Delay-Limited Cooperative Communication with Reliability Constraints in Wireless Networks

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***Abstract*—We investigate optimal resource allocation for delay-limited cooperative communication in time varying wireless networks. Motivated by real-time applications that have stringent delay constraints, we develop a dynamic cooperation strategy that makes optimal use of network resources to achieve a target outage probability (reliability) for each user subject to average power constraints. Using the technique of Lyapunov optimization, we first present a general framework to solve this problem and then derive quasi-closed form solutions for several cooperative protocols proposed in the literature. Unlike earlier works, our scheme does not require prior knowledge of the statistical description of the packet arrival, channel state and node mobility processes and can be implemented in an online fashion.**

***Keywords*—Cooperative Communication, Delay-Limited Communication, Mobile Ad-Hoc Networks, Reliability, Resource Allocation, Lyapunov Optimization**

I. Introduction

There is growing interest in the idea of utilizing cooperative communication [2]–[7] to improve the performance of wireless networks with time varying channels. The motivation comes from the work on MIMO systems [25] which shows that employing multiple antennas on a wireless node can offer substantial benefits. However, this may be infeasible in small-sized devices due to space limitations. Cooperative communication has been proposed as a means to achieve the benefits of traditional MIMO systems using *distributed single antenna* nodes. Much recent work in this area promises significant gains in several metrics of interest (such as diversity [4] [5], capacity [6]–[10], energy efficiency [11], [12], etc.) over conventional methods. We refer the interested reader to a recent comprehensive survey [2] and its references.

The main idea behind cooperative communication can be understood by considering a simple 2-hop network consisting of a source s, its destination d and a set of m relay nodes as shown in Fig. 1. Suppose s has a packet to send to d in timeslot t. The channel gains for all links in this network are shown

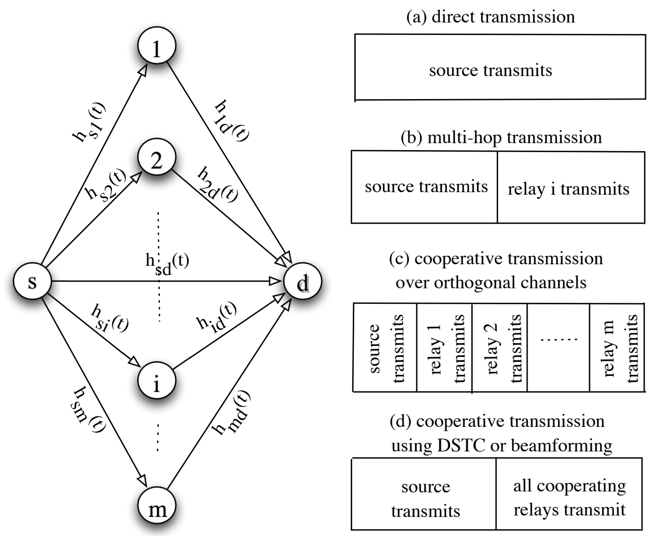

Fig. 1. Example 2-hop network with a source, destination and m relays. The time slot structures for different transmission strategies are also shown. Due to the half-duplex constraint, cooperative protocols need to operate in two phases. Hence, there is an inherent loss in the multiplexing gain under any such cooperative transmission strategy over direct transmission.

in the figure. In direct communication, s uses the full slot to transmit its packet to d over link $s-d$ as shown in Fig. 1(a). In conventional multi-hop relaying, s uses the first half of the slot to transmit its packet to a particular relay node i over link $s-i$ as shown in Fig. 1(b). If i can successfully decode the packet, it re-encodes and transmits it to d in the second half of the slot over link $i-d$. In both scenarios, to ensure reliable communication, the source and/or the relay must transmit at high power levels when the channel quality of any of the links involved is poor. However, note that due to the broadcast nature of wireless transmissions, other relay nodes may receive the signal from the transmission by s and can cooperatively relay it to d. The destination now receives multiple copies/signals and can use all of them jointly to decode the packet. Since these signals have been transmitted over independent paths, the probability that all of them have poor quality is significantly smaller. Cooperative communication protocols take advantage of this *spatial diversity gain* by making use of multiple relays for cooperative transmissions to increase reliability and/or reduce energy costs. This is different from traditional multi-hop relaying in which only one node is responsible for forwarding

This work was presented in part at the IEEE INFOCOM conference, Rio de Janeiro, Brazil, April 2009 [1].

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This material is supported in part by one or more of the following: the DARPA IT-MANET program grant W911NF-07-0028, the NSF Career grant CCF-0747525, and continuing through participation in the Network Science Collaborative Technology Alliance sponsored by the U.S. Army Research Laboratory.

at any time and in which the destination does not use multiple signals to decode a packet.

Because of the half-duplex nature of current wireless devices, a relay node cannot send and receive on the same channel simultaneously. Therefore, such cooperative communication protocols typically operate over a two phase slot structure as shown in Figs. 1(c) and 1(d). In the first phase, s transmits its packet to the set of relay nodes. In the second phase, a subset of these relays transmit their signals to d. Note that the destination may receive the source signal from the first phase as well. At the end of the second phase, the destination appropriately combines all of these received signals to decode the packet. The exact slot structure as well as the signals transmitted by the relays depend on the cooperative protocol being used[1]. For example, Fig. 1(c) shows the slot structure under a cooperative scheme that transmits over orthogonal channels. Specifically, the time slot is divided into $m + 1$ equal mini-slots. In phase one, the source transmits its packet in the first mini-slot. In the second phase, the relays transmit one after the other in their own mini-slots. Fig. 1(d) shows the slot structure under a cooperative scheme in which the cooperating relays use distributed space-time codes (DSTC) or a beamforming technique to transmit simultaneously in the second phase. It should be noted that due to this half-duplex constraint, there is an inherent loss in the multiplexing gain under any such cooperative transmission strategy over direct transmission. Therefore, it is important to develop algorithms that cooperate opportunistically.

In this work, we consider a mobile ad-hoc network with *delay-limited* traffic and cooperative communication. Many real-time applications (e.g., voice) have stringent delay constraints and fixed rate requirements. In slow fading environments (where decoding delay is of the order of the channel coherence time), it may not be possible to meet these delay constraints for every packet. However, these applications can often tolerate a certain fraction of lost packets or outages. A variety of techniques are used to combat fading and meet this target outage probability (including exploiting diversity, channel coding, ARQ, power control, etc.). Cooperative communication is a particularly attractive technique to improve reliability in such delay-limited scenarios since it can offer significant spatial diversity gains in addition to these techniques.

Much prior work on cooperative communication considers physical layer resource allocation for a static network, particularly in the case of a single source. Objectives such as minimizing sum power, minimizing outage probability, meeting a target SNR constraint, etc., are treated in this context [10]–[15]. We draw on this work in the development of *dynamic* resource allocation in a stochastic network with fading channels, node mobility, and random packet arrivals, where *opportunistic cooperation decisions* are required. Dynamic cooperation was also considered in the prior work [17] which investigates throughput optimality and queue stability in a multi-user network with static channels and randomly arriving traffic using the framework of Lyapunov drift. Our formulation is different and does not involve issues of queue stability. Rather, we consider a delay-limited scenario where each packet must either be transmitted in one slot, or dropped. This is similar to the concept of *delay-limited capacity* [18]. Also related to such scenarios is the notion of *minimum outage probability* [19]. These quantities are also investigated in the recent work [14] that considers a 3 node static network with Rayleigh fading and shows that opportunistic cooperation significantly improves the delay-limited capacity.

In this work, we use technique of Lyapunov optimization [23], [24] to develop a control algorithm that takes dynamic decisions for each new slot. Different from most work that applies this theory, our solution involves a 2-stage stochastic shortest path problem due to the cooperative relaying structure. This problem is non-convex and combinatorial in nature and does not admit closed form solutions in general. However, under several important and well known classes of physical layer cooperation models, we develop techniques for reducing the problem exactly to an m-stage set of convex programs. The convex programs themselves are shown to have quasi-closed form solutions and can be computed in real time for each slot, often involving simple water-filling strategies that also arise in related static optimization problems.

II. Basic Network Model

We consider a mobile ad-hoc network with delay-limited traffic over time varying fading channels. The network consists of a set $\mathcal{N}$ of nodes, all potentially mobile. All nodes are assumed to be within range of each other, and any node pair can communicate either through direct transmission or through a 2-phase cooperative transmission that makes use of other nodes as relays. The system operates in slotted time and the channel coefficient between nodes i and j in slot t is denoted by $h_{ij}(t)$. We assume a block fading model [25] for the channel coefficients so that their value remains fixed during a slot and changes from one slot to the other according to the distribution of the underlying fading and mobility processes.

For simplicity, we assume that the set $\mathcal{N}$ contains a single source-destination pair (s, d) and that all other nodes act simply as cooperative relays. This is similar to the single source assumption treated in [13]–[15] for static networks. We derive a dynamic cooperation strategy for this single source problem in Sec. IV that optimizes a weighted sum of reliability and power expenditure subject to individual reliability and average power constraints at the source and at all relays. This highlights the decisions involved from the perspective of a source node, and these decisions and the resulting solution structure are similar to the multiple source-destination pairs scenario operating under an orthogonal medium access scheme (such as TDMA or FDMA) studied later in Sec. VII. In the following, we denote the set of relay nodes by $\mathcal{R}$ and the set $\{s\} \cup \mathcal{R}$ by $\widehat{\mathcal{R}}$. All nodes $i \in \widehat{\mathcal{R}}$ have both long term average and instantaneous peak power constraints given by P_i^{avg} and P_i^{max} respectively.

We consider two models for the availability of the channel state information (CSI) at the source. The first is the *known channels, unknown statistics* model. Under this model, we assume that the channel gains between the source node and

[1] We consider several protocol examples in Sec. V.

its relay set and destination as well as the channel gains between the relays and the destination are known every slot. These could be obtained by sending pilot signals and receiving feedback. The exact description of CSI under this model depends on the cooperative communication protocol being used by the network. For example, when the source and relays transmit on orthogonal channels or make use of distributed space-time codes [4], [5], then the CSI information under this model can represent just the amplitude of the channel coefficients $|h_{ij}(t)|$. On the other hand, when a cooperative scheme such as beamforming is used [13], then the CSI information under this model represents the complete description of the fading coefficients that includes the phase information. We will present several examples of cooperative protocols in Sec. V where we highlight this distinction.

