Quality of Information Maximization for Wireless Networks via a Fully Separable Quadratic Policy

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***Abstract*—An information collection problem in a wireless network with random events is considered. Wireless devices report on each event using one of multiple reporting formats. Each format has a different quality and uses different data lengths. Delivering all data in the highest quality format can overload system resources. The goal is to make intelligent format selection and routing decisions to maximize time-averaged information quality subject to network stability. Lyapunov optimization theory can be used to solve such a problem by repeatedly minimizing the linear terms of a quadratic drift-plus-penalty expression. To reduce delays, this paper proposes a novel extension of this technique that preserves the quadratic nature of the drift minimization while maintaining a fully separable structure. In addition, to avoid high queuing delay, paths are restricted to at most two hops. The resulting algorithm can push average information quality arbitrarily close to optimum, with a tradeoff in queue backlog. The algorithm compares favorably to the basic drift-plus-penalty scheme in terms of backlog and delay.**

I. Introduction

THIS paper investigates dynamic scheduling and data format selection in a network where multiple wireless devices, such as smart phones, report information to a receiver station. The devices together act as a pervasive pool of information about the network environment. Such scenarios have been recently considered, for example, in applications of social sensing [2] and personal environment monitoring [3], [4]. Sending all information in the highest quality format can quickly overload network resources. Thus, it is often more important to optimize the *quality of information*, as defined by an end-user, rather than the raw number of bits that are sent. The case for quality-aware networking is made in [5], [6], [7]. Network management with quality of information awareness for wireless sensor networks is considered in [8]. More recently, quality metrics of accuracy and credibility are considered in [9], [10] using simplified models that do not consider the actual dynamics of a wireless network.

In this paper, we extend the quality-aware format selection problem in [10] to a dynamic network setting. We particularly focus on distributed algorithms for routing, scheduling, and format selection that jointly optimize quality of information. Specifically, we assume that random events occur over time in the network environment, and these can be sensed by one or more of the wireless devices, perhaps at different sensing qualities. At the transport layer, each device selects one of multiple reporting formats, such as a video clip at one of several resolution options, an audio clip, or a text message. Information quality depends on the selected format. For example, higher quality formats use messages with larger bit lengths. The resulting bits are handed to the network layer at each device and must be delivered to the receiver station over possibly time-varying channels.

We first consider the case where all devices transmit directly to the destination over uplink channels. Due to heterogeneous channel conditions, the delivery rates in this case may be limited. To improve performance, we next allow devices to relay their information through other devices that have more favorable connections to the destination. An example is a single-cell wireless network with multiple smart phones and one base station, where each smart phone has 4G capability for uplink transmission and Wi-Fi capability for device-to-device relay transmission.

Such a problem can be cast as a stochastic network optimization and solved using Lyapunov optimization theory. A "standard" method is to minimize a linear term in a quadratic drift-plus-penalty expression, which leads to *max-weight* type solutions [11], [12]. This can be shown to yield algorithms that converge to optimal average utility with a tradeoff in average queue size. The linearization is useful for enabling decisions to be separated at each device. However, it can lead to larger queue sizes and delays. In this work, we propose a novel method that uses a quadratic minimization for the drift-plus-penalty expression, yet still allows separability of the decisions. This results in an algorithm that maintains distributed format selection decisions across all devices, but reduces average delay. Similar to the standard (linearized) drift-plus-penalty methods, the transmission decisions can also be made in a distributed manner under suitable physical layer models, such as when channels are orthogonal.

Thus, the contributions of this paper are twofold: (i) We formulate an important quality-of-information problem for reporting information in wireless systems. This problem is of recent interest and can be used in other contexts where *data deluge* issues require selectivity in reporting of information.

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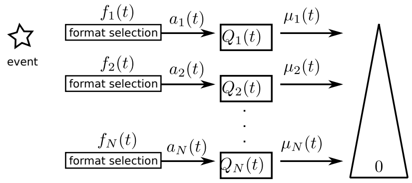

Fig. 1. An example network with N devices as queues $Q_n(t)$, and a receiver station.

(ii) We extend Lyapunov optimization theory by presenting a new algorithm that uses a quadratic minimization to reduce queue sizes while maintaining separability across decisions. This new technique is general and can be used to reduce queue sizes in other Lyapunov optimization problems.

The next section formulates the problem for an uplink network without relay capabilities. Section III derives the quadratic algorithm for this network, and Section IV analyzes and simulates its performance. Relay capabilities are introduced and analyzed in Sections V-VIII. To reduce delays, this paper restricts all paths to at most 2-hops, so that data can pass through at most one relay. This 2-hop restriction is not crucial to the analysis. Indeed, the same techniques can be used to treat multi-hop routing via the *backpressure* methodology [12][13], although we omit that extension for brevity.

II. SINGLE-HOP SYSTEM MODEL

Consider a network with N wireless devices that report information to a single receiver station. Let $\mathcal{N} = \{1, \ldots, N\}$ be the set of devices. The receiver station is not part of the set $\mathcal{N}$ and can be viewed as "device 0." A network with $N = 3$ devices is shown in Fig. 1. The system is slotted with fixed size slots $t \in \{0, 1, 2, \ldots\}$. Every slot, *format selection decisions* are made at the transport layer of each device, and *scheduling* decisions are made at the network layer.

A. Format selection

A new event can occur on each slot. Events are observed with different levels of quality at each device. For example, some devices may be physically closer to the event and hence can deliver higher quality. On slot t, each device $n \in \mathcal{N}$ selects a format $f_n(t)$ from a set of available formats $\mathcal{F} = \{0, 1, \ldots, F\}$. Format selection affects quality and data lengths of the reported information. To model this, the event on slot t is described by a vector of *event characteristics* $(r_n^{(f)}(t), a_n^{(f)}(t))|_{n\in\mathcal{N}, f\in\mathcal{F}}$. The value $r_n^{(f)}(t)$ is a numeric *reward* that is earned if device n uses format f to report on the event that occurs on slot t. The value $a_n^{(f)}(t)$ is the amount of data units required for this choice. This data is injected as arrivals to a network layer queue and must eventually be delivered to the receiver station (see Fig. 1). Each device n observes $(r_n^{(f)}(t), a_n^{(f)}(t))$ at the beginning of slot t and chooses a format $f_n(t)$. Define $r_n(t)$ and $a_n(t)$ as the resulting reward and data size:

$$r_n(t) \triangleq r_n^{(f_n(t))}(t) \ , \ a_n(t) \triangleq a_n^{(f_n(t))}(t)$$

If a device n does not observe the event on slot t (which might occur if it is physically too far from the event), then $(r_n^{(f)}(t), a_n^{(f)}(t)) = (0, 0)$ for all formats $f \in \mathcal{F}$. If no event occurs on slot t, then $(r_n^{(f)}(t), a_n^{(f)}(t)) = (0, 0)$ for all $n \in \mathcal{N}$ and $f \in \mathcal{F}$. To allow a device n not to report on an event, there is a *blank format* $0 \in \mathcal{F}$ such that $(r_n^{(0)}(t), a_n^{(0)}(t)) = (0, 0)$ for all slots t and all devices $n \in \mathcal{N}$.

Rewards $r_n(t)$ are assumed to be real numbers that satisfy $0 \leq r_n(t) \leq r_n^{(\max)}$ for all t, where $r_n^{(\max)}$ is a finite maximum. Data sizes $a_n(t)$ are non-negative integers that satisfy $0 \leq a_n(t) \leq a_n^{(\max)}$ for all t, where $a_n^{(\max)}$ is a finite maximum. The vectors $(r_n^{(f)}(t), a_n^{(f)}(t))|_{n\in\mathcal{N}, f\in\mathcal{F}}$ are independent and identically distributed (i.i.d.) over slots t, and have a joint probability distribution over devices n and formats f that is arbitrary (subject to the above boundedness assumptions). This distribution is not necessarily known.

A simple example is when there is no time-variation in the format selection process, so that the reward and bit length options $(r_n^{(f)}, a_n^{(f)})$ are the same for all time. This holds when each particular format always yields the same reward and has the same bit length. The model also treats cases when these values can change from slot to slot. This holds, for example, in a video streaming application where format selection options correspond to different video compression techniques. These can have variable bit outputs depending on the content of the current video frame. For simplicity, this paper assumes the random processes are i.i.d. over slots. This assumption is not crucial to the analysis, and the results can be extended to treat non-i.i.d. scenarios using techniques in [11].

B. Uplink scheduling

At each device $n \in \mathcal{N}$, the $a_n(t)$ units of data generated by format selection are put into *input queue* $Q_n(t)$. Each device communicates directly to the receiver station through (direct) *uplink transmission* as shown in Fig. 1. The amount that can be transmitted by uplink transmission at device n is denoted by $\mu_n(t)$. The $\mu_n(t)$ values are determined by the current *channel states* and the current transmission decisions. Specifically, define $\boldsymbol{\mu}(t) = (\mu_n(t))|_{n\in\mathcal{N}}$ as the transmission vector, and define $\boldsymbol{\eta}(t)$ as a vector of current channel states in the network at time t. It is assumed that $\boldsymbol{\mu}(t)$ is chosen every slot t within a set $\mathcal{U}_{\boldsymbol{\eta}(t)}$ that depends on the observed $\boldsymbol{\eta}(t)$.

The sets $\mathcal{U}_{\boldsymbol{\eta}(t)}$ are assumed to restrict transmissions to non-negative and bounded rates, so that $0 \leq \mu_n(t) \leq \mu_n^{(\max)}$ for all $n \in \mathcal{N}$ and all t. Additional structure of the sets $\mathcal{U}_{\boldsymbol{\eta}(t)}$ can be imposed to model the physical transmission capabilities of the network. A special case is when all uplink channels are orthogonal and the set $\mathcal{U}_{\boldsymbol{\eta}(t)}$ can be decomposed into a set product of individual options for each uplink channel:

$$\mathcal{U}_{\boldsymbol{\eta}(t)} = \mathcal{U}_{1,\eta_1(t)} \times \mathcal{U}_{2,\eta_2(t)} \times \ldots \times \mathcal{U}_{N,\eta_N(t)}$$

where, in this case, $\eta_n(t)$ represents the component of the current channel state vector $\boldsymbol{\eta}(t)$ associated with channel n.

The dynamics of input queue $Q_n(t)$ are:

$$Q_n(t+1) = \max[Q_n(t) - \mu_n(t), 0] + a_n(t), \qquad (1)$$

which assumes that newly arriving data $a_n(t)$ cannot be transmitted on slot t. As a minor technical detail that is useful later, the $\max[\ldots, 0]$ operator above allows $\mu_n(t)$ to be greater than $Q_n(t)$.

C. Stochastic network optimization

Here we define the problem of maximizing time-averaged quality of information subject to queue stability. We use the following stability definition [12]:

Definition 1: Queue $\{X(t) : t \in \{0, 1, 2, \ldots\}\}$ is strongly stable if

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left[X(\tau)\right] < \infty$$

Intuitively, this means that a queue is strongly stable if its average backlog is finite. A network is defined to be strongly stable if all of its queues are strongly stable.

