evaluated at uncertainty strengths $\sigma_{\mathrm{sim}} = 0, 0.01, \ldots, 0.1$.

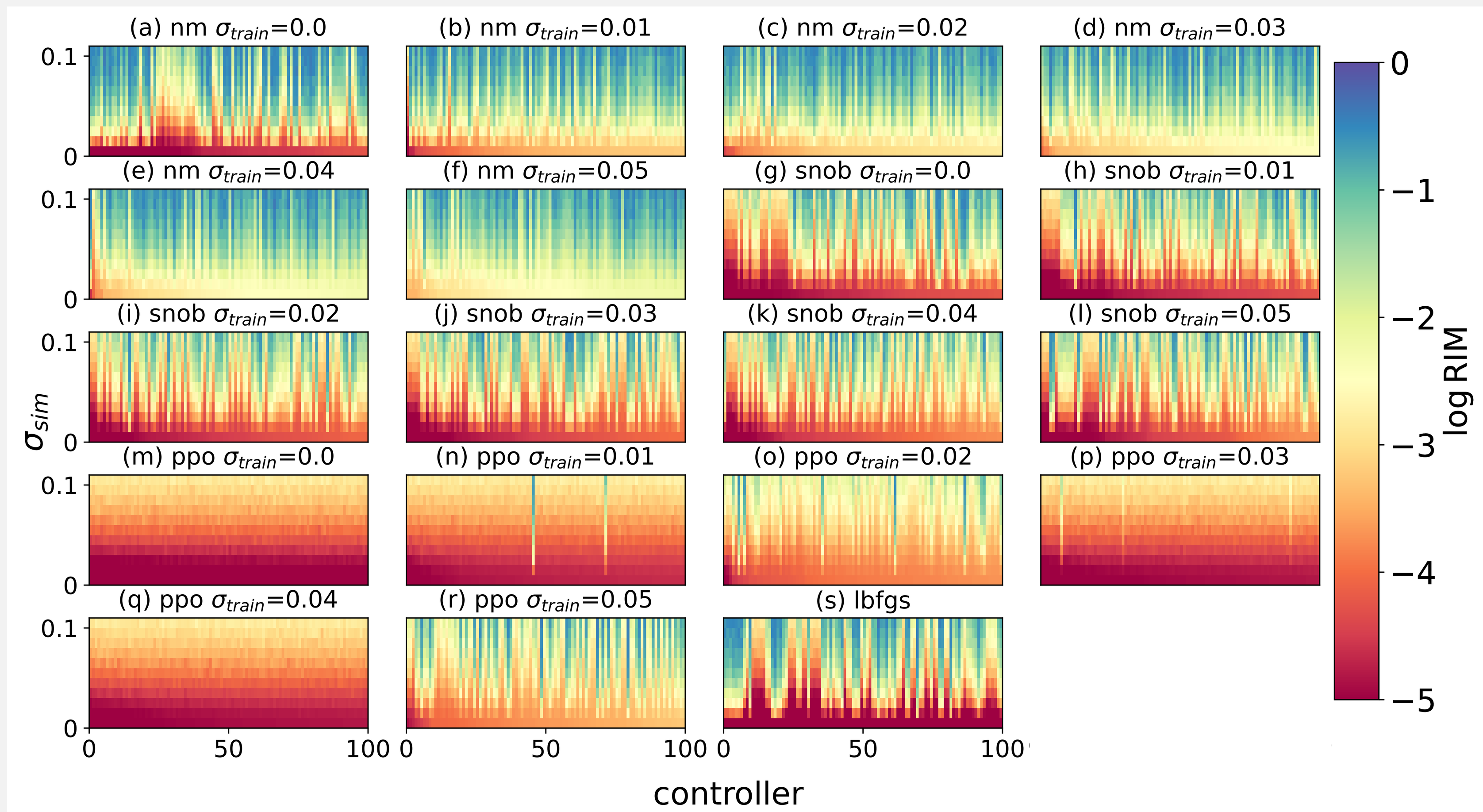

Figure 3: Top 100 controllers sorted (left to right) by $1 - \mathcal{F}$ for the uncertainty level $\sigma_{\mathrm{sim}} = 0$.