This model has been considered in prior works [13]–[16] on power allocation in static networks where, in addition to the current channel gains, a knowledge of the distribution governing the fading process is assumed. In our work, under this *known channels, unknown statistics* model, we do not assume any knowledge of the distributions governing the evolution of the channel states, mobility processes, or traffic. Thus, our algorithm and its optimality properties hold for a very general class of channel and mobility models that satisfy certain ergodicity requirements (to be made precise later).

The second model we consider is the *unknown channels, known statistics* model. In this case, we assume that the current set of potential relay nodes is known on each slot t, but the exact channel realizations between the source and these relays, and the relays and the destination, are unknown. Rather, we assume only that the *statistics* of the fading coefficients are known between the source and current relays, and the current relays and destination. However, we still do not require knowledge of the distributions governing the arriving traffic or the mobility pattern (which affects the set of relays we will see in future slots). This is in contrast to prior works that have considered resource allocation in the presence of partial CSI only for static networks.

For both models, we use $\mathcal{T}(t)$ to represent the collection of all channel state information known in slot t. For the known channels, unknown statistics model, when the source and relays transmit on orthogonal channels or make use of distributed space-time codes, $\mathcal{T}(t)$ represents the collection of amplitudes of current channel coefficients $|h_{ij}(t)|$ between the source and relays and relays and destination in slot t. When beamforming is used, then $\mathcal{T}(t)$ represents the complete description of channel coefficients that includes the phase information. For the unknown channels, known statistics model, $\mathcal{T}(t)$ represents the set of all nodes that are available on slot t for relaying and the distribution of the fading coefficients.

We assume that $\mathcal{T}(t)$ lies in a space of finite but arbitrarily large size and evolves according to an ergodic process with a well defined steady state distribution. This variation in channel state information affects the reliability and power expenditure associated with the direct and cooperative transmission modes that are discussed in Sec. II-B.

A. Example of Channel State Information Models

As an example of these models, suppose the nodes move in a cell-partitioned network according to a Markovian random walk (see also Fig. 2 in Sec. VIII on Simulations). Each slot, a node may decide to stay in its current cell or move to an adjacent cell according to the probability distribution governing the random walk. Suppose that each slot, the set of potential relays consists only of nodes in either the same cell or an adjacent cell of the source. Further, suppose the network uses a cooperative protocol that operates over orthogonal channels. Assume that the channel gains between nodes in the same cell are distributed according to a Rayleigh fading model with a particular mean and variance, while the gains for nodes in adjacent cells are Rayleigh distributed with a different mean and variance. Under the known channels, unknown statistics model, the $\mathcal{T}(t)$ information is the set of current gains $|h_{ij}(t)|$, and knowledge of the Rayleigh distribution and statistics (such as mean and variance) is not needed. Under the unknown channels, known statistics model, the $\mathcal{T}(t)$ information is the set of nodes currently in the same and adjacent cells of the source, and we assume we know that the fading distribution is Rayleigh, and we know the corresponding means and variances. However, neither model requires knowledge of the mobility model or the traffic rates.

B. Control Options

Suppose the slot size is normalized to integer slots $t \in \{0, 1, 2, \ldots, \}$. In each slot, the source s receives new packets for its destination d according to an i.i.d. Bernoulli process $A_s(t)$ of rate λ_s. Each packet is assumed to be R bits long and has a *strict* delay constraint of 1 slot. Thus, a packet not served within 1 slot of its arrival is dropped. Further, packets that are not successfully received by their destinations due to channel errors are not retransmitted. The source node has a minimum time-average reliability requirement specified by a fraction ρ_s which denotes the fraction of packets that were transmitted successfully. In any slot t, if source s has a new packet for transmission, it can use one of the following transmission modes (Fig. 1):

1) Transmit directly to d using the full slot
2) Transmit to d using traditional relaying over two hops
3) Transmit cooperatively with the set $\mathcal{R}$ of relay nodes using the two phase slot structure
4) Stay idle (so that the packet gets dropped)

We consider all of these transmission modes because, depending on the current channel conditions and energy costs in slot t, it might be better to choose one over the other. For example, due to the half-duplex constraint, direct transmission using the full slot might be preferable to cooperative transmission over two phases on slots when the source-destination link quality is good. Note that this is similar to the much studied framework of opportunistic transmission scheduling in time varying channels. Further, even in the special case of static channels, the optimal strategy may involve a mixture of these modes of operation to meet the target reliability and average power constraints.

Let $\mathcal{I}^\eta(t)$ denote the collective control action in slot t under some policy η that includes the choice of the transmission mode at the source, power allocations for the source and all relevant relays, and any additional physical layer choices such as modulation and coding. Specifically, we have:

$$\mathcal{I}^\eta(t) = [\text{mode choice}, \boldsymbol{P}^\eta(t), \text{other PHY layer choices}]$$

where the mode choice refers to one of the 4 transmission modes for the source, and where $\boldsymbol{P}^\eta(t)$ is the collection of $P_i^\eta(t)$ for all nodes $i \in \widehat{\mathcal{R}}$ with $P_i^\eta(t)$ representing the power allocation for node i in slot t. Note that $P_i^\eta(t) = 0$ for all i under transmission mode 4 (idle). If the source s chooses mode 1, we have $P_i^\eta(t) = 0$ for all relay nodes $i \in \mathcal{R}$, whereas if s chooses mode 2, we have $P_i^\eta(t) > 0$ for at most one relay $i \in \mathcal{R}$. Note that under any feasible policy η, $P_i^\eta(t)$ must satisfy the instantaneous peak power constraint every slot for all i. Also note that under the cooperative transmission option, the power allocation for the source node and the relays corresponds to the first and second phase respectively. Thus, the source is active in the first phase while the relays are active in the second phase. We denote the set of all valid power allocations by $\mathcal{P}$ and define $\mathcal{C}$ as the set of all valid control actions:

$$\mathcal{C} = \{1, 2, 3, 4\} \times \{\mathcal{P}\} \times \{\text{other PHY layer choices}\}$$

The success/failure outcome of the control action is represented by an indicator random variable $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ that depends on the current control action and channel state. Successful transmission of a packet is usually a complicated function of the transmission mode chosen, the associated power allocations and channel states, as well as physical layer details like modulation, coding/decoding scheme, etc. In this work, the particular physical layer actions are included in the $\mathcal{I}^\eta(t)$ decision variable. Specifically, given a control action $\mathcal{I}^\eta(t)$ and a channel state $\mathcal{T}(t)$, the outcome is defined as follows:

$$\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t)) \triangleq \begin{cases} 1 & \text{if a packet transmitted by } s \text{ in slot} \\ & t \text{ is successfully received by } d \\ 0 & \text{else} \end{cases} \tag{1}$$

Note that $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ is a random variable, and its conditional expectation given $(\mathcal{I}^\eta(t), \mathcal{T}(t))$ is equal to the success probability under the given physical layer channel model. Use of this abstract indicator variable allows a unified treatment that can include a variety of physical layer models. Under the known channels, unknown statistics model, $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ can be a determinisitic $0/1$ function based on the known channel state and control action. Specific examples for this model are considered in Sec. V. Under the unknown channels, known statistics model (where $\mathcal{T}(t)$ represents only the set of current possible relays and the fading statistics), we assume we know the value of $Pr[\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t)) = 1]$ under each possible control action $\mathcal{I}^\eta(t)$. This model is considered in Sec. VI. Under both models, we assume that explicit ACK/NACK information is received at the end of each slot, so that the source knows the value of $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$. For notational convenience, in the rest of the paper, we use $\Phi_s^\eta(t)$ instead of $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ noting that the dependence on $(\mathcal{I}^\eta(t), \mathcal{T}(t))$ is implicit.

C. Discussion of Basic Model

The basic model described above extends prior work on 2-phase cooperation in static networks to a mobile environment, and treats the important example scenario where a team of nodes move in a tight cluster but with possible variation in the relative locations of nodes within the cluster. We note that our model and results are applicable to the special case of a static network as well. Another example scenario captured by our model is an OFDMA-based cellular network with multiple users that have both inter-cell and intra-cell mobility. In each slot, a set of transmitters is determined in each orthogonal channel (for example, based on a predetermined TDMA schedule, or dynamically chosen by the base station). The remaining nodes can potentially act as cooperative relays in that slot.

The basic model treats scenarios in which a source node can transmit to its destination, possibly with the help of multiple relay nodes, in 2 stages. While this is a simplifying assumption, the framework developed here can be applied to more general scenarios in which, in a single slot, cooperative relaying over K stages is performed (for some $K > 2$) using multi-hop cooperative techniques (e.g., [20], [21]).

III. Control Objective

Let α_s and β_i for $i \in \widehat{\mathcal{R}}$ be a collection of non-negative weights. Then our objective is to design a policy η that solves the following *stochastic optimization problem*:

$$\begin{aligned}
\text{Maximize:} \quad & \alpha_s \bar{r}_s^\eta - \sum_{i \in \widehat{\mathcal{R}}} \beta_i \bar{e}_i^\eta \\
\text{Subject to:} \quad & \bar{r}_s^\eta \geq \rho_s \lambda_s \\
& \bar{e}_i^\eta \leq P_i^{avg} \ \forall\, i \in \widehat{\mathcal{R}} \\
& 0 \leq P_i^\eta(t) \leq P_i^{max} \ \forall\, i \in \widehat{\mathcal{R}},\ \forall t \\
& \mathcal{I}^\eta(t) \in \mathcal{C} \ \forall t
\end{aligned} \tag{2}$$

where $\bar{r}_s^\eta$ is the time average reliability for source s under policy η and is defined as:

$$\bar{r}_s^\eta \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{\Phi_s^\eta(\tau)\right\} \tag{3}$$

and $\bar{e}_i^\eta$ is the time average power usage of node i under η:

$$\bar{e}_i^\eta \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{P_i^\eta(\tau)\right\} \tag{4}$$

Here, the expectation is with respect to the possibly randomized control actions that policy η might take. The α_s and β_i weights allow us to consider several different objectives. For example, setting $\alpha_s = 0$ and $\beta_i = 1$ for all i reduces (2) to the problem of minimizing the average sum power expenditure subject to minimum reliability and average power constraints. This objective can be important in the multiple source scenario when the resources of the relays must be shared across many users. Setting all of these weights to 0 reduces (2) to a feasibility problem where the objective is to provide minimum reliability guarantees subject to average power constraints.