Define $y_0(t)$ as the total quality of information from format selection on slot t:

$$y_0(t) \triangleq \sum_{n\in\mathcal{N}} r_n(t)$$

Define the upper bound $y_0^{(\max)} \triangleq \sum_{n\in\mathcal{N}} r_n^{(\max)}$. The time-averaged total information quality is

$$\bar{y}_0 \triangleq \liminf_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left[y_0(\tau)\right].$$

The objective is to solve:

$$\begin{aligned} \text{Maximize} \quad & \bar{y}_0 & (2)\\ \text{Subject to} \quad & \text{Network is strongly stable} \\ & f_n(t) \in \mathcal{F} \text{ for all } t, \text{ and all } n \in \mathcal{N} \\ & \boldsymbol{\mu}(t) \in \mathcal{U}_{\boldsymbol{\eta}(t)} \text{ for all } t \end{aligned}$$

This problem is always feasible because stability is trivially achieved if all devices always select the blank format.

III. Dynamic algorithm of the uplink network

This section derives a novel *quadratic policy* to solve problem (2).

A. Lyapunov optimization

Let $\boldsymbol{Q}(t) = (Q_n(t))|_{n\in\mathcal{N}}$ represent the vector of all queues in the system. Define a quadratic *Lyapunov function*:

$$L(t) \triangleq \frac{1}{2} \sum_{n\in\mathcal{N}} Q_n(t)^2$$

Define $L(t+1) - L(t)$ as the *Lyapunov drift*. In order to maximize $\bar{y}_0$ in (2), the *drift-plus-penalty* function $L(t+1) - L(t) - Vy_0(t)$ is considered, where $V \geq 0$ is a constant that determines a tradeoff between queue size and proximity to optimality.[1] Later, this is used to prove stability. Intuitively, when queue lengths grow large beyond certain values, the drift becomes negative and the system is stable because the negative drift tends to reduce total queue lengths.

[1]The minus sign in front of $Vy_0(t)$ comes from the fact that the quality of information is viewed as a negative penalty.

From (1) and the definition of $y_0(t)$, the drift-plus-penalty expression is given by:

$$\begin{aligned} & L(t+1) - L(t) - Vy_0(t) \\ &= \frac{1}{2} \sum_{n\in\mathcal{N}} \left[Q_n(t+1)^2 - Q_n(t)^2 - 2Vr_n(t)\right] \\ &= \frac{1}{2} \sum_{n\in\mathcal{N}} \left[\left(\max[Q_n(t) - \mu_n(t), 0] + a_n(t)\right)^2 - 2Vr_n(t)\right] \\ &\quad - \frac{1}{2} \sum_{n\in\mathcal{N}} Q_n(t)^2 \qquad (3) \end{aligned}$$

Ideally, every slot t one would like to observe the current queue values $\boldsymbol{Q}(t)$ and select decision variables $r_n(t)$, $a_n(t)$, $\mu_n(t)$ to minimize the above expression over all possible decision options for that slot. Since the $Q_n(t)$ values are fixed in this decision, this amounts to minimizing the first summation term in the expression above. However, the quadratic nature of the above expression couples all decision variables. Thus, such an algorithm would not allow for format selection decisions to be distributed across devices, and would not allow format selection and transmission scheduling to be separated.

A standard simplification seeks to minimize the following linearized approximation of the above expression [11]:

$$\sum_{n\in\mathcal{N}} \left[Q_n(t)(a_n(t) - \mu_n(t)) - Vr_n(t)\right] \qquad (4)$$

This expression is a separable sum over individual devices. Minimization of this expression every slot results in the *drift-plus-penalty algorithm* [11]. This allows a clean separation of format selection and transmission decisions, and allows format selection to be distributed across all devices. It is known that using this linearized approximation does not hinder asymptotic stability or time average quality. However, intuitively, one expects that something is lost by only using the linear approximation. Often, this loss translates into larger queue sizes. The next section develops a novel alternative method that preserves the quadratic nature of the minimization while maintaining a clean separation across decision variables.

B. The separable quadratic policy

Lemma 1: Suppose a and μ are non-negative constants such that $a \leq a^{(\max)}$ and $\mu \leq \mu^{(\max)}$. Then for any $x \geq 0$:

$$\begin{aligned} & \left(\max\left[x - \mu, 0\right] + a\right)^2 - x^2 \\ & \leq (x-\mu)^2 + (x+a)^2 - 2x^2 \qquad (5) \end{aligned}$$

Proof: Note that $\max[x-\mu, 0]^2 \leq (x-\mu)^2$. Thus:

$$\begin{aligned} & \left(\max[x-\mu, 0] + a\right)^2 - x^2 \\ & \leq (x-\mu)^2 + a^2 + 2a\max[x-\mu, 0] - x^2 \\ & \leq (x-\mu)^2 + a^2 + 2ax - x^2 \\ & = (x-\mu)^2 + (x+a)^2 - 2x^2 \end{aligned}$$

■

Using the result of Lemma 1 in (3) gives:

$$\begin{aligned} &L(t+1) - L(t) - Vy_0(t) \\ &\leq \frac{1}{2}\sum_{n\in\mathcal{N}} \left[(Q_n(t) - \mu_n(t))^2 + (Q_n(t) + a_n(t))^2\right] \\ &\quad - \frac{1}{2}\sum_{n\in\mathcal{N}} 2Vr_n(t) - \sum_{n\in\mathcal{N}} Q_n(t)^2 \end{aligned} \tag{6}$$

Our novel separable quadratic policy observes the queue values $Q_n(t)$ every slot t and makes format selection and transmission decisions to minimize the right-hand-side of the expression (6). That is, $\boldsymbol{\mu}(t)$ and $f_n(t)$ decisions are made to solve the following optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \textstyle\sum_{n\in\mathcal{N}}\left[(Q_n(t)-\mu_n(t))^2+(Q_n(t)+a_n(t))^2-2Vr_n(t)\right] \qquad (7) \\ &\text{Subject to} \quad \boldsymbol{\mu}(t)\in\mathcal{U}_{\boldsymbol{\eta}(t)}, \quad f_n(t)\in\mathcal{F} \quad \forall n\in\mathcal{N} \\ &\qquad\qquad a_n(t)=a_n^{(f_n(t))}(t),\ r_n(t)=r_n^{(f_n(t))}(t) \quad \forall n\in\mathcal{N} \end{aligned}$$

where weights $Q_n(t)$ act as given constants in the above optimization problem. The queues are then updated via (1) and the procedure is repeated for the next slot.

Intuitively, every time slot, the bound (6) is minimized by the quadratic policy, so its value is smaller than that resulting from applying any other policies. This will become clear in Section IV.

C. Separability

The control algorithm (7) can be simplified by exploiting the separable structure as follows: Every slot t, each device $n \in \mathcal{N}$ observes input queue $Q_n(t)$ and options $(r_n^{(f)}(t), a_n^{(f)}(t))|_{f\in\mathcal{F}}$. It then chooses a format $f_n(t)$ according to the *admission-control problem*:

$$\begin{aligned} &\text{Minimize} \quad \left[Q_n(t) + a_n^{(f_n(t))}(t)\right]^2 - 2Vr_n^{(f_n(t))}(t) \\ &\text{Subject to} \quad f_n(t) \in \mathcal{F} \end{aligned} \tag{8}$$

This is solved easily by comparing each option $f_n(t) \in \mathcal{F}$. Intuitively, a large value of V allows more candidate formats to be selected. As an algorithm evolves, queue $Q_n(t)$ will enforce a system to select an optimal format at a particular time t. Note that these decisions are distributed across users and are separated from the uplink transmission rate decisions.

The *uplink-allocation* problem to determine transmission rates $\boldsymbol{\mu}(t)$ is:

$$\begin{aligned} &\text{Minimize} \quad \textstyle\sum_{n\in\mathcal{N}}[Q_n(t) - \mu_n(t)]^2 \\ &\text{Subject to} \quad \boldsymbol{\mu}(t) \in \mathcal{U}_{\boldsymbol{\eta}(t)}. \end{aligned} \tag{9}$$

This can be solved at the receiver station. Intuitively, a system minimizes a sum of the square of the remaining queue lengths, so a longer queue is treated with higher priority. When the difference between queues is small, those queues are treated fairly equally. If all uplink channels are orthogonal, the problem can be decomposed further so that each device n solves:

$$\begin{aligned} &\text{Minimize} \quad [Q_n(t) - \mu_n(t)]^2 \\ &\text{Subject to} \quad \mu_n(t) \in \mathcal{U}_{n,\eta_n(t)} \end{aligned}$$

where $\mathcal{U}_{n,\eta_n(t)}$ is a feasible set of $\mu_n(t)$ options. This chooses the uplink transmission rate which is the closest rate in $\mathcal{U}_{n,\eta_n(t)}$ to $Q_n(t)$. The algorithm is summarized in the algorithms below.

Algorithm 1: Distributed format selection

```
// Device side
foreach device n ∈ N do
    – Observe Q_n(t) and (r_n^(f)(t), a_n^(f)(t))|_{f∈F}
    – Select format f_n(t) according to (8)
end
```

Algorithm 2: Uplink resource allocation

```
// Receiver-station side
for receiver station 0 do
    – Observe (Q_n(t))|_{n∈N} and U_η(t)
    – Signal devices n ∈ N to make uplink transmission
      μ(t) according to (9)
end
```

To compare this approach to the standard drift-plus-penalty technique, consider the following example. Suppose the transmission rate set is given by:

$$\mathcal{U}_{\boldsymbol{\eta}(t)} = \left\{ \boldsymbol{\mu} \geq \mathbf{0} \,\middle|\, \sum_{n\in\mathcal{N}} \frac{\mu_n}{\eta_n(t)} \leq 1 \right\}$$

This allows for a division of either time or frequency resources over one slot, so that a fraction of the channel capacity can be devoted to one or more users simultaneously. The standard drift-plus-penalty approach of minimizing (4) over this set results in a *max-weight* decision that allocates the full channel to a single user n at the full rate $\eta_n(t)$. This is an inefficient use of resources if the queue backlog $Q_n(t)$ of device n is less than $\eta_n(t)$. In contrast, our separable quadratic policy never over-allocates resources: In this example it ensures that $\mu_n(t) \leq Q_n(t)$ for all slots t. Also, it often enables queues to be emptied more quickly by allowing multiple devices to transmit simultaneously.