Algorithmic RIM (ARIM) for RIM distribution comparison

$$\mathrm{ARIM} := \mathcal{W}_1(\mathcal{P}_{\sigma_{\mathrm{sim}}}(\mathrm{RIM}), \delta(\mathrm{RIM}-0)) = \mathbb{E}[\mathrm{RIM}_1] \tag{5}$$

We use the ARIM to compare empirical quantum controller acquisition schemes. Here, reinforcement learning (PPO) has a lower sample complexity for ARIM optimisation that is especially pronounced in a stochastic setting.

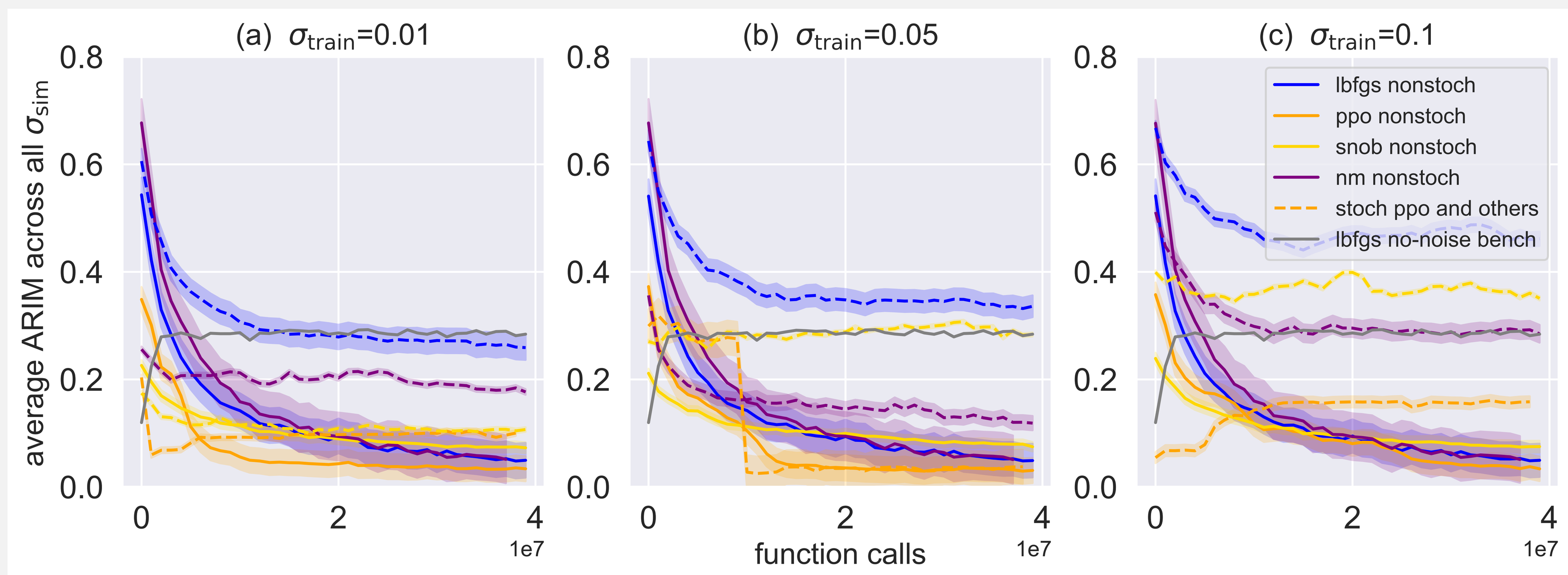

Figure 4: Asymptotic control algorithm ARIM performance when the number of fidelity function calls is unconstrained for derandomised (non-stochastic) and randomised optimisation objective function settings.