Problem (2) is similar to the general stochastic utility maximization problem presented in [23], [24]. Suppose (2) is feasible and let r_s^* and $e_i^*\ \forall i \in \widehat{\mathcal{R}}$ denote the optimal value of the objective function, potentially achieved by some arbitrary policy. Using the techniques developed in [22], it can be shown that it is sufficient to consider only the class of stationary, randomized policies that take control decisions purely as a (possibly random) function of the channel state information $\mathcal{T}(t)$ every slot to solve (2). However, computing the optimal stationary, randomized policy explicitly can be challenging and often impractical as it requires knowledge of arrival distributions, channel probabilities and mobility patterns in advance. Further, as pointed out earlier, even in the special case of a static channel, the optimal strategy may involve a mixture of direct transmission, multi-hop, and cooperative modes of operation, and the relaying modes must select different relay sets over time to achieve the optimal time average mixture.

However, the technique of Lyapunov optimization [23], [24] can be used to construct an alternate dynamic policy that overcomes these challenges and is provably optimal. Unlike the stationary, randomized policy, this policy does not need to be computed beforehand and can be implemented in an online fashion. In the known channels model, it does not need a-priori statistics of the traffic, channels, or mobility. In the unknown channels model, it does not need a-priori statistics of the traffic or mobility. We present this policy in the next section.

IV. Optimal Control Algorithm

In this section, we present a dynamic control algorithm that achieves the optimal solution r_s^* and $e_i^*\ \forall i \in \widehat{\mathcal{R}}$ to the stochastic optimization problem presented earlier. This algorithm is similar in spirit to the backpressure algorithms proposed in [23], [24] for problems of throughput and energy optimal networking in time varying wireless ad-hoc networks.

The algorithm makes use of a "reliability queue" $Z_s(t)$ for source s. Specifically, let $Z_s(t)$ be a value that is initialized to zero (so that $Z_s(0) = 0$), and that is updated at the end of every slot t according to the following equation:

$$Z_s(t+1) = \max[Z_s(t) - \Phi_s(t), 0] + \rho_s A_s(t) \tag{5}$$

where $A_s(t)$ is the number of arrivals to source s on slot t (being either 0 or 1), and $\Phi_s(t)$ is 1 if and only if a packet that arrived was successfully delivered (recall that ACK/NACK information gives the value of $\Phi_s(t)$ at the end of every slot t). Additionally, it also uses the following virtual power queues $\forall i \in \widehat{\mathcal{R}}$:

$$X_i(t+1) = \max[X_i(t) - P_i^{avg}, 0] + P_i(t) \tag{6}$$

All these queues are also initialized to 0 and updated at the end of every slot t according to the equation above. We note that these queues are virtual in that they do not represent any real backlog of data packets. Rather, they facilitate the control algorithm in achieving the time average reliability and energy constraints of (2) as follows. If a policy η stabilizes (5), then we must have that its service rate is no smaller than the input rate, i.e.,

$$\bar{r}_s^\eta = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{\Phi_s^\eta(\tau)\right\} \geq \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{\rho_s A_s(\tau)\right\} = \rho_s \lambda_s$$

Similarly, stabilizing (6) yields the following:

$$\bar{e}_i^\eta = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{P_i^\eta(\tau)\right\} \leq P_i^{avg}$$

where we have used definitions (3), (4). This technique of turning time-average constraints into queueing stability problems was first used in [22].

To stabilize these virtual queues and optimize the objective function in (2), the algorithm operates as follows. Let $\boldsymbol{Q}(t) = (Z_s(t), X_i(t))\ \forall i \in \widehat{\mathcal{R}}$ denote the collection of these queues in timeslot t. Every slot t, given $\boldsymbol{Q}(t)$ and the current channel state $\mathcal{T}(t)$, it chooses a control action $\mathcal{I}^*(t)$ that minimizes the following stochastic metric (for a given control parameter $V \geq 0$):

$$\begin{aligned}
\text{Minimize:} \quad & (X_s(t) + V\beta_s)\mathbb{E}\left\{P_s(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\} \\
& + \sum_{i\in\mathcal{R}} (X_i(t) + V\beta_i)\mathbb{E}\left\{P_i(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\} \\
& - (Z_s(t) + V\alpha_s)\mathbb{E}\left\{\Phi_s(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\} \\
\text{Subject to:} \quad & 0 \leq P_i(t) \leq P_i^{max}\ \forall i \in \widehat{\mathcal{R}} \\
& \mathcal{I}(t) \in \mathcal{C}
\end{aligned} \tag{7}$$

After implementing $\mathcal{I}^*(t)$ and observing the outcome, the virtual queues are updated using (5), (6). Recall that there are no actual queues in the system. Our algorithm enforces a strict 1-slot delay constraint so that $\Phi_s(t) = 0$ if the packet is not successfully delivered after 1 slot. The virtual queues $X_i(t), Z_s(t)$ are maintained only in software and act as known weights in the optimization (7) that guide decisions towards achieving our time average power and reliability goals. The control action $\mathcal{I}^*(t)$ that optimizes (7) affects the powers $P_i(t)$ allocated and the $\Phi_s(t)$ value according to (1).

The above optimization is a 2-stage *stochastic shortest path* problem [26] where the two stages correspond to the two phases of the underlying cooperative protocol. Specifically, when s decides to use the option of transmitting cooperatively, the cost incurred in the first stage is given by the first term $(X_s(t) + V\beta_s)\mathbb{E}\left\{P_s(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\}$. The cost incurred during the second stage is given by $\sum_{i\in\mathcal{R}}(X_i(t) + V\beta_i)\mathbb{E}\left\{P_i(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\}$ and at the end of this stage, we get a reward of $(Z_s(t) + V\alpha_s)\mathbb{E}\left\{\Phi_s(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\}$. The transmission outcome $\Phi_s(t)$ depends on the power allocation decisions in *both* phases which makes this problem different from greedy strategies (e.g., [17], [22]). In order to determine the optimal strategy in slot t, the source s computes the minimum cost of (7) for all transmission modes described earlier and chooses one with the least cost.

Note that this problem is unconstrained since the long term time average reliability and power constraints do not appear explicitly as in the original problem. These are implicitly captured by the virtual queue values. Further, its solution uses the value of the *current* channel state $\mathcal{T}(t)$ and does not require knowledge of the statistics that govern the evolution of the channel state process. Thus, the control strategy involves implementing the solution to the sequence of such unconstrained problems every slot and updating the queue values according to (5), (6). Assuming i.i.d. $\mathcal{T}(t)$ states, the following theorem characterizes the performance of this dynamic control algorithm A similar statement can be made for more general Markov modulated $\mathcal{T}(t)$ using the techniques of [23], [24]. For simplicity, here we consider the i.i.d. case.

Theorem 1: (Algorithm Performance) Suppose all queues are initialized to 0. Then, implementing the dynamic algorithm (7) every slot stabilizes all queues, thereby satisfying the minimum reliability and time-average power constraints, and guarantees the following performance bounds (for some $\epsilon > 0$ that depends on the slackness of the feasibility constraints):

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\left\{Z_s(\tau)\right\} \leq \frac{B + V(\alpha_s + \sum_{i\in\widehat{\mathcal{R}}} \beta_i P_i^{max})}{\epsilon}$$

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \sum_{i\in\widehat{\mathcal{R}}} \mathbb{E}\left\{X_i(\tau)\right\} \leq \frac{B + V(\alpha_s + \sum_{i\in\widehat{\mathcal{R}}} \beta_i P_i^{max})}{\epsilon}$$

Further, the time average utility achieved for any $V \geq 0$ satisfies:

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\left\{\alpha_s \Phi_s(\tau) - \sum_{i\in\widehat{\mathcal{R}}} \beta_i P_i(\tau)\right\} \geq \zeta^* - \frac{B}{V}$$

where $\zeta^* \triangleq \alpha_s r_s^* - \sum_{i\in\widehat{\mathcal{R}}} \beta_i e_i^*$ is the optimal value of the objective in (2) and $B \triangleq \frac{1+\lambda_s^2\rho_s^2 + \sum_{i\in\widehat{\mathcal{R}}} (P_i^{avg})^2 + (P_i^{max})^2}{2}$.

Proof: Appendix A. □

Thus, one can get within $O(1/V)$ of the optimal values by increasing V at the cost of an $O(V)$ increase in the virtual queue backlogs. The size of these queues affects the time required for the time average values to converge to the desired performance.

In the following sections, we investigate the basic 2-stage resource allocation problem (7) in detail and present solutions for two widely studied classes of cooperative protocols proposed in the literature: Decode-and-Forward (DF) and Amplify-and-Forward (AF) [4], [5]. These protocols differ in the way the transmitted signal from the first phase is processed by the cooperating relays. In DF, a relay fully decodes the signal. If the packet is received correctly, it is re-encoded and transmitted in the second phase. In AF, a relay simply retransmits a scaled version of the received analog signal. We refer to [4], [5] for an introduction to the working of these protocols and to [13] for derivations of expressions for the mutual information achieved by them.

Let $m = |\mathcal{R}|$. In the following, we assume a Gaussian channel model with a total bandwidth W and unit noise power per dimension. We use the information theoretic definition of a transmission failure (an outage event) as discussed in [18], [19]. Here, an outage occurs when the total instantaneous mutual information is smaller than the rate R at which data is being transmitted.

We first consider the known channels, unknown statistics model for CSI. In this scenario, (7) becomes a 2-stage *deterministic shortest path problem* because the outcome $\Phi_s(t)$ due to any control decision and its power allocation can be computed beforehand. Specifically, $\Phi_s(t) = 1$ when the resulting total mutual information exceeds R and $\Phi_s(t) = 0$ otherwise. Further, this outcome is a function of control actions taken over two stages when cooperative transmission is used. This resulting problem is combinatorial and non-convex and does not admit closed-form solutions in general. However, for these protocols, we can reduce it to a set of simpler convex programs for which we can derive quasi-closed form solutions. Then in Sec. VI, we consider the case when only the statistics of the channel gains are known. In this case, the outcome $\Phi_s(t)$ is random function of the control actions (taken over the two stages in case of cooperative transmission) and (7) becomes a 2-stage *stochastic dynamic program*. While standard dynamic programming techniques can be used to compute the optimal solution, they are typically computationally intensive. Therefore, for this case, we present a Monte Carlo simulation based technique to efficiently solve the resulting dynamic program.