IV. Performance and simulation of the uplink network

Compare the separable quadratic policy with any other policy. Let $(f_n(\tau))|_{n\in\mathcal{N}}, \boldsymbol{\mu}(\tau)$ be the decision variables from the quadratic policy, which is a solution of problem (7), and $r_n(t) \triangleq r_n^{(f_n(t))}(t)$, $a_n(t) \triangleq a_n^{(f_n(t))}(t)$. Also let $(\hat{f}_n(\tau))|_{n\in\mathcal{N}}, \hat{\boldsymbol{\mu}}(\tau)$ be decision variables from any other policy, and $\hat{r}_n(t) \triangleq r_n^{(\hat{f}_n(t))}(t), \hat{a}_n(t) \triangleq a_n^{(\hat{f}_n(t))}(t)$. Because the quadratic policy makes decisions that minimize the right-hand-side of (6), we have at every slot τ:

$$\begin{aligned}
&L(\tau+1) - L(\tau) - Vy_0(\tau) \\
&\quad \leq \frac{1}{2}\sum_{n\in\mathcal{N}}\left[(Q_n(\tau) - \hat{\mu}_n(\tau))^2 + (Q_n(\tau) + \hat{a}_n(\tau))^2\right] \\
&\qquad - \frac{1}{2}\sum_{n\in\mathcal{N}} 2V\hat{r}_n(\tau) - \sum_{n\in\mathcal{N}} Q_n(\tau)^2 \\
&\quad = \sum_{n\in\mathcal{N}}\left[Q_n(\tau)(\hat{a}_n(\tau) - \hat{\mu}_n(\tau)) - V\hat{r}_n(\tau)\right] \\
&\qquad + \frac{1}{2}\sum_{n\in\mathcal{N}}\left[(\hat{\mu}_n(\tau))^2 + (\hat{a}_n(\tau))^2\right]
\end{aligned}$$

Therefore:

$$\begin{aligned}
&L(\tau+1) - L(\tau) - Vy_0(\tau) \\
&\quad \leq C + \sum_{n\in\mathcal{N}}\left[Q_n(\tau)(\hat{a}_n(\tau) - \hat{\mu}_n(\tau)) - V\hat{r}_n(\tau)\right] \qquad (10)
\end{aligned}$$

where the constant C is defined:

$$C \triangleq \frac{1}{2}\sum_{n\in\mathcal{N}}\left[(\mu_n^{(\max)})^2 + (a_n^{(\max)})^2\right] \qquad (11)$$

Now define $\omega(t)$ as a concatenated vector of all random events observed on slot t:

$$\omega(t) \triangleq [\boldsymbol{\eta}(t); (r_n^{(f)}(t), a_n^{(f)}(t))|_{n\in\mathcal{N}, f\in\mathcal{F}}]$$

As discussed in Section II, vector $\omega(t)$ is i.i.d. over slots according to some (possibly unknown) probability distribution. The components of $\omega(t)$ on a given slot t can be arbitrarily correlated. Define an *ω-only policy* as one that makes a (possibly randomized) choice of decision variables based only on the observed $\omega(t)$ (and hence independently of queue backlogs). We now customize an important theorem from [11].

Theorem 1: For any $\delta > 0$ there exists an ω-only policy that chooses all controlled variables $(f_n^*(t))|_{n\in\mathcal{N}}, \boldsymbol{\mu}^*(t)$ such that:

$$\mathbb{E}\left[y_0^*(t)\right] \geq y_0^{(\text{opt})} - \delta \qquad (12)$$

$$\mathbb{E}\left[a_n^*(t) - \mu_n^*(t)\right] \leq \delta \quad \text{for all } n \in \mathcal{N} \qquad (13)$$

where $y_0^{(\text{opt})}$ is the optimal solution of problem (2). Also, $y_0^*(t) \triangleq \sum_{n\in\mathcal{N}} r_n^*(t)$ when $r_n^*(t) \triangleq r_n^{(f_n^*(t))}(t)$ and $a_n^*(t) \triangleq a_n^{(f_n^*(t))}(t)$.

We additionally assume all constraints of the network can be achieved with ϵ slackness [11]. In other words, there exists a policy that, at every queue, has average transmission rate higher than average arrival rate.

Assumption 1: There are values $\epsilon > 0$ and $0 \leq y_0^{(\epsilon)} \leq y_0^{(\max)}$ and an ω-only policy choosing all controlled variables $(f_n^*(t))|_{n\in\mathcal{N}}, \boldsymbol{\mu}^*(t)$ that satisfies:

$$\mathbb{E}\left[y_0^*(t)\right] = y_0^{(\epsilon)} \qquad (14)$$

$$\mathbb{E}\left[a_n^*(t) - \mu_n^*(t)\right] \leq -\epsilon \quad \text{for all } n \in \mathcal{N} \qquad (15)$$

A. Performance analysis

Since our quadratic algorithm satisfies the bound (10), where the right-hand-side is in terms of any alternative policy $(\hat{f}_n(t))\big|_{n\in\mathcal{N}}, \hat{\boldsymbol{\mu}}(t)$, it holds for any ω-only policy $(f_n^*(t))\big|_{n\in\mathcal{N}}, \boldsymbol{\mu}^*(t)$. Substituting an ω-only policy into (10) and taking expectations gives:

$$\begin{aligned}
&\mathbb{E}\left[L(\tau+1) - L(\tau) - Vy_0(\tau)\right] \\
&\quad \leq C + \textstyle\sum_{n\in\mathcal{N}} \mathbb{E}\left[Q_n(\tau)(a_n^*(\tau) - \mu_n^*(\tau)) - Vr_n^*(\tau)\right] \\
&\quad = C + \textstyle\sum_{n\in\mathcal{N}} \left[\mathbb{E}\left[Q_n(\tau)\right]\mathbb{E}\left[a_n^*(\tau) - \mu_n^*(\tau)\right] - V\mathbb{E}\left[r_n^*(\tau)\right]\right]
\end{aligned} \qquad (16)$$

where we have used the fact that $Q_n(\tau)$ and $(a_n^*(\tau) - \mu_n^*(\tau))$ are independent under an ω-only policy.

Theorem 2: Assume queues are initially empty, so that $Q_n(0) = 0$ for all n, and that Assumption 1 holds. Then the time-averaged total quality of information $\overline{y}_0$ is within $O(1/V)$ of optimality under the separable quadratic policy, while the total queue backlog is $O(V)$.

This theorem is proven in the next two subsections.

1) Quality of Information vs. V: Using the ω-only policy from (12)–(13) in the right-hand-side of (16) gives:

$$\begin{aligned}
&\mathbb{E}\left[L(\tau+1) - L(\tau) - Vy_0(\tau)\right] \\
&\qquad \leq C - V\left(y_0^{(\text{opt})} - \delta\right) + \delta \textstyle\sum_{n\in\mathcal{N}} \mathbb{E}\left[Q_n(\tau)\right]
\end{aligned}$$

This inequality is valid for every $\delta > 0$. Therefore

$$\mathbb{E}\left[L(\tau+1) - L(\tau) - Vy_0(\tau)\right] \leq C - Vy_0^{(\text{opt})}$$

Summing from $\tau = 0$ to $t-1$:

$$\mathbb{E}\left[L(t) - L(0) - V\textstyle\sum_{\tau=0}^{t-1} y_0(\tau)\right] \leq Ct - Vty_0^{(\text{opt})}$$

Using $L(t) \geq 0$, $L(0) = 0$ and dividing by $-Vt$ gives:

$$\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left[y_0(\tau)\right] \geq -\frac{C}{V} + y_0^{(\text{opt})} \qquad (17)$$

The above holds for all $t > 0$. Taking a limit as $t \to \infty$ shows that $\overline{y}_0$ is at least $y_0^{(\text{opt})} - C/V$, where the gap C/V can be made arbitrarily small by increasing the V parameter.

2) Total Queue Backlog vs. V: Now consider the existence of an ω-only policy that satisfies Assumption 1. Using (14)–(15) in the right-hand-side of (16) gives:

$$\begin{aligned}
&\mathbb{E}\left[L(\tau+1) - L(\tau) - Vy_0(\tau)\right] \\
&\qquad \leq C - Vy_0^{(\epsilon)} - \epsilon \textstyle\sum_{n\in\mathcal{N}} \mathbb{E}\left[Q_n(\tau)\right]
\end{aligned}$$

Thus:

$$\begin{aligned}
&\mathbb{E}\left[L(\tau+1) - L(\tau)\right] \\
&\qquad \leq C + V\left(y_0^{(\max)} - y_0^{(\epsilon)}\right) - \epsilon \textstyle\sum_{n\in\mathcal{N}} \mathbb{E}\left[Q_n(\tau)\right]
\end{aligned}$$

Summing from $\tau = 0$ to $t-1$ gives:

$$\begin{aligned}
&\mathbb{E}\left[L(t) - L(0)\right] \\
&\quad \leq \left(C + V(y_0^{(\max)} - y_0^{(\epsilon)})\right)t - \epsilon \textstyle\sum_{\tau=0}^{t-1}\sum_{n\in\mathcal{N}} \mathbb{E}\left[Q_n(\tau)\right]
\end{aligned}$$

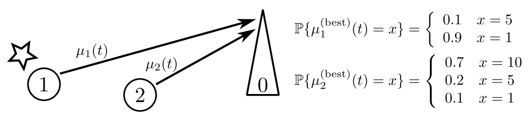

Fig. 2. Small network with orthogonal channels and distributions

Using $L(t) \geq 0$, $L(0) = 0$, and rearranging terms above gives:

$$\frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{n\in\mathcal{N}}\mathbb{E}\left[Q_n(\tau)\right] \leq \frac{C + V(y_0^{(\max)} - y_0^{(\epsilon)})}{\epsilon} \tag{18}$$

The above holds for all $t > 0$. Taking a limit as $t \to \infty$ shows that total time-average expected queue backlog is bounded by a constant that is $O(V)$. In particular, this bound implies that every queue is strongly stable.

The V parameter in (17) and (18) affects the performance tradeoff $[O(1/V), O(V)]$ between quality of information and total queue backlog. These results are similar to those that can be derived under the standard max-weight algorithm [11][12]. However, simulation in the next section shows significant reduction of queue backlog under the quadratic policy. Note that our proofs are inspired by the techniques in [11][12].

In addition to the above tradeoff, it is possible to show that queues $Q_n(t)$ are *deterministically* bounded by a constant that is $O(V)$. This is skipped for brevity, but is shown more generally for the 2-hop problem in Section VII-B.

B. Simulation

Simulation under the proposed quadratic policy and the standard max-weight policy is performed over a small network in Fig. 2. The network contains two devices, $\mathcal{N} = \{1, 2\}$. An event occurs in every slot with probability 0.3. Device 1 is closer to the event, but device 2 is closer to the receiver station. Due to this, the uplink channel distribution for device 2 is better than that of device 1, as shown in Fig. 2. We assume the uplink channels are orthogonal.

The constraints are $\mu_n(t) \in \{0, \ldots, \mu_n^{(\text{best})}(\eta_n(t))\}$ for $n \in \mathcal{N}$. The feasible set of formats is $\mathcal{F} = \{0, 1, \ldots, 6\}$ with constant options given by

$$\begin{array}{ll}
(r_1^{(0)}, a_1^{(0)}) = (0, 0) & (r_2^{(0)}, a_2^{(0)}) = (0, 0) \\
(r_1^{(1)}, a_1^{(1)}) = (15, 10) & (r_2^{(1)}, a_2^{(1)}) = (1.5, 10) \\
(r_1^{(2)}, a_1^{(2)}) = (45, 40) & (r_2^{(2)}, a_2^{(2)}) = (4.5, 40) \\
(r_1^{(3)}, a_1^{(3)}) = (65, 70) & (r_2^{(3)}, a_2^{(3)}) = (6.5, 70) \\
(r_1^{(4)}, a_1^{(4)}) = (75, 100) & (r_2^{(4)}, a_2^{(4)}) = (7.5, 100) \\
(r_1^{(5)}, a_1^{(5)}) = (90, 200) & (r_2^{(5)}, a_2^{(5)}) = (9.0, 200) \\
(r_1^{(6)}, a_1^{(6)}) = (110, 400) & (r_2^{(6)}, a_2^{(6)}) = (11.0, 400)
\end{array}$$

whenever there is an event. In particular, the rewards associated with device 1 are ten times larger than those of device 2.

The separable quadratic policy minimizes (7) every slot, while the max-weight policy minimizes (4). The time-averaged quality of information for the two policies is shown in Fig. 3a. From the plot, the values of $\bar{y}_0$ under both policies converge

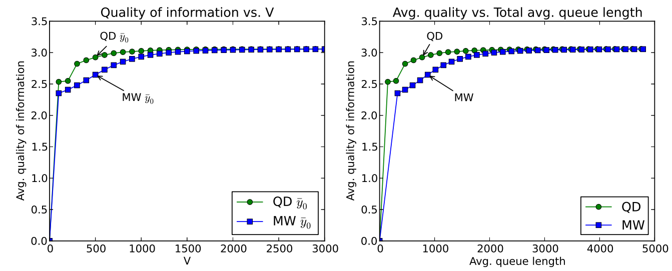

Fig. 3. Quality of information versus V and averaged queue lengths under the quadratic (QD) and max-weight (MW) policies

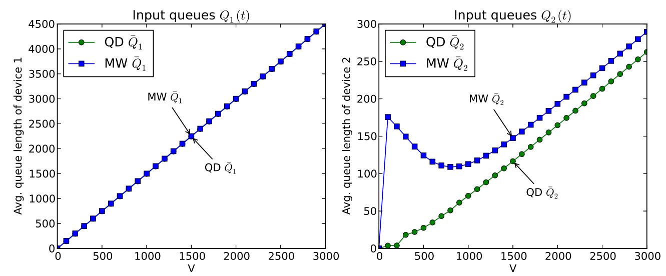

Fig. 4. Averaged backlog in queues versus V under the quadratic and max-weight policies

to optimality following the $O(1/V)$ performance bound. The averaged total rewards from the quadratic policy converges faster than that from the max-weight policy.

Fig. 4 reveals queue lengths in the inputs under the quadratic and max-weight policies. At the same V, the quadratic policy yields smaller or equal queue lengths compared to the cases under the max-weight policy. The plot also shows the growth of queue lengths with parameter V, which follows the $O(V)$ bound of the queue length.

Fig. 3b shows that the quadratic policy can achieve near optimality with significantly smaller total system backlog compared to the case under the max-weight policy. This shows a significant advantage, which in turn affects buffer size and packet delay.

V. System model with relay

The simulation scenario in the previous section only allows direct transmission to the destination, which limits device 1 from reporting high-quality information. In this section, every device has the choice of either transmitting to the destination via the direct uplink channel, or transmitting to a neighboring device that will act as a relay. These uplink and device-to-device transmissions are assumed to be orthogonal, e.g., 4G and Wi-Fi. Although muti-hop relaying is possible, the considered system restricts to at most two hops (i.e., paths to the destination use at most one relay). This allows tighter control over network delay.

The definitions of format selection and uplink scheduling in Section II-A and Section II-B are still valid and are used for this new model. However, the input queue $Q_n(t)$ will be redefined when routing is introduced.

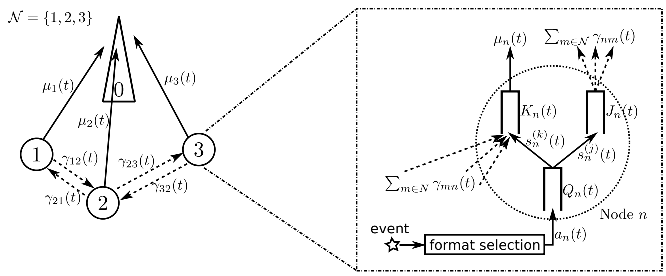

Fig. 5. An example network consists of devices with uplink and relay capabilities and a receiver station.

A. Routing and scheduling

At each device $n \in \mathcal{N}$, the $a_n(t)$ units of data generated by format selection are put into *input queue* $Q_n(t)$. To ensure all data takes at most two hops to the receiver station, the data in each queue $Q_n(t)$ is internally routed to one of two queues $K_n(t)$ and $J_n(t)$, respectively holding data for uplink and relay transmission (see Fig. 5). Data in queue $K_n(t)$ must be transmitted directly to the receiver station, while data in queue $J_n(t)$ can be transmitted to another device m, but is then placed in queue $K_m(t)$ for that device.[2] This is conceptually similar to the hop-count based queue architecture in [14].

In each slot t, let $s_n^{(k)}(t)$ and $s_n^{(j)}(t)$ represent the amount of data in $Q_n(t)$ that can be internally moved to $K_n(t)$ and $J_n(t)$, respectively, as illustrated in Fig. 5. These decision variables are chosen within sets $\mathcal{S}_n^{(k)}$ and $\mathcal{S}_n^{(j)}$, respectively, where:

$$\begin{aligned}\mathcal{S}_n^{(k)} &\triangleq \{0,1,\ldots,s_n^{(k)(\max)}\}\\ \mathcal{S}_n^{(j)} &\triangleq \{0,1,\ldots,s_n^{(j)(\max)}\}\end{aligned}$$

where $s_n^{(k)(\max)}$, $s_n^{(j)(\max)}$ are finite maximum values.[3]

The new dynamics of $Q_n(t)$ are

$$Q_n(t+1) = \max[Q_n(t) - s_n^{(k)}(t) - s_n^{(j)}(t), 0] + a_n(t). \quad (19)$$

The $s_n^{(k)}(t)$ and $s_n^{(j)}(t)$ decisions are selected by an algorithm, but the *actual* $s_n^{(k)(\text{act})}(t)$ and $s_n^{(j)(\text{act})}(t)$ data units moved from $Q_n(t)$ can be any values that satisfy:

$$s_n^{(k)(\text{act})}(t) + s_n^{(j)(\text{act})}(t) = \min[Q_n(t), s_n^{(k)}(t) + s_n^{(j)}(t)] \quad (20)$$

$$0 \le s_n^{(k)(\text{act})}(t) \le s_n^{(k)}(t) \quad (21)$$

$$0 \le s_n^{(j)(\text{act})}(t) \le s_n^{(j)}(t) \quad (22)$$

Again, wireless transmission is assumed to be channel-aware, and decision options are determined by a vector $\boldsymbol{\eta}(t)$ of *current channel states* in the network, which now includes channels for both uplink and device-to-device transmission. Let $\mu_n(t)$ be the uplink rate from device n to the destination, and let $\boldsymbol{\mu}(t)$ be the vector of these values. Let $\gamma_{nm}(t)$ be the amount of data selected for relay transmission from device n to device m, and let $\boldsymbol{\gamma}(t) = (\gamma_{nm}(t))|_{n,m\in\mathcal{N}}$. Assume $\gamma_{nn}(t) = 0$ for every t and n. Transmissions to relays are assumed to be orthogonal to the uplink transmissions. Every slot t, the vectors $\boldsymbol{\mu}(t)$ and $\boldsymbol{\gamma}(t)$ are chosen within sets $\mathcal{U}_{\boldsymbol{\eta}(t)}$ and $\mathcal{A}_{\boldsymbol{\eta}(t)}$, respectively (where $\mathcal{U}$ and $\mathcal{A}$ stand for *uplink* and *ad-hoc relay*, respectively). If each relay channel is orthogonal then set $\mathcal{A}_{\boldsymbol{\eta}(t)}$ can be decomposed into a set product of individual options for each relay link, where each option depends on the component of $\boldsymbol{\eta}(t)$ that represents its own relay channel.

[2] It is possible to extend the model to allow at most H hops by replacing $J_n(t)$ with $J_{n,2}(t),\ldots,J_{n,H}(t)$ where $J_{n,h}(t)$ carries data that must be delivered to the receiver station within h hops. This extension increases complexity linearly in H.

[3] These upper bounds are necessary for the performance analysis. In practice they are not required.

The dynamics of relay queue $J_n(t)$ are:

$$J_n(t+1) = \max\left[J_n(t) - \textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}(t) + s_n^{(j)(\text{act})}(t), 0\right]. \quad (23)$$

As before, the *actual* amount of data $\gamma_{nm}^{(\text{act})}(t)$ satisfies:

$$\textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}^{(\text{act})}(t) = \min\Big(J_n(t) + s_n^{(j)(\text{act})}(t), \sum_{m\in\mathcal{N}}\gamma_{nm}(t)\Big) \quad (24)$$

$$0 \le \gamma_{nm}^{(\text{act})}(t) \le \gamma_{nm}(t) \qquad \text{for } m \in \mathcal{N} - \{n\}. \quad (25)$$

The dynamics of uplink queue $K_n(t)$ are:

$$K_n(t+1) = \max\left[K_n(t) - \mu_n(t) + s_n^{(k)(\text{act})}(t), 0\right] + \textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}^{(\text{act})}(t), \quad (26)$$

The $J_n(t)$ and $K_n(t)$ dynamics assume the incoming data $s_n^{(j)}(t)$ and $s_n^{(k)}(t)$ can be transmitted out on the same slot t, since moving data between internal buffers of the same device incurs negligible delay.

Notice that all data transmitted to a relay is placed in the uplink queue of that relay (which ensures all paths take at most two hops). The queueing equations (23) and (26) involve actual amounts of data, but they can be bounded using (21), (22) and (25) as

$$J_n(t+1) \le \max\left[J_n(t) - \textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}(t) + s_n^{(j)}(t), 0\right] \quad (27)$$

$$K_n(t+1) \le \max\left[K_n(t) - \mu_n(t) + s_n^{(k)}(t), 0\right] + \textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}(t). \quad (28)$$

The queue dynamics (19), (27), (28) do not require the actual variables $s_n^{(j)(\text{act})}$, $s_n^{(k)(\text{act})}(t)$, $\gamma_{nm}^{(\text{act})}(t)$, and are the only ones needed in the rest of the paper.