V. Known Channels, Unknown Statistics

Recall that in order to determine the optimal control action in any slot t, we must choose between the four modes of operation as discussed in Sec. II: (1) direct transmission, (2) multi-hop relay, (3) cooperative, and (4) idle. Let $c_i(t)$ and $I_i(t)$ denote the optimal cost of the metric (7), and the corresponding action that achieves that metric, assuming that mode $i \in \{1, 2, 3, 4\}$ is chosen in slot t. Every slot, the algorithm computes $c_i(t)$ and $I_i(t)$ for each mode and then implements the mode i and the resulting action $I_i(t)$ that minimizes cost. Note that the cost $c_4(t)$ for the idle mode is trivially 0.

The minimum cost for direct transmission can be computed as follows. When the source transmits directly, we have $P_i(t) = 0 \ \forall i \in \mathcal{R}$. The minimum cost $c_1(t)$ associated with a *successful* direct transmission ($\Phi_s(t) = 1$) can be obtained by solving the following convex problem[2]:

$$\begin{aligned} \text{Minimize:} \quad & \Big(X_s(t) + V\beta_s\Big)P_s(t) - Z_s(t) - V\alpha_s \\ \text{Subject to:} \quad & W\log_2\Big(1 + \frac{P_s(t)}{W}|h_{sd}(t)|^2\Big) \geq R \\ & 0 \leq P_s(t) \leq P_s^{max} \end{aligned} \tag{8}$$

where the constraint $W\log\Big(1 + \frac{P_s(t)}{W}|h_{sd}(t)|^2\Big) \geq R$ represents the fact that to get $\Phi_s(t) = 1$, the mutual information must exceed R. It is easy to see that if there is a feasible solution to the above, then for minimum cost, this

[2]Note that the term $-Z_s(t) - V\alpha_s$ in the objective is a constant in any given slot and does not affect the solution. However, we keep it to compare the net cost between all modes of operation.

constraint must be met with equality. Using this, the minimum cost corresponding to the direct transmission mode is given by: $\Big(X_s(t) + V\beta_s\Big)P_s^{dir}(t) - Z_s(t) - V\alpha_s$ if $P_s^{dir}(t) = \frac{W}{|h_{sd}(t)|^2}(2^{R/W} - 1) \leq P_s^{max}$. Otherwise, direct transmission is infeasible and so we set $c_1(t) = +\infty$. In this case, direct transmission will not be considered as the idle mode cost $c_4(t) = 0$ is strictly better, but we must also compare with the costs $c_2(t)$ and $c_3(t)$.

To compute the minimum cost $c_2(t)$ associated with multi-hop transmission, note that in this case, the slot is divided into two parts (Fig. 1(b)) and $P_i(t) > 0$ for at most one $i \in \mathcal{R}$. This strategy is a special case of the Regenerative DF protocol over orthogonal channels (to be discussed next) that uses only 1 relay and in which the destination does not use signals received from the first stage for decoding. Therefore, the optimal cost for this can be calculated using the procedure for the Regenerative DF case by imposing the single relay constraint and setting $h_{sd}(t) = 0$.

Below we present the computation of the minimum cost $c_3(t)$ for the cooperative transmission mode under several protocols. We first consider examples of cooperative schemes that either operate over orthogonal channels or make use of space-time codes. In this setting, as discussed in Sec. II, the CSI information $\mathcal{T}(t)$ refers to the amplitude of the channel coefficients $|h_{ij}(t)|$. Then, in Sec. V-F we consider a beamforming based scheme that requires knowledge of *both* amplitude and phase information.

In what follows, we drop the time subscript (t) for notational convenience. Also, we use $\log$ to mean $\log_2$ throughout the rest of the paper.

A. Regenerative DF, Orthogonal Channels

Here, the source and relays are each assigned an orthogonal channel of equal size. An example slot structure is shown in Fig. 1(c) in which the entire slot is divided into $m + 1$ equal mini-slots. In the first phase of the protocol, s transmits the packet in its slot using power P_s. In the second phase, a subset $\mathcal{U} \subset \mathcal{R}$ of relays that were successful in reliably decoding the packet, re-encode it using the *same* code book and transmit to the destination on their channels with power P_i (where $i \in \mathcal{U}$). Given such a set $\mathcal{U}$, the total mutual information under this protocol is given by [13]:

$$\frac{W}{m} \log\Big(1 + \frac{mP_s}{W}|h_{sd}|^2 + \sum_{i\in\mathcal{U}} \frac{mP_i}{W}|h_{id}|^2\Big)$$

This is derived by assuming that the receiver uses Maximal Ratio Combining to process the signals. As seen in the expression for the mutual information, such an orthogonal structure increases the SNR, but utilizes only a fraction of the available degrees of freedom leading to reduced multiplexing gain.

Define binary variables x_i to be 1 if relay i can reliably decode the packet after the first stage and 0 else. Then, for this protocol, (7) is equivalent to the following optimization problem:

$$\begin{aligned}
\text{Minimize:}\ & (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{R}}(X_i + V\beta_i)P_i - Z_s - V\alpha_s \\
\text{Subject to:}\ & \frac{W}{m}\log\Big(1 + \frac{mP_s}{W}|h_{sd}|^2 + \sum_{i\in\mathcal{R}} x_i \frac{mP_i}{W}|h_{id}|^2\Big) \geq R \\
& \frac{W}{m}\log\Big(1 + \frac{mP_s}{W}|h_{si}|^2\Big) \geq x_i R \\
& 0 \leq P_s \leq P_s^{max} \\
& 0 \leq P_i \leq P_i^{max}, x_i \in \{0,1\}\ \forall i \in \mathcal{R}
\end{aligned} \tag{9}$$

The variables x_i capture the requirement that a relay can cooperatively transmit in the second stage only if it was successful in reliably decoding the packet using the first stage transmission. A similar setup is considered in [13] but it treats the limiting case when W goes to infinity. Because of the integer constraints on x_i, (9) is non-convex. However, we can exploit the structure of this protocol to reduce the above to a set of $m + 1$ subproblems as follows. We first order the relays in decreasing order of their $|h_{si}|^2$ values. Define $\mathcal{U}_k$ as the set that contains the first k (where $0 \leq k \leq m$) relays from this ordering. Let $P_s^{\mathcal{U}_k}$ denote the minimum source power required to ensure that all relays in $\mathcal{U}_k$ can reliably decode the packet after the first stage. We note that for all values of P_s in the range $(P_s^{\mathcal{U}_k}, P_s^{\mathcal{U}_{k+1}})$, the relay set that can reliably decode remains the same, i.e., $\mathcal{U}_k$. Thus, we need to consider only $m+1$ subproblems, one for each $\mathcal{U}_k$. The subproblem for any set $\mathcal{U}_k$ is given by:

$$\begin{aligned}
\text{Minimize:}\ & (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{U}_k}(X_i + V\beta_i)P_i - Z_s - V\alpha_s \\
\text{Subject to:}\ & \frac{W}{m}\log\Big(1 + \frac{mP_s}{W}|h_{sd}|^2 + \sum_{i\in\mathcal{U}_k} \frac{mP_i}{W}|h_{id}|^2\Big) \geq R \\
& P_s^{\mathcal{U}_k} \leq P_s \leq P_s^{max} \\
& 0 \leq P_i \leq P_i^{max} \qquad \forall i \in \mathcal{U}_k
\end{aligned} \tag{10}$$

This can easily be expressed as the following LP:

$$\begin{aligned}
\text{Minimize:}\ & (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{U}_k}(X_i + V\beta_i)P_i - Z_s - V\alpha_s \\
\text{Subject to:}\ & P_s|h_{sd}|^2 + \sum_{i\in\mathcal{U}_k} P_i|h_{id}|^2 \geq \theta \\
& P_s^{\mathcal{U}_k} \leq P_s \leq P_s^{max} \\
& 0 \leq P_i \leq P_i^{max} \qquad \forall i \in \mathcal{U}_k
\end{aligned} \tag{11}$$

where $\theta = \frac{W}{m}(2^{Rm/W} - 1)$. The solution to the LP above has a greedy structure where we start by allocating increasing power to the nodes (including s) in decreasing order of the value of $\frac{|h_{id}|^2}{(X_i + V\beta_i)}$ (where $i \in \mathcal{U}_k \cup \{s\}$) till any constraint is met.

Therefore, for this protocol, the optimal solution to finding the cost $c_3(t)$ associated with the cooperative transmission mode in (7) can be computed by solving (11) for each $\mathcal{U}_k$ and picking the one with the least cost. It is interesting to note that if we impose a constraint on the sum total power of the relays instead of individual node constraints, then due to the greedy nature of the solution to (11), it is optimal to select at most 1 relay for cooperation. Specifically, this relay is the one that has the highest value of $\frac{|h_{id}|^2}{(X_i+V\beta_i)}$.