Assume the relay transmissions have bounded rates. Specifically, let $\gamma_{nm}^{(\max)}$ be finite maximum values of $\gamma_{nm}(t)$. Further, assume that for each $n \in \mathcal{N}$, $s_n^{(k)(\max)} \ge \mu_n^{(\max)}$ and $s_n^{(j)(\max)} \ge \sum_{m\in\mathcal{N}}\gamma_{nm}^{(\max)}$, so that the maximum amount that can be internally shifted is at least as much as the maximum amount that can be transmitted.

B. Stochastic network optimization

From Section II-C, recall that $y_0(t)$ is the total quality of information from format selection on slot t, and its upper bound is $y_0^{(\max)}$. The time-averaged total information quality is $\bar{y}_0$. The objective is to solve:

$$\begin{aligned}
\text{Maximize} \quad & \bar{y}_0 \\
\text{Subject to} \quad & \text{Network is strongly stable} \\
& f_n(t) \in \mathcal{F} \text{ for all } t \text{ and all } n \in \mathcal{N} \\
& \boldsymbol{\mu}(t) \in \mathcal{U}_{\boldsymbol{\eta}(t)} \text{ for all } t \\
& \boldsymbol{\gamma}(t) \in \mathcal{A}_{\boldsymbol{\eta}(t)} \text{ for all } t \\
& s_n^{(k)}(t) \in \mathcal{S}_n^{(k)} \text{ for all } t \text{ and all } n \in \mathcal{N} \\
& s_n^{(j)}(t) \in \mathcal{S}_n^{(j)} \text{ for all } t \text{ and all } n \in \mathcal{N}
\end{aligned}$$

As before, this problem is always feasible.

VI. Dynamic algorithm

To apply the separable quadratic technique to this problem, we require the following lemma, which is an extension of Lemma 1. The notation $\mathbb{R}$ denotes the real numbers, and $\mathbb{R}_+$ denotes the non-negative reals.

Lemma 2: Let $y_i \in \mathbb{R}$ and $z_j \in \mathbb{R}_+$ for $i \in \{1, 2, \dots, Y\}$ and $j \in \{1, 2, \dots, Z\}$, where Y and Z are non-negative integers. Assume that $|y_i| \leq y_i^{(\max)}$ and $z_j \leq z_j^{(\max)}$. Then for any $x \in \mathbb{R}_+$,

$$\begin{aligned}
&\left[\max\left(x + \textstyle\sum_{i=1}^{Y} y_i, 0\right) + \textstyle\sum_{j=1}^{Z} z_j\right]^2 - x^2 \\
&\leq \textstyle\sum_{i=1}^{Y}(x + y_i)^2 + \textstyle\sum_{j=1}^{Z}(x + z_j)^2 - (Y + Z)x^2 + D
\end{aligned} \tag{29}$$

where

$$D \triangleq \left[\textstyle\sum_{i=1}^{Y} y_i^{(\max)} + \textstyle\sum_{j=1}^{Z} z_j^{(\max)}\right]^2$$

Proof: Lemma 2 is proven in the Appendix. ■

A. Lyapunov optimization

Let $\boldsymbol{\Theta}(t) = (Q_n(t), K_n(t), J_n(t))|_{n\in\mathcal{N}}$ represent a vector of all queues in the system. The quadratic Lyapunov function becomes:

$$L(t) \triangleq \tfrac{1}{2} \textstyle\sum_{n\in\mathcal{N}} \left[Q_n^2(t) + K_n^2(t) + J_n^2(t)\right]$$

Using queuing dynamics (19), (27), and (28), the drift-plus-penalty expression is bounded by (30) below. Then, using relation (29), the bound becomes (31).

$$\begin{aligned}
&L(\tau+1) - L(\tau) - V y_0(\tau) \\
&\leq \tfrac{1}{2} \textstyle\sum_{n\in\mathcal{N}} \Big\{ \left[\max(Q_n(\tau) - s_n^{(k)}(\tau) - s_n^{(j)}(\tau), 0) + a_n(\tau)\right]^2 - Q_n(\tau)^2 \\
&\quad + \left[\max(K_n(\tau) - \mu_n(\tau) + s_n^{(k)}(\tau), 0) + \textstyle\sum_{m\in\mathcal{N}} \gamma_{mn}(\tau)\right]^2 - K_n(\tau)^2 \\
&\quad + \left[\max(J_n(\tau) - \textstyle\sum_{m\in\mathcal{N}} \gamma_{nm}(\tau) + s_n^{(j)}(\tau), 0)\right]^2 - J_n(\tau)^2 - 2Vr_n(\tau) \Big\}
\end{aligned} \tag{30}$$

$$\begin{aligned}
&\leq \tfrac{1}{2} \textstyle\sum_{n\in\mathcal{N}} \Big\{ \left[Q_n(\tau) - s_n^{(k)}(\tau)\right]^2 + \left[Q_n(\tau) - s_n^{(j)}(\tau)\right]^2 + [Q_n(\tau) + a_n(\tau)]^2 \\
&\quad + [K_n(\tau) - \mu_n(\tau)]^2 + \left[K_n(\tau) + s_n^{(k)}(\tau)\right]^2 + \textstyle\sum_{m\in\mathcal{N}} [K_n(\tau) + \gamma_{mn}(\tau)]^2 \\
&\quad + \textstyle\sum_{m\in\mathcal{N}} [J_n(\tau) - \gamma_{nm}(\tau)]^2 + [J_n(\tau) + s_n^{(j)}(\tau)]^2 - 2Vr_n(\tau) + D_n(\tau) \Big\}
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
D_n(\tau) \triangleq & -3Q_n^2(\tau) - (2+N)K_n^2(\tau) - (1+N)J_n^2(\tau) \\
& + (s_n^{(k)(\max)} + s_n^{(j)(\max)} + a_n^{(\max)})^2 \\
& + (\mu_n^{(\max)} + s_n^{(k)(\max)} + \textstyle\sum_{m\in\mathcal{N}} \gamma_{mn}^{(\max)})^2 \\
& + (\textstyle\sum_{m\in\mathcal{N}} \gamma_{nm}^{(\max)} + s_n^{(j)(\max)})^2 \\
& - (a_n^{(\max)})^2 - 2(s_n^{(j)(\max)})^2
\end{aligned}$$

Thus, every time t, the quadratic policy observes current queue backlogs $\boldsymbol{\Theta}(t)$ and randomness $\omega(t)$ and makes a decision according to the following minimization problem.

$$\begin{aligned}
\text{Minimize} \quad & \textstyle\sum_{n\in\mathcal{N}} \Big\{ \left[Q_n(t) - s_n^{(k)}(t)\right]^2 + \left[Q_n(t) - s_n^{(j)}(t)\right]^2 \\
& + [Q_n(t) + a_n(t)]^2 + [K_n(t) - \mu_n(t)]^2 + \left[K_n(t) + s_n^{(k)}(t)\right]^2 \\
& + \textstyle\sum_{m\in\mathcal{N}} [K_n(t) + \gamma_{mn}(t)]^2 + \textstyle\sum_{m\in\mathcal{N}} [J_n(t) - \gamma_{nm}(t)]^2 \\
& + \left[J_n(t) + s_n^{(j)}(t)\right]^2 - 2Vr_n(t) \Big\} \\
\text{Subject to} \quad & s_n^{(k)}(t) \in \mathcal{S}_n^{(k)},\ s_n^{(j)}(t) \in \mathcal{S}_n^{(j)} \quad \forall n \in \mathcal{N} \\
& f_n(t) \in \mathcal{F}, r_n(t) = r_n^{(f_n(t))}(t),\ a_n(t) = a_n^{(f_n(t))}(t) \quad \forall n \in \mathcal{N} \\
& \boldsymbol{\gamma}(t) \in \mathcal{A}_{\boldsymbol{\eta}(t)},\ \boldsymbol{\mu}(t) \in \mathcal{U}_{\boldsymbol{\eta}(t)}
\end{aligned} \tag{32}$$

As a result, the policy leads to a separated control algorithm specified in the next section. The performace tradeoff and deterministic bounds are proven in Section VII.

B. Separability

The control algorithm is derived from problem (32) by separately minimizing each sum of terms.

At every slot t, each device $n \in \mathcal{N}$ observes input queue $Q_n(t)$ and options $(r_n^{(f)}(t), a_n^{(f)}(t))|_{f\in\mathcal{F}}$. It then chooses a format $f_n(t)$ according to the *admission-control problem*:

$$\begin{aligned}
\text{Minimize} \quad & \left[Q_n(t) + a_n^{(f_n(t))}(t)\right]^2 - 2Vr_n^{(f_n(t))}(t) \\
\text{Subject to} \quad & f_n(t) \in \mathcal{F}
\end{aligned} \tag{33}$$

This is solved easily by comparing each option $f_n(t) \in \mathcal{F}$. This problem is similar to (8), and the same intuition applies.

Each device n moves data from its input queue to its uplink queue according to the *uplink routing problem*

$$\begin{aligned}
\text{Minimize} \quad & \left[Q_n(t) - s_n^{(k)}(t)\right]^2 + \left[K_n(t) + s_n^{(k)}(t)\right]^2 \\
\text{Subject to} \quad & s_n^{(k)}(t) \in \mathcal{S}_n^{(k)}.
\end{aligned} \tag{34}$$

This can be solved in a closed form by letting $I_K^+(t) \triangleq \left\lceil \frac{Q_n(t) - K_n(t)}{2} \right\rceil$, $I_K^-(t) \triangleq \left\lfloor \frac{Q_n(t) - K_n(t)}{2} \right\rfloor$ and $g_K(x, t) = [Q_n(t) - x]^2 + [K_n(t) + x]^2$. Then choose

$$s_n^{(k)}(t) = \begin{cases} s_n^{(k)(\max)} & , Q_n(t) - K_n(t) \geq 2s_n^{(k)(\max)} \\ \operatorname{argmin}_{x\in\left\{I_K^+(t), I_K^-(t)\right\}} g_K(x,t) & , 0 < Q_n(t) - K_n(t) < 2s_n^{(k)(\max)} \\ 0 & , Q_n(t) - K_n(t) \leq 0 \end{cases} \tag{35}$$

Intuitively, the amount $s_n^{(k)}(t)$ is half of the difference between queues $Q_n(t)$ and $K_n(t)$ that does not exceed $s_n^{(k)(\max)}$.

Also each device n moves data from its input queue to its relay queue according to the *relay routing problem*

$$\begin{aligned} \text{Minimize} \quad & \left[Q_n(t) - s_n^{(j)}(t)\right]^2 + \left[J_n(t) + s_n^{(j)}(t)\right]^2. \quad (36) \\ \text{Subject to} \quad & s_n^{(j)}(t) \in \mathcal{S}_n^{(j)} \end{aligned}$$

Again, let $I_J^+(t) \triangleq \left\lceil \frac{Q_n(t)-J_n(t)}{2} \right\rceil$, $I_J^-(t) \triangleq \left\lfloor \frac{Q_n(t)-J_n(t)}{2} \right\rfloor$ and $g_J(x,t) = [Q_n(t) - x]^2 + [J_n(t) + x]^2$. Then choose

$$s_n^{(j)}(t) = \quad (37)$$
$$\begin{cases} s_n^{(j)(\max)} & , \ Q_n(t) - J_n(t) \geq 2 s_n^{(j)(\max)} \\ \arg\min_{x \in \{I_J^+(t), I_J^-(t)\}} g_J(x,t) & , \ 0 < Q_n(t) - J_n(t) < 2 s_n^{(j)(\max)} \\ 0 & , \ Q_n(t) - J_n(t) \leq 0 \end{cases}$$

An intuition of decision $s_n^{(j)}(t)$ is similar to the one from the case of $s_n^{(k)}(t)$. Note that the solutions from the quadratic policy are "smoother" as compared to the solutions from the max-weight policy that would choose "bang-bang" decisions of either 0 or $s_n^{(k)(\max)}$ for $s_n^{(k)}(t)$ (and 0 or $s_n^{(j)(\max)}$ for $s_n^{(j)}(t)$).

The *uplink allocation* problem determining uplink transmission of every node $n \in \mathcal{N}$ is

$$\begin{aligned} \text{Minimize} \quad & \textstyle\sum_{n \in \mathcal{N}} [K_n(t) - \mu_n(t)]^2 \quad (38) \\ \text{Subject to} \quad & \boldsymbol{\mu}(t) \in \mathcal{U}_{\boldsymbol{\eta}(t)}. \end{aligned}$$

This can be solved at the receiver station and is similar to (9), so the same intuition follows. If all uplink channels are orthogonal, the problem can be decomposed further to be solved at each device n by

$$\begin{aligned} \text{Minimize} \quad & [K_n(t) - \mu_n(t)]^2 \quad (39) \\ \text{Subject to} \quad & \mu_n(t) \in \mathcal{U}_{n,\boldsymbol{\eta}(t)}, \end{aligned}$$

where $\mathcal{U}_{n,\boldsymbol{\eta}(t)}$ is a feasible set of $\mu_n(t)$. An optimal uplink transmission rate is the closest rate in $\mathcal{U}_{n,\boldsymbol{\eta}(t)}$ to $K_n(t)$.

The *relay allocation* problem determining relay transmission of every node $n \in \mathcal{N}$ is

$$\begin{aligned} \text{Minimize} \quad & \textstyle\sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{N}} \Big\{ [K_n(t) + \gamma_{mn}(t)]^2 \\ & \qquad + [J_n(t) - \gamma_{nm}(t)]^2 \Big\} \quad (40) \\ \text{Subject to} \quad & \boldsymbol{\gamma}(t) \in \mathcal{A}_{\boldsymbol{\eta}(t)}. \end{aligned}$$

Intuitively, the decision $\gamma_{nm}(t)$ is made to balance the difference between queues $K_m(t)$ and $J_n(t)$ while transmission resource is shared among devices.

If channels are orthogonal so the sets have a product form, then the decisions are separable across transmission links (n,m) for $n \in \mathcal{N}, m \in \mathcal{N}$ as

$$\begin{aligned} \text{Minimize} \quad & [K_m(t) + \gamma_{nm}(t)]^2 + [J_n(t) - \gamma_{nm}(t)]^2 \quad (41) \\ \text{Subject to} \quad & \gamma_{nm}(t) \in \mathcal{A}_{nm,\boldsymbol{\eta}(t)}, \end{aligned}$$

where $\mathcal{A}_{nm,\boldsymbol{\eta}(t)}$ is a feasible set of $\gamma_{nm}(t)$. The closed form solution of this problem is

$$\gamma_{nm}(t) = \quad (42)$$
$$\begin{cases} \gamma_{nm}^{(\max)} & , \ J_n(t) - K_m(t) \geq 2\gamma_{nm}^{(\max)} \\ \text{argmin}_{x \in \{I_A^+(t), I_A^-(t)\}} g_A(x,t) & , \ 0 < J_n(t) - K_m(t) < 2\gamma_{nm}^{(\max)} \\ 0 & , \ J_n(t) - K_m(t) \leq 0 \end{cases}$$

where $I_A^+(t) \triangleq \text{argmin}_{a \in \mathcal{A}_{nm,\boldsymbol{\eta}(t)}} \left| a - \frac{J_n(t)-K_m(t)}{2} \right|$ and $I_A^-(t) \triangleq \text{argmin}_{a \in \mathcal{A}_{nm,\boldsymbol{\eta}(t)} - \{I_A^+(t)\}} \left| a - \frac{J_n(t)-K_m(t)}{2} \right|$ and $g_A(x,t) = [J_n(t) - x]^2 + [K_m(t) + x]^2$.

Again, due to the structure of $\mathcal{U}_{\boldsymbol{\eta}(t)}$ and $\mathcal{A}_{\boldsymbol{\eta}(t)}$, orthogonality assumption may not hold, but the subproblems are always fully separable as a result of the novel quadratic policy.

C. Algorithm

At every time slot t, our algorithm has two parts: device side and receiver-station side, which are summarized in the algorithms below.

Algorithm 3: Distributed format selection and routing

```
// Device side
foreach device n ∈ N do
    – Observe Q_n(t), K_n(t) and J_n(t)
    – Observe (r_n^(f)(t), a_n^(f)(t))|_{f∈F}
    – Select format f_n(t) according to (33)
    – Move data from Q_n(t) to K_n(t) and J_n(t) with
      s_n^(k)(act)(t), s_n^(j)(act)(t) satisfying (20)-(22) with
      values of s_n^(k)(t), s_n^(j)(t) calculated from (35) and
      (37).
end
```

Algorithm 4: Uplink and Relay resource allocation

```
// Receiver-station side
for receiver station 0 do
    – Observe (K_n(t), J_n(t))|_{n∈N}
    – Observe U_η(t) and A_η(t)
    – Signal devices n ∈ N to make uplink transmission
      μ(t) according to (38)
    – Signal devices n ∈ N to relay data γ(t) according
      to (40)
end
```

After these processes, queues $Q_n(t+1), K_n(t+1)$ and $J_n(t+1)$ are updated via (19), (23), (26).

VII. Stability and performance bounds

Compare the quadratic policy with any other policy. Let $(f_n(\tau), s_n^{(k)}(\tau), s_n^{(j)}(\tau))|_{n \in \mathcal{N}}, \boldsymbol{\mu}(\tau), \boldsymbol{\gamma}(\tau)$ be the decision variables from the quadratic policy, and $r_n(t) \triangleq r_n^{(f_n(t))}(t)$, $a_n(t) \triangleq a_n^{(f_n(t))}(t)$. Also let $(\hat{f}_n(\tau), \hat{s}_n^{(k)}(\tau), \hat{s}_n^{(j)}(\tau))|_{n \in \mathcal{N}}, \hat{\boldsymbol{\mu}}(\tau), \hat{\boldsymbol{\gamma}}(\tau)$ be decision variables from any other policy, and $\hat{r}_n(t) \triangleq r_n^{(\hat{f}_n(t))}(t), \hat{a}_n(t) \triangleq a_n^{(\hat{f}_n(t))}(t)$. Because the quadratic policy minimizes the right-hand-side of (31), the drift-plus-penalty expression under the quadratic policy satisfies:

$$L(\tau+1)-L(\tau)-Vy_0(\tau)$$
$$\leq \tfrac{1}{2}\sum_{n\in\mathcal{N}}\Big\{\big[Q_n(\tau)-s_n^{(k)}(\tau)\big]^2+\big[Q_n(\tau)-s_n^{(j)}(\tau)\big]^2+[Q_n(\tau)+a_n(\tau)]^2 +[K_n(\tau)-\mu_n(\tau)]^2+\big[K_n(\tau)+s_n^{(k)}(\tau)\big]^2+\sum_{m\in\mathcal{N}}[K_n(\tau)+\gamma_{mn}(\tau)]^2 +\sum_{m\in\mathcal{N}}[J_n(\tau)-\gamma_{nm}(\tau)]^2+[J_n(\tau)+s_n^{(j)}(\tau)]^2-2Vr_n(\tau)+D_n(\tau)\Big\} \quad (43)$$
$$\leq \tfrac{1}{2}\sum_{n\in\mathcal{N}}\Big\{\big[Q_n(\tau)-\hat{s}_n^{(k)}(\tau)\big]^2+\big[Q_n(\tau)-\hat{s}_n^{(j)}(\tau)\big]^2+[Q_n(\tau)+\hat{a}_n(\tau)]^2 +[K_n(\tau)-\hat{\mu}_n(\tau)]^2+\big[K_n(\tau)+\hat{s}_n^{(k)}(\tau)\big]^2+\sum_{m\in\mathcal{N}}[K_n(\tau)+\hat{\gamma}_{mn}(\tau)]^2 +\sum_{m\in\mathcal{N}}[J_n(\tau)-\hat{\gamma}_{nm}(\tau)]^2+[J_n(\tau)+\hat{s}_n^{(j)}(\tau)]^2-2V\hat{r}_n(\tau)+D_n(\tau)\Big\}. \quad (44)$$

where (43) is a restatement of (31). It follows that

$$\begin{aligned}L(\tau+1)-L(\tau)-Vy_0(\tau) &\leq \textstyle\sum_{n\in\mathcal{N}}\Big\{Q_n(\tau)\left[\hat{a}_n(\tau)-\hat{s}_n^{(k)}(\tau)-\hat{s}_n^{(j)}(\tau)\right]\\ &+K_n(\tau)\left[\hat{s}_n^{(k)}(\tau)+\textstyle\sum_{m\in\mathcal{N}}\hat{\gamma}_{mn}(\tau)-\hat{\mu}_n(\tau)\right]\\ &+J_n(\tau)\left[\hat{s}_n^{(j)}(\tau)-\textstyle\sum_{m\in\mathcal{N}}\hat{\gamma}_{nm}(\tau)\right]\\ &-V\hat{r}_n(\tau)\Big\}+E \end{aligned} \quad (45)$$

where E is a suitable constant that does not depend on V. In particular, it can be shown that:

$$\begin{aligned}E \triangleq \frac{1}{2}\textstyle\sum_{n\in\mathcal{N}}\Big\{&\left[s_n^{(k)(\max)}+s_n^{(j)(\max)}+a_n^{(\max)}\right]^2\\ &+\left[s_n^{(k)(\max)}+\mu_n^{(\max)}+\textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}^{(\max)}\right]^2\\ &+\left[s_n^{(j)(\max)}+\textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}^{(\max)}\right]^2\\ &+2(s_n^{(k)(\max)})^2+2(s_n^{(j)(\max)})^2+2(a_n^{(\max)})^2\\ &+2(\mu_n^{(\max)})^2+(\textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}^{(\max)})^2+(\textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}^{(\max)})^2\Big\}\end{aligned}$$

The derivations (43)–(45) show that applying the quadratic policy to the drift-plus-penalty expression leads to the bound (45) which is valid for every other control policy.

As discussed in Section V, $\omega(t)$ is i.i.d. over slots. Define an ω-only policy as one that makes a (possibly randomized) choice of decision variables based only on the observed $\omega(t)$. Then we customize an important theorem from [11].