B. Non-Regenerative DF, Orthogonal Channels

This protocol is similar to Regenerative DF protocol discussed in Sec. V-A. The only difference is that here, in the second stage, the subset $\mathcal{U} \subset \mathcal{R}$ relays that were successful in reliably decoding the packet re-encode it using *independent* code books. In this case, the total mutual information is given by [13]:

$$\frac{W}{m}\log\left(1+\frac{mP_s}{W}|h_{sd}|^2\right)+\sum_{i\in\mathcal{R}}\frac{W}{m}\log\left(1+x_i\frac{mP_i}{W}|h_{id}|^2\right)$$

Using the same definition of binary variables x_i as in Sec.V-A, we can express (7) for this protocol as an optimization problem that resembles (9). Similar to the Regenerative DF over orthogonal channels case, we can then reduce this to a set of $m+1$ subproblems, one for each $\mathcal{U}_k$. The subproblem for set $\mathcal{U}_k$ is given by:

$$\begin{aligned}
&\text{Minimize: } (X_s+V\beta_s)P_s+\sum_{i\in\mathcal{U}_k}(X_i+V\beta_i)P_i-Z_s-V\alpha_s\\
&\text{Subject to:}\\
&\log\left(1+\frac{mP_s}{W}|h_{sd}|^2\right)+\sum_{i\in\mathcal{U}_k}\log\left(1+\frac{mP_i}{W}|h_{id}|^2\right)\geq\frac{mR}{W}\\
&P_s^{\mathcal{U}_k}\leq P_s\leq P^{max}\\
&0\leq P_i\leq P^{max}\qquad \forall i\in\mathcal{U}_k
\end{aligned}\tag{12}$$

The above problem is convex and we can use the KKT conditions to get the optimal solution (see Appendix B for details). Define $[x]_0^{P^{max}}\triangleq\min[\max(x,0),P^{max}]$. Then the solution to the subproblem for set $\mathcal{U}_k$ is given by:

$$\begin{aligned}
P_s^*(\mathcal{U}_k)&=\left[\frac{\nu^*}{X_s+V\beta_s}-\frac{W}{m|h_{sd}|^2}\right]_{P_s^{\mathcal{U}_k}}^{P_s^{max}}\\
P_i^*(\mathcal{U}_k)&=\left[\frac{\nu^*}{X_i+V\beta_i}-\frac{W}{m|h_{id}|^2}\right]_0^{P_i^{max}}\ \forall i\in\mathcal{U}_k
\end{aligned}\tag{13}$$

where $\nu^*\geq 0$ is chosen so that the total mutual information constraint is met with equality. Therefore, the optimal solution for the cost $c_3(t)$ in (7) for this protocol can be computed by solving (13) for each $\mathcal{U}_k$ and picking one with the least cost. We note that the solution above has a water-filling type structure that is typical of related resource allocation problems in static settings.

C. AF, Orthogonal Channels

In this protocol, the source and relays are again assigned an orthogonal channel of equal size. An example slot structure is shown in Fig. 1(c). However, instead of trying to decode the packet, the relays amplify and forward the received signal from the first stage. The total mutual information under this protocol is given by [13] [15]:

$$\frac{W}{m}\log\left(1+\frac{mP_s}{W}\left(|h_{sd}|^2+\sum_{i\in\mathcal{R}}\psi_i\right)\right)$$

where $\psi_i\triangleq\frac{P_i|h_{si}|^2|h_{id}|^2}{P_s|h_{si}|^2+P_i|h_{id}|^2+\frac{W}{m}}$. Using this, we can express (7) for this model as follows.

$$\begin{aligned}
&\text{Minimize: } (X_s+V\beta_s)P_s+\sum_{i\in\mathcal{R}}(X_i+V\beta_i)P_i-Z_s-V\alpha_s\\
&\text{Subject to: } \frac{W}{m}\log\left(1+\frac{mP_s}{W}\left(|h_{sd}|^2+\sum_{i\in\mathcal{R}}\psi_i\right)\right)\geq R\\
&\qquad 0\leq P_s\leq P_s^{max}\\
&\qquad 0\leq P_i\leq P_i^{max}\ \forall i\in\mathcal{R}
\end{aligned}\tag{14}$$

This problem is non-convex. However, if we fix the source power P_s, then it becomes convex in the other variables. This reduction has been used in [15] as well, although it considers a static scenario with the objective of minimizing instantaneous outage probability. After fixing P_s, we can compute the optimal relay powers for this value of P_s by solving the following:

$$\begin{aligned}
\text{Minimize:}\quad &\sum_{i\in\mathcal{R}}(X_i+V\beta_i)P_i-Z_s-V\alpha_s\\
\text{Subject to:}\quad &P_s|h_{sd}|^2+\sum_{i\in\mathcal{R}}P_s\psi_i\geq\theta\\
&0\leq P_i\leq P_i^{max}\qquad\forall i\in\mathcal{R}
\end{aligned}\tag{15}$$

where $\theta=\frac{W}{m}(2^{Rm/W}-1)$. The first constraint can be simplified as:

$$\begin{aligned}
P_s|h_{sd}|^2+\sum_{i\in\mathcal{R}}P_s\psi_i=&P_s(|h_{sd}|^2+\sum_{i\in\mathcal{R}}|h_{si}|^2)\\
&-\sum_{i\in\mathcal{R}}\frac{P_s^2|h_{si}|^4+P_s|h_{si}|^2\frac{W}{m}}{P_s|h_{si}|^2+P_i|h_{id}|^2+\frac{W}{m}}
\end{aligned}$$

Since we have fixed P_s, we can express (15) as:

$$\begin{aligned}
\text{Minimize:}\quad &\sum_{i\in\mathcal{R}}(X_i+V\beta_i)P_i-Z_s-V\alpha_s\\
\text{Subject to:}\quad &\sum_{i\in\mathcal{R}}\frac{P_s^2|h_{si}|^4+P_s|h_{si}|^2\frac{W}{m}}{P_s|h_{si}|^2+P_i|h_{id}|^2+\frac{W}{m}}\leq\theta'\\
&0\leq P_i\leq P_i^{max}\qquad\forall i\in\mathcal{R}
\end{aligned}\tag{16}$$

where $\theta'=P_s(|h_{sd}|^2+\sum_{i\in\mathcal{R}_s}|h_{si}|^2)-\theta$. Using the KKT conditions, the solution the above convex optimization problem is given by (see Appendix C for details):

$$P_i^*=\left[\sqrt{\frac{\nu^*(P_s^2|h_{si}|^4+P_s|h_{si}|^2\frac{W}{m})}{(X_i+V\beta_i)|h_{id}|^2}}-\frac{P_s|h_{si}|^2+\frac{W}{m}}{|h_{id}|^2}\right]_0^{P_i^{max}}$$

where $\nu^*\geq 0$ is chosen so that the second constraint is met with equality. We note that this solution has a water-filling type structure as well. Therefore, to compute the optimal solution to (7) for this protocol, we would have to solve the above for each value of $P_s \in [0, P_s^{max}]$. In practice, this computation can be simplified by considering only a discrete set of values for P_s. Because we have derived a simple closed form expression for each P_s, it is easy to compare these values over, say, a discrete list of 100 options in $[0, P_s^{max}]$ to pick the best one, which enables a very accurate approximation to optimality in real time.

D. DF with DSTC

In this protocol, all the cooperating relays in the second stage use an appropriate distributed space-time code (DSTC) [5] so that they can transmit simultaneously on the same channel. The slot structure under this scheme is shown in Fig.1(d). Suppose in the first phase of the protocol, s transmits the packet in the first half of the slot using power P_s. In the second phase, a subset $\mathcal{U} \subset \mathcal{R}$ of relays that were successful in reliably decoding the packet, re-encode it using a DSTC and transmit to the destination with power P_i (where $i \in \mathcal{U}$) in the second half of the slot. Given such a set $\mathcal{U}$, the total mutual information under this protocol is given by [4]:

$$\frac{W}{2}\log\Big(1 + \frac{2P_s}{W}|h_{sd}|^2 + \sum_{i\in\mathcal{U}}\frac{2P_i}{W}|h_{id}|^2\Big)$$

The factor of 2 appears because only half of the slot is being used for transmission. As seen in the expression above, unlike the earlier examples, this protocol does not suffer from reduced multiplexing gains due to orthogonal channels.

We can now express (7) for this protocol as follows. Define binary variables x_i to be 1 if relay i can reliably decode the packet after the first stage and 0 else. Then, for this protocol, (7) is equivalent to the following optimization problem:

$$\begin{aligned}
\text{Minimize: } & (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{R}}(X_i + V\beta_i)P_i - Z_s - V\alpha_s \\
\text{Subject to: } & \frac{W}{2}\log\Big(1 + \frac{2P_s}{W}|h_{sd}|^2 + \sum_{i\in\mathcal{R}} x_i\frac{2P_i}{W}|h_{id}|^2\Big) \geq R \\
& \frac{W}{2}\log\Big(1 + \frac{2P_s}{W}|h_{si}|^2\Big) \geq x_i R \\
& 0 \leq P_s \leq P_s^{max} \\
& 0 \leq P_i \leq P_i^{max}, x_i \in \{0,1\}\ \forall i \in \mathcal{R}
\end{aligned} \tag{17}$$

By comparing the above with (9), it can be seen that the computation of minimum cost under this protocol follows the same procedure as described in Sec. V-A of solving $m+1$ subproblems, each an LP, by ordering the relays greedily and hence we do not repeat it.

E. AF with DSTC

Here, all cooperating relays use amplify and forward along with DSTC. The total mutual information under this protocol is given by:

$$\frac{W}{2}\log\left(1 + \frac{2P_s}{W}\Big(|h_{sd}|^2 + \sum_{i\in\mathcal{R}}\psi_i\Big)\right)$$

where $\psi_i = \frac{P_i|h_{si}|^2|h_{id}|^2}{P_s|h_{si}|^2 + P_i|h_{id}|^2 + \frac{W}{2}}$. Using this, we can express (7) for this model as follows.

$$\begin{aligned}
\text{Minimize: } & (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{R}}(X_i + V\beta_i)P_i - Z_s - V\alpha_s \\
\text{Subject to: } & \frac{W}{2}\log\left(1 + \frac{mP_s}{W}\Big(|h_{sd}|^2 + \sum_{i\in\mathcal{R}}\psi_i\Big)\right) \geq R \\
& 0 \leq P_s \leq P_s^{max} \\
& 0 \leq P_i \leq P_i^{max}\ \forall i \in \mathcal{R}
\end{aligned} \tag{18}$$

This is similar to (14) and thus, we fix P_s and use a similar reduction to get a convex optimization problem whose solution can be derived using KKT conditions and is given by:

$$P_i^* = \left[\sqrt{\frac{\nu^*(P_s^2|h_{si}|^4 + P_s|h_{si}|^2\frac{W}{2})}{(X_i + V\beta_i)|h_{id}|^2}} - \frac{P_s|h_{si}|^2 + \frac{W}{2}}{|h_{id}|^2}\right]_0^{P_i^{max}}$$

where $\nu^* \geq 0$ is chosen so that the constraint on the total mutual information at the destination is met with equality.