Theorem 3: For any $\delta > 0$ there exists an ω-only policy that chooses all control variables $(f_n^*(t), s_n^{(k)*}(t), s_n^{(j)*}(t))|_{n\in\mathcal{N}}, \boldsymbol{\mu}^*(t), \boldsymbol{\gamma}^*(t)$ such that for all $n\in\mathcal{N}$:

$$\mathbb{E}\left[y_0^*(t)\right] \geq y_0^{(\text{opt})}-\delta \quad (46)$$
$$\mathbb{E}\left[a_n^*(t)-s_n^{(k)*}(t)-s_n^{(j)*}(t)\right]\leq\delta \quad (47)$$
$$\mathbb{E}\left[s_n^{(k)*}(t)+\textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}^*(t)-\mu_n^*(t)\right]\leq\delta \quad (48)$$
$$\mathbb{E}\left[s_n^{(j)*}(t)-\textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}^*(t)\right]\leq\delta \quad (49)$$

where $y_0^{(\text{opt})}$ is the optimal solution of the new problem defined in Section V-B. Also, $y_0^*(t)\triangleq\sum_{n\in\mathcal{N}}r_n^*(t)$ when $r_n^*(t)\triangleq r_n^{(f_n^*(t))}(t)$ and $a_n^*(t)\triangleq a_n^{(f_n^*(t))}(t)$.

We additionally assume all constraints of the network can be achieved with ϵ slackness:

Assumption 2: There are values $\epsilon>0$ and $0\leq y_0^{(\epsilon)}\leq y_0^{(\max)}$ and an ω-only policy choosing all control variables $(f_n^*(t), s_n^{(k)*}(t), s_n^{(j)*}(t))|_{n\in\mathcal{N}}, \boldsymbol{\mu}^*(t), \boldsymbol{\gamma}^*(t)$ that satisfies for all $n\in\mathcal{N}$:

$$\mathbb{E}\left[y_0^*(t)\right]=y_0^{(\epsilon)} \quad (50)$$
$$\mathbb{E}\left[a_n^*(t)-s_n^{(k)*}(t)-s_n^{(j)*}(t)\right]\leq-\epsilon \quad (51)$$
$$\mathbb{E}\left[s_n^{(k)*}(t)+\textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}^*(t)-\mu_n^*(t)\right]\leq-\epsilon \quad (52)$$
$$\mathbb{E}\left[s_n^{(j)*}(t)-\textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}^*(t)\right]\leq-\epsilon. \quad (53)$$

A. Performance analysis

Since our quadratic algorithm satisfies the bound (45), where the right-hand-side is in terms of any alternative policy $\left(\hat{f}_n(t), \hat{s}_n^{(k)}(t), \hat{s}_n^{(j)}(t)\right)\Big|_{n\in\mathcal{N}}, \hat{\boldsymbol{\mu}}(t), \hat{\boldsymbol{\gamma}}(t)$, it holds for any ω-only policy $\left(f_n^*(t), s_n^{(k)*}(t), s_n^{(j)*}(t)\right)\Big|_{n\in\mathcal{N}}, \boldsymbol{\mu}^*(t), \boldsymbol{\gamma}^*(t)$. Substituting an ω-only policy into (45) and taking expectations gives:

$$\begin{aligned}\mathbb{E}&\left[L(\tau+1)-L(\tau)-Vy_0(\tau)\right]\\ &\leq\textstyle\sum_{n\in\mathcal{N}}\Big\{\mathbb{E}\left[Q_n(\tau)\right]\mathbb{E}\left[a_n^*(\tau)-s_n^{(k)*}(\tau)-s_n^{(j)*}(\tau)\right]\\ &\quad+\mathbb{E}\left[K_n(\tau)\right]\mathbb{E}\left[s_n^{(k)*}(\tau)+\textstyle\sum_{m\in\mathcal{N}}\gamma_{mn}^*(\tau)-\mu_n^*(\tau)\right]\\ &\quad+\mathbb{E}\left[J_n(\tau)\right]\mathbb{E}\left[s_n^{(j)*}(\tau)-\textstyle\sum_{m\in\mathcal{N}}\gamma_{nm}^*(\tau)\right]\\ &\quad-V\mathbb{E}\left[r_n^*(\tau)\right]\Big\}+E\end{aligned} \quad (54)$$

where we have used the fact that queue backlogs on slot t are independent of the control decision variables of an ω-only policy on that slot.

Theorem 4: If Assumption 2 holds, then the time-averaged total quality of information $\bar{y}_0$ is within $O(1/V)$ of optimality under the quadratic policy, while the total queue backlog grows with $O(V)$.

Proof: Theorem 4 is proven by substituting the ω-only policies from Theorem 3 and Assumption 2 into the right-hand-side of (54). The analysis is almost identical to that given for the uplink problem and details are omitted for brevity. ∎

B. Deterministic bounds of queue lengths

Here we show that, in addition to the average queue size bounds derived in the previous subsection, our algorithm also yields deterministic worst-case queue size bounds. Practically, these bounds can be used to determined memory requirement of a system for a particular value of V.

For each device $n\in\mathcal{N}$, define β_n as the maximum possible value of the expression:

$$\frac{2Vr_n^{(f)}(t)-(a_n^{(f)}(t))^2}{2a_n^{(f)}(t)}$$

over all slots t and all formats $f\in\mathcal{F}$ for which $a_n^{(f)}(t)\neq 0$. Define:

$$Q_n^{(\max)}\triangleq\beta_n+a_n^{(\max)} \text{ for } n\in\mathcal{N}$$
$$K_n^{(\max)}\triangleq\max_{m\in\mathcal{N}}\left[Q_m^{(\max)}\right]+\sum_{m\in\mathcal{N}}\gamma_{mn}^{(\max)}+s_n^{(\max)}.$$

Theorem 5: Under the separable quadratic policy, for all devices $n \in \mathcal{N}$ and all slots $t \geq 0$, we have:

$$Q_n(t) \leq Q_n^{(\max)} \tag{55}$$

$$J_n(t) \leq Q_n^{(\max)} \tag{56}$$

$$K_n(t) \leq K_n^{(\max)} \tag{57}$$

provided that these inequalities hold at $t = 0$.

The bounds (55)–(57) are proven in the next subsections.

1) Input Queue: From the admission-control problem (33), if $(r_n(t), a_n(t)) = (0, 0)$, then the objective value of the problem is $Q_n(t)^2$. Therefore, device n only chooses $(r_n(t), a_n(t))$ such that $a_n(t) \neq 0$ when:

$$(Q_n(t) + a_n(t))^2 - 2Vr_n(t) \leq Q_n(t)^2$$

This is equivalent to:

$$2Q_n(t)a_n(t) + a_n(t)^2 - 2Vr_n(t) \leq 0$$

$$\begin{aligned} Q_n(t) &\leq \frac{2Vr_n(t) - a_n(t)^2}{2a_n(t)} \\ &\leq \beta_n \end{aligned}$$

This implies that $Q_n(t)$ can only increase when $Q_n(t) \leq \beta_n$, and it receives no new data otherwise. It follows that for all slots t:

$$0 \leq Q_n(t) \leq \beta_n + a_n^{(\max)}$$

provided that this holds for slot $t = 0$. This proves (55).

2) Relay Queue: Fix t and assume for each device $n \in \mathcal{N}$ that $J_n(t) \leq Q_n^{(\max)}$ for this slot t. From the closed form solution (37) and queue equation (23), there are three cases to consider.

i) When $Q_n(t) - J_n(t) \leq 0$, then $s_n^{(j)}(t) = 0$, and

$$\begin{aligned} J_n(t+1) &\leq \max\left[J_n(t) + s_n^{(j)}(t), 0\right] \\ &= J_n(t) \leq Q_n^{(\max)}. \end{aligned}$$

ii) When $Q_n(t) - J_n(t) \geq 2s_n^{(j)(\max)}$ (or $J_n(t) \leq Q_n(t) - 2s_n^{(j)(\max)}$), then $s_n^{(j)}(t) = s_n^{(j)(\max)}$, and

$$\begin{aligned} J_n(t+1) &\leq \max\left[J_n(t) + s_n^{(j)}(t), 0\right] \\ &\leq \max\left[Q_n(t) - s_n^{(j)(\max)}, 0\right] \\ &\leq Q_n(t) \leq Q_n^{(\max)}. \end{aligned}$$

iii) When $0 < Q_n(t) - J_n(t) < 2s_n^{(j)(\max)}$, then $s_n^{(j)}(t) \leq \left\lceil \frac{Q_n(t) - J_n(t)}{2} \right\rceil$, and

$$\begin{aligned} J_n(t+1) &\leq \max\left[J_n(t) + s_n^{(j)}(t), 0\right] \\ &\leq \max\left[\left\lceil \frac{Q_n(t) + J_n(t)}{2} \right\rceil, 0\right] \\ &\leq Q_n(t) \leq Q_n^{(\max)}. \end{aligned}$$

Thus, given that $J_n(0) \leq Q_n^{(\max)}$, $J_n(t) \leq Q_n^{(\max)}$ for all $t \geq 0$ by mathematical induction.

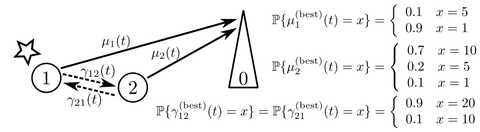

Fig. 6. Small network with independent channels and distributions

3) Uplink Queue: To provide a general upper bound for the uplink queue, we assume that all relay channels are orthogonal. This implies every device $n \in \mathcal{N}$ can transmit and receive relayed data simultanously.

Fix t and assume $K_n(t) \leq K_n^{(\max)}$ for this slot t. Then consider $K_n(t+1)$ from (26).

i) When $K_n(t) \geq \max_{m \in \mathcal{N}} \left[Q_m^{(\max)}\right]$, from (35) and (42), it follows that $s_n^{(k)}(t) = 0$ and $\gamma_{mn}(t) = 0$ for all $m \in \mathcal{N}$, so $K_n(t+1) \leq K_n(t) \leq K_n^{(\max)}$.

ii) When $K_n(t) < \max_{m \in \mathcal{N}} \left[Q_m^{(\max)}\right]$, then this queue may received data $s_n^{(k)}(t)$ and $\gamma_{mn}(t)$ for some $m \in \mathcal{N}$, so

$$\begin{aligned} K_n(t+1) &\leq \max\left[K_n(t) + s_n^{(k)}(t), 0\right] + \sum_{m \in \mathcal{N}} \gamma_{mn}(t) \\ &\leq K_n(t) + s_n^{(k)(\max)} + \sum_{m \in \mathcal{N}} \gamma_{mn}^{(\max)} \\ &\leq K_n^{(\max)}. \end{aligned}$$

Thus, given $K_n(0) \leq K_n^{(\max)}$, $K_n(t) \leq K_n^{(\max)}$ for all $t \geq 0$ by mathematical induction.