F. DF with Beamforming

Finally, we consider an example where the cooperating relays use coherent beamforming to steer their signals towards the destination. Note that this strategy needs both the channel gain amplitude and phase information, in contrast to the strategies in the previous subsections, which only required channel gain amplitude information. The slot structure under this scheme is shown in Fig.1(d). Suppose in the first phase of the protocol, s transmits the packet in the first half of the slot using power P_s. In the second phase, a subset $\mathcal{U} \subset \mathcal{R}$ of relays that were successful in reliably decoding the packet use beamforming to transmit to the destination with power P_i (where $i \in \mathcal{U}$) in the second half of the slot. Given such a set $\mathcal{U}$, the total mutual information under this protocol is given by [13]:

$$\frac{W}{2}\log\left(1 + \frac{2P_s}{W}|h_{sd}|^2 + \frac{2}{W}\big(\sum_{i\in\mathcal{U}}\sqrt{P_i}|h_{id}|\big)^2\right)$$

Using the same definition of binary variables x_i as in Sec.V-A , we can express (7) for this protocol as an optimization problem that resembles (9). As before, we can then reduce this to a set of $m+1$ subproblems, one for each $\mathcal{U}_k$. The subproblem for set $\mathcal{U}_k$ is given by:

$$\begin{aligned}
\text{Minimize:} & (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{U}_k}(X_i + V\beta_i)P_i - Z_s - V\alpha_s \\
\text{Subject to:} & \log\Big(1 + \frac{2P_s}{W}|h_{sd}|^2 + \frac{2}{W}\big(\sum_{i\in\mathcal{U}}\sqrt{P_i}|h_{id}|\big)^2\Big) \geq \frac{2R}{W} \\
& P_s^{\mathcal{U}_k} \leq P_s \leq P^{max} \\
& 0 \leq P_i \leq P^{max} \qquad \forall i \in \mathcal{U}_k
\end{aligned} \tag{19}$$

The above problem is convex and we can use the KKT conditions to get the optimal solution in quasi-closed form (see Appendix D for details).

VI. Unknown Channels, Known Statistics

We next consider the solution to (7) when the source does not know the current channel gains and is only aware of their statistics. In this case, (7) becomes a 2-stage stochastic dynamic program. For brevity, here we focus on its solution for the cooperative transmission mode.

Suppose the source uses power P_s in the first stage. Let ω denote the outcome of this transmission. This lies in a space Ω of possible network states which is assumed to be of a finite but arbitrarily large size. For example, in the DF protocol, ω might represent the set of relay nodes that received the packet successfully after the first stage as well as the mutual information accumulated so far at the destination. For AF, ω can represent the SNR value at each relay node and at the destination.

Let $J_1^*(P_s, \omega)$ be the optimal cost-to-go function for the 2-stage dynamic program (7) given that the source uses power P_s in the first stage and the network state is ω at the beginning of the second stage. Let J_0^* denote the optimal cost-to-go function starting from the first stage. Also, let $\mathcal{R}(\omega)$ denote the set of relay nodes that can take part in cooperative transmission when the network state in ω. We define the following probabilities. Let $f(P_s, \omega)$ be the probability that the outcome of the first stage is ω when the source uses power P_s. Also, let $g(\overrightarrow{P}_{\mathcal{R}(\omega)}, P_s, \omega)$ be the probability that the receiver gets the packet successfully when relays in $\mathcal{R}(\omega)$ use a power allocation $\overrightarrow{P}_{\mathcal{R}(\omega)}$ and the source uses power P_s. Note that these probabilities are obtained by taking expectation over all channel state realizations. We assume these are obtained from the knowledge of the channel statistics.

Using these definitions, we can now write the Bellman optimality equations [26] for this dynamic program $\forall \omega \in \Omega$:

$$J_0^* = \min_{P_s} \Big[(X_s + V\beta_s)P_s + \sum_{\omega \in \Omega} f(P_s, \omega) J_1^*(P_s, \omega)\Big] \quad (20)$$

$$J_1^*(P_s, \omega) = \min_{\overrightarrow{P}_{\mathcal{R}(\omega)}} \Big[\sum_{i \in \mathcal{R}(\omega)} (X_i + V\beta_i)P_i - (Z_s + V\alpha_s) g(\overrightarrow{P}_{\mathcal{R}(\omega)}, P_s, \omega)\Big] \quad (21)$$

While this can be solved using standard dynamic programming techniques, it has a computational complexity that grows with the state space size Ω and can be prohibitive when this is large. We therefore present an alternate method based on the idea of Monte Carlo simulation.

A. Simulation Based Method

Suppose the transmitter performs the following simulation. Fix a source power P_s. Define $J_0^*(P_s)$ as the optimal cost-to-go function *given* that the source uses power P_s. Note that this is simply the expression on the right hand side of (20) with P_s fixed. Simulate the outcome of a transmission at this power n times independently using the values of $f(P_s, \omega)$. Let $\omega_j \in \Omega$ denote the outcome of the j^{th} simulation. For each generated outcome ω_j, compute the optimal cost-to-go function $J_1^*(P_s, \omega_j)$ by solving (21) (this could be done using the knowledge of $g(\overrightarrow{P}_{\mathcal{R}(\omega)}, P_s, \omega)$ either analytically or numerically). Use this to update $J_0^{est}(P_s, n)$, which is an *estimate* of $J_0^*(P_s)$ for a given P_s after n iterations and is defined as follows:

$$J_0^{est}(P_s, n) = (X_s + V\beta_s)P_s + \frac{1}{n}\sum_{j=1}^{n} J_1^*(P_s, \omega_j) \quad (22)$$

We now show that, for a given P_s, $J_0^{est}(P_s, n)$ can be pushed arbitrarily close to the optimal cost-to-go function $J_0^*(P_s)$ by increasing n. Since we have fixed P_s, from (20), we have:

$$J_0^*(P_s) = (X_s + V\beta_s)P_s + \sum_{\omega \in \Omega} f(P_s, \omega) J_1^*(P_s, \omega)$$

Define the following indicator random variables for each simulation j and $\forall \omega \in \Omega$:

$$1_\omega(P_s, j) = \begin{cases} 1 & \text{if the outcome of simulation } j \text{ is } \omega \\ 0 & \text{else} \end{cases}$$

Note that by definition $\mathbb{E}\{1_\omega(P_s, j)\} = f(P_s, \omega)$. Therefore, we can express $J_0^{est}(P_s, n)$ in terms of these indicator variables as follows:

$$J_0^{est}(P_s, n) = (X_s + V\beta_s)P_s + \frac{1}{n}\sum_{j=1}^{n}\sum_{\omega \in \Omega} 1_\omega(P_s, j) J_1^*(P_s, \omega)$$

We note that $\Big(\sum_{\omega \in \Omega} 1_\omega(P_s, j) J_1^*(P_s, \omega)\Big)$ are i.i.d. random variables with mean $\mu = \sum_{\omega \in \Omega} f(P_s, \omega) J_1^*(P_s, \omega)$ and variance $\sigma^2 = \sum_{\omega \in \Omega} f(P_s, \omega)(J_1^*(P_s, \omega))^2 - \mu^2$. Using Chebyshev's inequality, we get for any $\epsilon > 0$:

$$Pr\Big[|\frac{1}{n}\sum_{j=1}^{n}\Big(\sum_{\omega \in \Omega} 1_\omega(P_s, j) J_1^*(P_s, \omega)\Big) - \mu| \geq \epsilon\Big] \leq \frac{\sigma^2}{n\epsilon^2}$$

This shows that the value of the estimate quickly converges to the optimal cost-to-go value. Thus, this method can be used to get a good estimate of the optimal cost-to-go function for a fixed value of P_s in a reasonable number of steps.

VII. Multi Source-Destination Pairs

In this section, we extend the basic model of Sec. II to the case when there are multiple source-destination pairs in the network. Let the set of source and destination nodes be given by $\mathcal{S}$ and $\mathcal{D}$ respectively. We consider the case when all source-destination pairs have orthogonal channels[3]. In particular, we assume that in each slot, a medium access process $\chi(t)$ determines which source nodes get transmission opportunities. For simplicity, we assume that at most one source transmits in a slot. This models situations where there might be a pseudo-random TDMA schedule that determines a unique transmitter node every slot. It also models situations where the source nodes use a contention-resolution mechanism

[3]For the non-orthogonal scenario, there will two sources of outages: transmission failure at the physical layer and delay violation due to contention in medium access. Hence, MAC scheduling in addition to physical layer resource allocation must be considered. This is not the focus of the current work.

such as CSMA. Our model can be extended to scenarios where more than one source node can transmit, potentially over orthogonal frequency channels.

Let $s(t) = s(\chi(t)) \in \mathcal{S}$ be the source node that gets a transmission opportunity in slot t. Then, the optimal resource allocation framework developed in Sec. IV can be applied as follows. A virtual reliability queue is defined for each source node $s \in \mathcal{S}$ and is updated as in (5). Note that in slots where a source node s does not get a transmission opportunity, $\Phi_s(t) = 0$. We assume that each incoming packet gets one transmission opportunity so that the delay constraint of 1 slot per packet only measures the transmission delay and not the queueing delay that would be incurred due to contention. Similarly, a virtual power queue is maintained for each node as in (6) including the source nodes and relay nodes. Note that in this model, it is possible for a source node to act as a relay for another source node when it is not transmitting its own data. We denote the set of relay nodes (that includes such source nodes) in slot t as $\mathcal{R}(t)$.

Then the optimal control algorithm operates as follows. Let $\boldsymbol{Q}(t)$ denote the collection of all virtual queues in timeslot t. Every slot, given $\boldsymbol{Q}(t)$ and any channel state $\mathcal{T}(t)$, it chooses a control action $\mathcal{I}_{s(t)}$ that minimizes the following stochastic metric (for a given control parameter $V \geq 0$):

$$\begin{aligned}
\text{Minimize:} \quad & (X_{s(t)} + V\beta_{s(t)})\mathbb{E}\left\{P_{s(t)}|\boldsymbol{Q}(t), \mathcal{T}(t)\right\} \\
& + \sum_{i\in\mathcal{R}(t)} (X_i(t) + V\beta_i)\mathbb{E}\left\{P_i(t)|\boldsymbol{Q}(t), \mathcal{T}(t)\right\} \\
& - (Z_{s(t)} + V\alpha_{s(t)})\mathbb{E}\left\{\Phi_{s(t)}|\boldsymbol{Q}(t), \mathcal{T}(t)\right\} \\
\text{Subject to:} \quad & 0 \leq P_{s(t)} \leq P_{s(t)}^{max} \\
& 0 \leq P_i(t) \leq P_i^{max} \; \forall i \in \mathcal{R}(t) \\
& \mathcal{I}_{s(t)} \in \mathcal{C}
\end{aligned} \tag{23}$$

This problem can be solved using the techniques described for the single source case.