For comparison, using a technique in [11], the deterministic upper bounds of queues $Q_n^{(\text{mw})}(t), K_n^{(\text{mw})}(t)$, and $J_n^{(\text{mw})}(t)$ under the max-weight algorithm are respectively given without proofs due to space limit.

$$\begin{aligned} Q_n^{(\text{mw})(\max)} &\triangleq \max_{t, f \in \mathcal{F} | a_n^{(f)}(t) \neq 0} \left[\frac{Vr_n^{(f)}(t)}{a_n^{(f)}(t)}\right] + a_n^{(\max)} \\ J_n^{(\text{mw})(\max)} &\triangleq Q_n^{(\text{mw})(\max)} + s_n^{(j)(\max)} \\ K_n^{(\text{mw})(\max)} &\triangleq \max\left[Q_n^{(\text{mw})(\max)}, \left\{J_m^{(\text{mw})(\max)}\right\}_{m \in \mathcal{N}}\right] + s_n^{(k)(\max)} \\ &\quad + \sum_{m \in \mathcal{N}} \gamma_{mn}^{(\max)}. \end{aligned}$$

It is easy to see that the deterministic bounds from the quadratic policy are smaller than the bounds from the max-weight algorithm.

VIII. Simulation

Simulation under the proposed quadratic policy and the standard max-weight policy is performed over a small network of Fig. 6. This is the same network as considered in the simulation of the pure uplink problem (without relaying capabilities) from Fig. 2. To compare results with and without relaying, we use the same assumptions on event probability, formats, and uplink channel conditions as Section IV-B. Now, each device has another as its neighbor. We assume all uplink and relay channels are orthogonal.

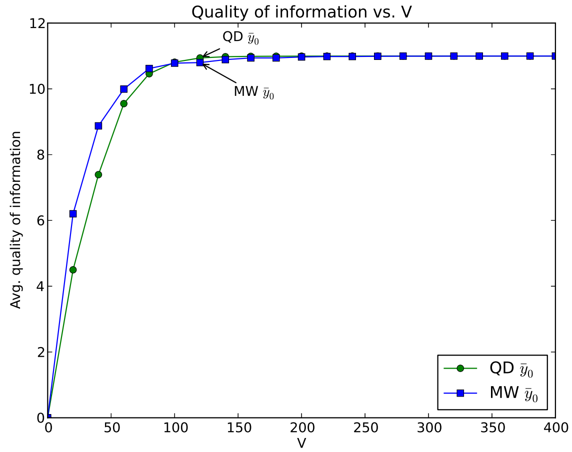

Fig. 7. Quality of information versus V under the quadratic (QD) and max-weight (MW) policies

For relay transmissions, the constraints are $\gamma_{12}(t) \in \{0, \ldots, \gamma_{12}^{(\text{best})}(\boldsymbol{\eta}(t))\}$ and $\gamma_{21}(t) \in \{0, \ldots, \gamma_{21}^{(\text{best})}(\boldsymbol{\eta}(t))\}$. Then set $s_n^{(k)(\max)} = s_n^{(j)(\max)} = 30$.

The simulation is performed according to the algorithm in Section VI-C. The time-averaged quality of information under the quadratic and max-weight policies are shown in Fig. 7. From the plot, the values of $\bar{y}_0$ under both policies converge to optimality following the $O(1/V)$ performance bound. The optimal time-averaged quality of information in this relaying system is significantly higher than that of the pure uplink system (compare Figs. 7 and Fig. 3a). Indeed, in this example, the time average utility increases by more than a factor of 3 when relaying is allowed. This gain is intuitive, because additional relay capability allows device 2 to relay device 1's information which has higher quality.

Fig. 8abc reveals queue lengths in the input, uplink, and relay queues of device 1 under the quadratic and max-weight policies. At the same V, the quadratic policy reduces queue lengths by a significant constant compared to the cases under the max-weight policy. The plot also shows the growth of queue lengths with parameter V, which follows the $O(V)$ bound of the queue length.

Fig. 9 shows that the quadratic policy can achieve near optimality with significantly smaller total system backlog compared to the case under the max-weight policy. This shows a significant advantage, which in turn affects buffer size and packet delay.

Another larger network shown in Fig. 10 is simulated to observe convergence of the proposed algorithm. The probability of event occurrence is 0.3. Channel distributions are configured in Fig. 10. The feasible set of formats is $\mathcal{F} = \{0, 1, 2, 3\}$ with constant options given by, for all $n \in \mathcal{N}$, $(r_n^{(0)}, a_n^{(0)}) = (0, 0)$, $(r_n^{(1)}, a_n^{(1)}) = (10, 10)$, $(r_n^{(2)}, a_n^{(2)}) = (15, 50)$, $(r_n^{(3)}, a_n^{(3)}) = (20, 100)$ whenever there is an event. For $V = 800$, the time-averaged quality of information is 25.00 after 10^6 time slots as shown in the upper plot of Fig. 11. The lower plot in Fig. 11 illustrates the early period of the simulation to illustrate convergence time.

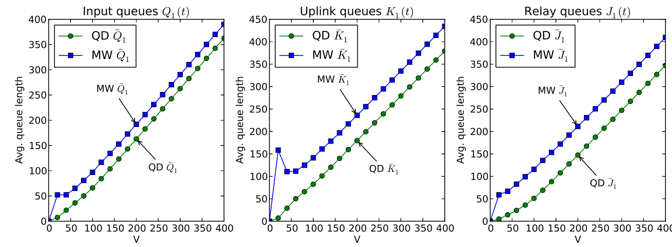

Fig. 8. Averaged backlog in device 1's queues versus V under the quadratic (QD) and max-weight (MW) policies

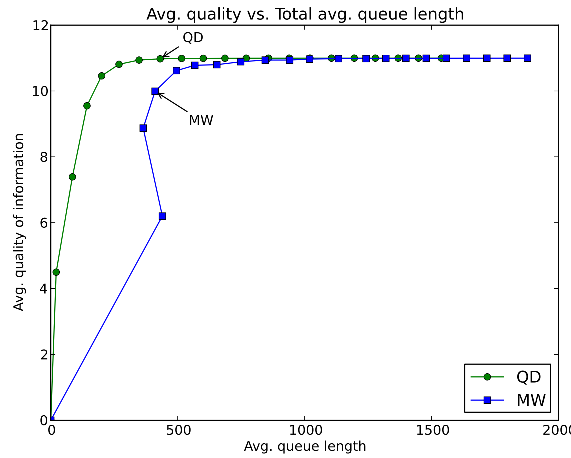

Fig. 9. The system obtains average quality of information while having average total queue length

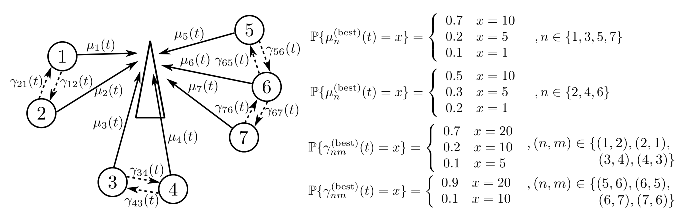

Fig. 10. Larger network with independent channels with distributions shown

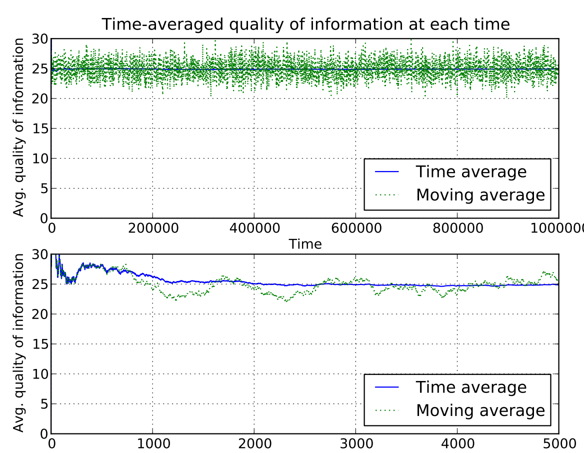

Fig. 11. Convergence of time-averaged quality of information. The interval of the moving average is 500 slots.

IX. CONCLUSION

We studied information quality maximization in a system with uplink and two-hop relaying capabilities. From Lyapunov optimization theory, we proposed a novel quadratic policy having a separable property, which leads to a distributed mechanism of format selection. In comparison to the standard max-weight policy, our policy leads to an algorithm that reduces queue backlog by a significant constant. Further, it was shown that device-to-device relaying can significantly increase total quality of information as compared to a network that does not allow relaying.

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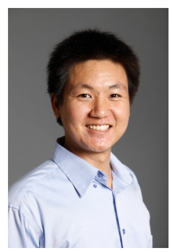
APPENDIX A
PROOF OF LEMMA 2

Let $x \geq 0$, $y_i \in \mathbb{R}$ and $z_j \in \mathbb{R}_+$ for $i \in \{1, 2, \ldots, Y\}$ and $j \in \{1, 2, \ldots, Z\}$. Assume that $|y_i| \leq y_i^{(\max)}$ and $z_j \leq z_j^{(\max)}$ for each i and j. Then:

$$
\begin{aligned}
&\left(\max\left[x+\sum_{i=1}^{Y} y_i, 0\right]+\sum_{j=1}^{Z} z_j\right)^2 - x^2 \\
&\leq \left(x+\sum_{i=1}^{Y} y_i\right)^2 + \left(\sum_{j=1}^{Z} z_j\right)^2 + 2\sum_{j=1}^{Z} z_j \left(x+\sum_{i=1}^{Y} |y_i|\right) - x^2 \\
&= 2x\sum_{i=1}^{Y} y_i + \left(\sum_{i=1}^{Y} y_i\right)^2 + \left(\sum_{j=1}^{Z} z_j\right)^2 + 2\sum_{j=1}^{Z} z_j \left(x+\sum_{i=1}^{Y} |y_i|\right) \\
&= \sum_{i=1}^{Y} (x+y_i)^2 + \sum_{j=1}^{Z} (x+z_j)^2 - \sum_{i=1}^{Y} y_i^2 - \sum_{j=1}^{Z} z_j^2 - (Y+Z)x^2 \\
&\quad + \left(\sum_{i=1}^{Y} y_i\right)^2 + \left(\sum_{j=1}^{Z} z_j\right)^2 + 2\left(\sum_{j=1}^{Z} z_j\right)\left(\sum_{i=1}^{Y} |y_i|\right) \\
&\leq \sum_{i=1}^{Y} (x+y_i)^2 + \sum_{j=1}^{Z} (x+z_j)^2 - (Y+Z)x^2 \\
&\quad + \left(\sum_{i=1}^{Y} |y_i| + \sum_{j=1}^{Z} z_j\right)^2 - \sum_{i=1}^{Y} y_i^2 - \sum_{j=1}^{Z} z_j^2
\end{aligned}
$$

and the sum of the final three terms is upper bounded by the D constant from Lemma 2.



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