VIII. Simulations

We simulate the dynamic control algorithm (7) in an ad-hoc network with 3 stationary sources and 7 mobile relays as shown in Fig. 2. Every slot, the sources receive new packets destined for the base station according to an i.i.d. Bernoulli process of rate λ and each packet has a delay constraint of 1 slot. The sources are assumed to have orthogonal channels and can transmit either directly or cooperatively with a subset of the relays in their vicinity. We impose a cell-partitioned structure so that a source can only cooperate with the relays that are in the same cell in that slot. The relays move from one cell to the other according to a Markovian random walk. In the simulation, at the end of every slot, a relay decides to stay in its current cell with probability 0.8, else decides to move to an adjacent cell with probability 0.2 (where any of the feasible adjacent cells are equally likely).

We assume a Rayleigh fading model. The amplitude squares of the instantaneous gains on the links involving a source, the set of relays in its cell in that slot and the base station are exponentially distributed random variables with mean 1. All power values are normalized with respect to the average noise power. All nodes have an average power constraint of 1 unit and a maximum power constraint of 10 units.

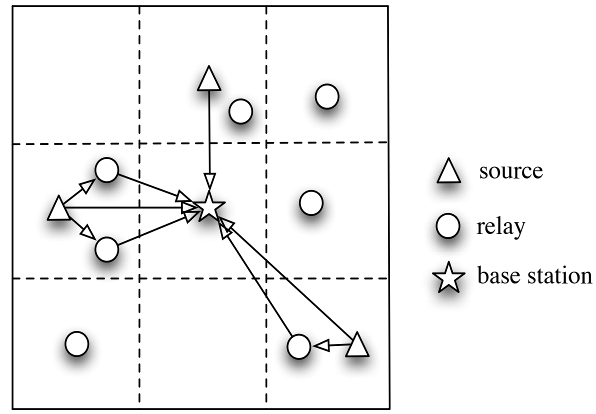

Fig. 2. A snapshot of the example network used in simulation.

We consider the Regenerative DF cooperative protocol over orthogonal channels and implement the optimal resource allocation strategy as computed in (11) for this network. In the first experiment, we consider the objective of minimizing the average sum power expenditure in the network given a minimum reliability constraint $\rho_s = 0.98$ and input rate $\lambda_s = 0.5$ packets/slot for all sources. For this, we set $\alpha_s = 0$ and $\beta_i = 1$. Fig. 3 shows the average sum power for different values of the control parameter V. It is seen that this value converges to 2.6 units for increasing values of V, as predicted by the performance bounds on the time average utility in Theorem 1. Fig. 4 shows the resulting average reliability queue occupancy. It is seen to increase linearly in V, again as predicted by the bound on the time average queue backlog in Theorem 1. We emphasize again that there are no actual queues in the system, and all successfully delivered packets have a delay exactly equal to 1 slot. The fact that all reliability queues are stable ensures that we are indeed meeting or exceeding the 98% reliability constraint. Indeed, in our simulations we found reliability to be almost exactly equal to the 98% constraint, as expected in an algorithm designed to minimize average power subject to this constraint. We further note that the instantaneous reliability queue value $Z(t)$ represents the worst case "excess" packets that did not meet the reliability constraints over any interval ending at time t, so that maintaining small $Z(t)$ (with a small V) makes the timescales over which the time average reliability constraints are satisfied smaller.

In the second experiment, we choose both $\alpha_s = 0$ and $\beta_i = 0$ so that (2) becomes a feasibility problem. We fix the average and peak power values to 1 and 10 respectively and implement (11) for different rate-reliability pairs. In Table I, we show whether these are feasible or not under three resource allocation strategies: direct transmission, always cooperative transmission and dynamic cooperation (that corresponds to implementing the solution to (11) every slot). It can be seen that dynamic cooperation significantly increases the feasible

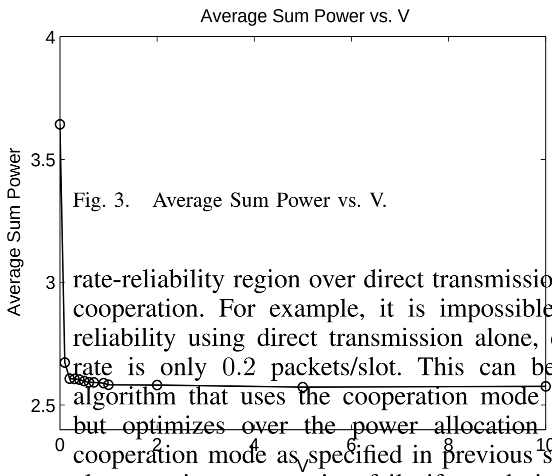

Fig. 3. Average Sum Power vs. V.

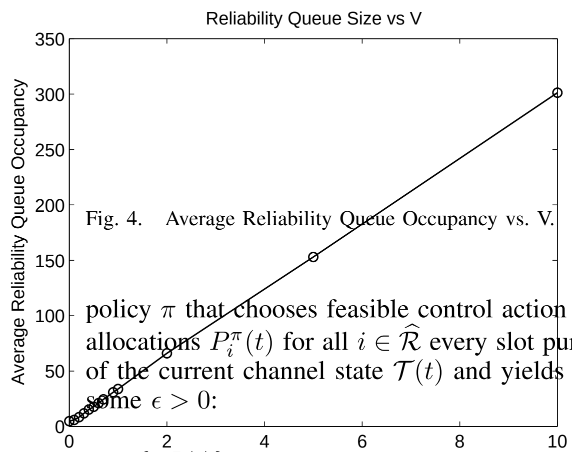

Fig. 4. Average Reliability Queue Occupancy vs. V.

rate-reliability region over direct transmission as well as s cooperation. For example, it is impossible to achieve reliability using direct transmission alone, even if the tr rate is only 0.2 packets/slot. This can be achieved by algorithm that uses the cooperation mode (mode 3) alw but optimizes over the power allocation decisions of cooperation mode as specified in previous sections. How..., always using cooperation fails if we desire 98% reliability, but using our optimal policy that dynamically mixes between the different modes, and chooses efficient power allocation decisions in each mode, can achieve 98% reliability, even at increased rates up to 0.6 packets/slot.

IX. Conclusions

In this paper, we considered the problem of optimal resource allocation for delay-limited cooperative communication in a mobile ad-hoc network. Using the technique of Lyapunov optimization, we developed dynamic cooperation strategies that make optimal use of network resources to achieve a target outage probability (reliability) for each user subject to average power constraints. Our framework is general enough to be applicable to a large class of cooperative protocols. In particular, in this paper, we derived quasi-closed form solutions for several variants of the Decode-and-Forward and Amplify-and-Forward strategies.

Appendix A: Proof of Theorem 1

Here, we prove Theorem 1 by comparing the Lyapunov drift of the dynamic control algorithm (7) with that of an optimal stationary, randomized policy. Let r_s^* and e_i^* $\forall i \in \widehat{\mathcal{R}}$ denote the optimal value of the objective in (2). Then we have the following fact[4]:

Existence of an Optimal Stationary, Randomized Policy: Assuming i.i.d. $\mathcal{T}(t)$ states, there exists a stationary randomized policy π that chooses feasible control action $\mathcal{I}^\pi(t)$ and power allocations $P_i^\pi(t)$ for all $i \in \widehat{\mathcal{R}}$ every slot purely as a function of the current channel state $\mathcal{T}(t)$ and yields the following for some $\epsilon > 0$:

$$\mathbb{E}\left\{\Phi_s^\pi(t)\right\} \geq \rho_s \lambda_s + \epsilon \tag{24}$$

$$\mathbb{E}\left\{P_i^\pi(t)\right\} + \epsilon \leq P_i^{avg} \tag{25}$$

$$\alpha_s \mathbb{E}\left\{\Phi_s^\pi(t)\right\} - \sum_{i\in\mathcal{R}} \beta_i \mathbb{E}\left\{P_i^\pi(t)\right\} = \alpha_s r_s^* - \sum_{i\in\mathcal{R}} \beta_i e_i^* \tag{26}$$

Let $\boldsymbol{Q}(t) = (Z_s(t), X_i(t))$ $\forall i \in \widehat{\mathcal{R}}$ represent the collection of these queue backlogs in timeslot t. We define a quadratic Lyapunov function:

$$L(\boldsymbol{Q}(t)) \triangleq \frac{1}{2}\Big[Z_s^2(t) + \sum_{i\in\widehat{\mathcal{R}}} X_i^2(t)\Big]$$

Also define the conditional Lyapunov drift $\Delta(\boldsymbol{Q}(t))$ as follows:

$$\Delta(\boldsymbol{Q}(t)) \triangleq \mathbb{E}\left\{L(\boldsymbol{Q}(t+1)) - L(\boldsymbol{Q}(t)) | \boldsymbol{Q}(t)\right\}$$

Using queueing dynamics (5), (6), the Lyapunov drift under any control policy can be computed as follows:

$$\begin{aligned}\Delta(\boldsymbol{Q}(t)) \leq\ & B - Z_s(t)\mathbb{E}\left\{\Phi_s(t) - \rho_s A_s(t)|\boldsymbol{Q}(t)\right\} \\ & - \sum_{i\in\widehat{\mathcal{R}}} X_i(t)\mathbb{E}\left\{P_i^{avg} - P_i(t)|\boldsymbol{Q}(t)\right\}\end{aligned} \tag{27}$$

where $B = \frac{1+\lambda_s^2\rho_s^2+\sum_{i\in\widehat{\mathcal{R}}}(P_i^{avg})^2+(P^{max})^2}{2}$.

For a given control parameter $V \geq 0$, we subtract a "reward" metric $V\mathbb{E}\left\{\alpha_s\Phi_s(t) - \sum_{i\in\widehat{\mathcal{R}}}\beta_i P_i(t)|\boldsymbol{Q}(t)\right\}$ from both sides

[4]This can be shown using the techniques developed in [22].

TABLE I. TABLE SHOWING THE FEASIBILITY OF DIFFERENT RATE-RELIABILITY PAIRS.

(rate, reliability) = (λ_s, ρ_s)	(0.1, 0.9)	(0.2, 0.9)	(0.2, 0.95)	(0.5, 0.95)	(0.5, 0.98)	(0.6, 0.98)	(0.7, 0.99)
direct transmission	✓	✓	x	x	x	x	x
always cooperate	✓	✓	✓	✓	x	x	x
optimal strategy	✓	✓	✓	✓	✓	✓	x

of the above inequality to get the following:

$$\begin{aligned}\Delta(\boldsymbol{Q}(t)) - V\mathbb{E}\left\{\alpha_s\Phi_s(t) - \sum_{i\in\widehat{\mathcal{R}}}\beta_iP_i(t)|\boldsymbol{Q}(t)\right\} \le\ & B \\ - Z_s(t)\mathbb{E}\left\{\Phi_s(t) - \rho_sA_s(t)|\boldsymbol{Q}(t)\right\}& \\ - \sum_{i\in\widehat{\mathcal{R}}}X_i(t)\mathbb{E}\left\{P_i^{avg} - P_i(t)|\boldsymbol{Q}(t)\right\}& \\ - V\mathbb{E}\left\{\alpha_s\Phi_s(t) - \sum_{i\in\widehat{\mathcal{R}}}\beta_iP_i(t)|\boldsymbol{Q}(t)\right\}& \end{aligned} \tag{28}$$

From the above, it can be seen that the dynamic control algorithm (7) is designed to take a control action that minimizes the right hand side of (28) over all possible options every slot, including the stationary policy π. Thus, using (24), (25), (26), we can write the above as:

$$\begin{aligned}\Delta(\boldsymbol{Q}(t)) - V\mathbb{E}\left\{\alpha_s\Phi_s(t) - \sum_{i\in\widehat{\mathcal{R}}}\beta_iP_i(t)|\boldsymbol{Q}(t)\right\} \le B& \\ - Z_s(t)\epsilon - \sum_{i\in\widehat{\mathcal{R}}}X_i(t)\epsilon - V\alpha_sr_s^* - \sum_{i\in\widehat{\mathcal{R}}}\beta_ie_i^*&\end{aligned} \tag{29}$$

Theorem 1 now follows by a direct application of the Lyapunov optimization Theorem [23].

APPENDIX B – SOLUTION TO NON-REGENERATIVE DF ORTHOGONAL USING KKT CONDITIONS

We ignore the constant terms in the objective. It is easy to see that the first constraint in (12) must be met with equality. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} =& (X_s + V\beta_s)P_s + \sum_{i\in\mathcal{U}_k}(X_i + V\beta_i)P_i - \lambda_s(P_s - P_s^{\mathcal{U}_k}) \\ &- \sum_{i\in\mathcal{U}_k}\lambda_iP_i + \beta_s(P_s - P_s^{max}) + \sum_{i\in\mathcal{U}_k}\beta_i(P_i - P_i^{max}) \\ &+ \nu\Big[\log(1+\theta_sP_s) + \sum_{i\in\mathcal{U}_k}\log(1+\theta_iP_i) - \frac{mR}{W}\Big]\end{aligned}$$

where $\theta_s = \frac{m}{W}|h_{sd}|^2, \theta_i = \frac{m}{W}|h_{id}|^2$. The KKT conditions are:

$$\lambda_s^*(P_s^* - P_s^{\mathcal{U}_k}) = 0 \qquad \lambda_i^*P_i^* = 0$$
$$\beta_s^*(P_s^* - P_s^{max}) = 0 \qquad \beta_i^*(P_i^* - P_i^{max}) = 0$$
$$\lambda_s^*, \lambda_i^*, \beta_s^*, \beta_i^* \ge 0$$
$$(X_s + V\beta_s) - \lambda_s^* + \beta_s^* + \frac{\nu^*\theta_s}{1+\theta_sP_s^*} = 0$$
$$(X_i + V\beta_i) - \lambda_i^* + \beta_i^* + \frac{\nu^*\theta_i}{1+\theta_iP_i^*} = 0$$

If $\nu^* > 0$, then we must have that $\lambda_s^* - \beta_s^* > 0$ and $\lambda_i^* - \beta_i^* > 0$ for all i. This would mean that $P_s^* = P_s^{\mathcal{U}_k}$ and $P_i^* = 0$. For some $\nu^* \le 0$, we have three cases:

1) If $\lambda_i^* = \beta_i^*$, we get $P_i^* = \frac{-\nu^*}{X_i+V\beta_i} - \frac{1}{\theta_i}$
2) If $\lambda_i^* > \beta_i^*$, then we must have $\lambda_i^* > 0$ and we get $P_i^* = 0$
3) If $\lambda_i^* < \beta_i^*$, then we must have $\beta_i^* > 0$ and we get $P_i^* = P_i^{max}$

Similar results can be obtained for P_s^*. Combining these, we get:

$$P_s^* = \left[\frac{-\nu*}{X_s+V\beta_s} - \frac{1}{\theta_s}\right]_{P_s^{\mathcal{U}_k}}^{P_s^{max}} \quad P_i^* = \left[\frac{-\nu*}{X_i+V\beta_i} - \frac{1}{\theta_i}\right]_0^{P_i^{max}}$$

where $[X]_0^{P_{max}}$ denotes $\min[\max(X,0), P_{max}]$

APPENDIX C – SOLUTION TO AF ORTHOGONAL USING KKT CONDITIONS

It is easy to see that the first constraint in (16) must be met with equality. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} =& \sum_{i\in\mathcal{R}_s}(X_i + V\beta_i)P_i - \sum_{i\in\mathcal{R}_s}\lambda_iP_i + \sum_{\in\mathcal{R}_s}\beta_i(P_i - P_i^{max}) \\ &+ \nu\Big[\sum_{\in\mathcal{R}_s}\frac{P_s^2|h_{si}|^4 + P_s|h_{si}|^2W/m}{|h_{si}|^2P_s + |h_{id}|^2P_i + W/m} - \theta'\Big]\end{aligned}$$

The KKT conditions are:

$$\lambda_i^*P_i^* = 0 \qquad \beta_i^*(P_i^* - P_i^{max}) = 0 \qquad \lambda_i^*, \beta_i^* \ge 0$$
$$(X_i + V\beta_i) - \lambda_i^* + \beta_i^* = \frac{\nu^*|h_{id}|^2(P_s^2|h_{si}|^4 + P_s|h_{si}|^2W/m)}{(|h_{si}|^2P_s + |h_{id}|^2P_i^* + W/m)^2}$$

If $\nu^* < 0$, then we must have that $\lambda_i^* - \beta_i^* > 0$ for all i. This would mean that $P_i^* = 0$. For some $\nu^* \ge 0$, we have three cases:

1) If $\lambda_i^* = \beta_i^*$, we get $P_i^* = \sqrt{\frac{\nu^*(P_s^2|h_{si}|^4+P_s|h_{si}|^2W/m)}{(X_i+V\beta_i)|h_{id}|^2}} - \frac{P_s|h_{si}|^2+W/m}{|h_{id}|^2}$
2) If $\lambda_i^* > \beta_i^*$, then we must have $\lambda_i^* > 0$ and we get $P_i^* = 0$
3) If $\lambda_i^* < \beta_i^*$, then we must have $\beta_i^* > 0$ and we get $P_i^* = P_i^{max}$

Combining these, we get:

$$P_i^* = \left[\sqrt{\frac{\nu^*(P_s^2|h_{si}|^4+P_s|h_{si}|^2W/m)}{(X_i+V\beta_i)|h_{id}|^2}} - \frac{P_s|h_{si}|^2+W/m}{|h_{id}|^2}\right]_0^{P_i^{max}}$$

where $[X]_0^{P_{max}}$ denotes $\min[\max(X,0), P_{max}]$

APPENDIX D – SOLUTION TO DF BEAMFORMING USING KKT CONDITIONS

First, note that both the objective and the first constraint in (19) can be rewritten so that they are linear in P_s. Thus, we must have that $P_s^* \in \{P_s^{\mathcal{U}_k}, P_s^{max}\}$. Next, the Lagrangian for (19) is given by:

$$\begin{aligned}\mathcal{L} =&(X_s + V\beta_s)P_s + \sum_{i\in\mathcal{U}_k}(X_i + V\beta_i)P_i - \lambda_s(P_s - P_s^{\mathcal{U}_k})\\ &- \sum_{i\in\mathcal{U}_k}\lambda_i P_i + \beta_s(P_s - P_s^{max}) + \sum_{i\in\mathcal{U}_k}\beta_i(P_i - P_i^{max})\\ &+ \nu\big(P_s|h_{sd}|^2 + \ \big(\sum_{i\in\mathcal{U}}\sqrt{P_i}|h_{id}|\big)^2 - \theta\big)\end{aligned}$$

where $\theta = \frac{W}{2}(2^{2R/W} - 1)$. The KKT conditions are:

$$\begin{aligned}&\lambda_s^*(P_s^* - P_s^{\mathcal{U}_k}) = 0 \qquad \lambda_i^* P_i^* = 0\\ &\beta_s^*(P_s^* - P_s^{max}) = 0 \qquad \beta_i^*(P_i^* - P_i^{max}) = 0\\ &\lambda_s^*, \lambda_i^*, \beta_s^*, \beta_i^* \geq 0\\ &(X_s + V\beta_s) - \lambda_s^* + \beta_s^* + \nu^*|h_{sd}|^2 = 0\\ &(X_i + V\beta_i) - \lambda_i^* + \beta_i^* + \nu^*\frac{(\sum_{i\in\mathcal{U}_k}\sqrt{P_i^*}|h_{id}|)|h_{id}|}{\sqrt{P_i^*}} = 0\end{aligned}$$

Note that if $\lambda_i^* - \beta_i^* > 0$, then we have that $P_i^* = 0$. Similarly, if $\lambda_i^* - \beta_i^* < 0$, then we have that $P_i^* = P_i^{max}$. Finally, for all i for which $\lambda_i^* - \beta_i^* = 0$, the P_i^* satisfy the following set of linear equations for some ν^* such that the first constraint in (19) is met with equality:

$$(X_i + V\beta_i)\sqrt{P_i^*} + \nu^*|h_{id}|(\sum_{i\in\mathcal{U}_k}\sqrt{P_i^*}|h_{id}|) = 0$$

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