Energy-Optimal Scheduling with Dynamic Channel Acquisition in Wireless Downlinks

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Abstract—We consider a wireless base station serving L users through L time-varying channels. It is well known that opportunistic scheduling algorithms with full channel state information (CSI) can stabilize the system with any data rates within the capacity region. However, such opportunistic scheduling algorithms may not be energy efficient when the cost of channel acquisition is high and traffic rates are low. In particular, under the low traffic rate regime, it may be sufficient and more energy efficient to transmit data with no CSI, i.e., to transmit data blindly, since no power for channel acquisition is consumed. In general, we show strategies that probe channels in every slot or never probe channels in any slot are not necessarily optimal, and we must consider mixed strategies. We derive a unified scheduling algorithm that dynamically chooses to transmit data with full or no CSI based on queue backlog and channel statistics. Our methodology is general and can be naturally extended to include timing overhead due to channel acquisition, and to treat systems that allow any subset of channels to be measured. Through Lyapunov analysis, we show that the unified algorithm is throughput-optimal and stabilizes the downlink with optimal power consumption, balancing well between channel-aware and channel-blind transmission modes.

Index Terms—Stochastic control, queuing analysis, optimization, partial channel state information.

1 INTRODUCTION

To transmit data efficiently over wireless channels, it is important to accommodate time variations of channels (due to changing environments, multipath fading, and mobility, etc.) and the limited energy in wireless devices. In particular, the concept of *opportunistic scheduling* has been shown to enable the design of efficient control algorithms that boost supportable data rates to the limit. The intuition is that transmitting data only when channel states are good can increase the throughput of a wireless network with a limited energy budget.

The acquisition of channel states is central to opportunistic scheduling. Works in [2], [3], [4], [5], [6] focus on throughput/utility maximization with energy constraints in wireless networks, assuming that channel states are always known with negligible cost. In practical telecommunication systems, however, channel acquisition consumes power and time. To combat the power and timing overhead, the problem of scheduling with partial channel acquisition should be investigated. Related previous works include [7] and [8], which show that measuring all channels regularly may not be throughput optimal because of the trade-off between multiuser diversity gain and the associated timing overhead of channel probing. Kar et al. [9] study throughput-achieving algorithms when channel states are only measured every $T > 1$ slots. Gopalan et al. [10] develop a MaxWeight-type throughput-optimal policy in a wireless downlink, assuming that only a subset of channels, chosen from a fixed collection of subsets, can be observed at any time and only the channels with known states can serve packets. Works in [11], [12], [13] study the performance of a wireless downlink for which a channel state is only sent from a user to the base station when the associated channel quality exceeds some threshold. The works [14], [15], [16] develop optimal/near-optimal policies for joint partial channel probing and rate allocations to optimize a linear network utility.

In this paper, we consider channel measurement and scheduling algorithms for throughput and energy optimality. We extend the energy-optimal algorithm in [2], which assumes that perfect channel state information (CSI) is known at the beginning of each slot, by considering the case when there is a nonzero power cost to acquire CSI. This case has a larger decision space for control policies, including policies that probe channels in every slot, policies that never probe channels, and combinations of these. The problem discussed in [2] can be viewed as a special case of our system model, and the algorithm given there may no longer be optimal when channel measurement costs are considered.

For the sake of simple demonstrations, in the first part of the paper, we focus on a wireless downlink with the restriction that either all or none of the channel states are acquired in a slot, and we suppose the timing overhead due to channel acquisition is negligible. Later in Section 6, we show that our analysis can be naturally extended to models where these restrictions are relaxed.

The next section describes our mathematical model and provides simple motivating examples showing the necessity of using both channel-aware and channel-unaware transmission modes. Section 3 establishes the minimum average power for stability when scheduling allows for both modes. Section 4 presents our Dynamic Channel Acquisition algorithm (DCA) and proves that it yields stability with average power consumption that is arbitrarily close to optimal. Simulations verify that the algorithm efficiently mixes between channel-aware and channel-unaware modes, and adapts to different values of channel probing power, packet transmission power, and data arrival rates. Section 6 briefly discusses extensions to system models that

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allow any subset of channels to be measured and that incorporate timing overhead due to channel acquisition

In the rest of the paper, we use the following notations: For real vectors $\boldsymbol{a} = (a_1, \dots, a_L)$ and $\boldsymbol{b} = (b_1, \dots, b_L)$,

- $\boldsymbol{a} \le \boldsymbol{b}$ denotes that $a_i \le b_i$ for all i. $\boldsymbol{a} \ge \boldsymbol{b}$ is defined similarly.
- Define the product $\boldsymbol{a} \otimes \boldsymbol{b} \triangleq (a_1 b_1, \dots, a_L b_L)$.
- Let $\Pr\{A\}$ be the probability of event A occurring. Define a vector of probabilities
$$\boldsymbol{Pr}(\boldsymbol{a} \le \boldsymbol{b}) \triangleq (\Pr\{a_1 \le b_1\}, \dots, \Pr\{a_L \le b_L\}).$$
Probability vector $\boldsymbol{Pr}(\boldsymbol{a} \ge \boldsymbol{b})$ is defined similarly.
- Let $1_{[A]}$ be the indicator function where $1_{[A]} = 1$ if event A is true and 0 otherwise. Define the indicator vector $\mathbf{1}_{[\boldsymbol{a} \le \boldsymbol{b}]} \triangleq (1_{[a_1 \le b_1]}, \dots, 1_{[a_L \le b_L]})$.

2 System Model, Capacity Regions, and Motivating Examples

2.1 System Model

We consider a wireless base station serving L users through L time-varying channels. Time is slotted with normalized time slots $t = [t, t+1)$, $t \in \mathbb{Z}^+$. Data are measured in integer units of packets. Define $a_i(t)$ as the number of packet arrivals for user $i \in \{1, 2, \dots, L\}$ in slot t. Suppose $a_i(t)$ are independent for different i, i.i.d. over slots, and independent of channel state processes. Assume that $a_i(t)$ takes values in $\{0, 1, 2, \dots, A_{max}\}$ with mean $\mathbb{E}[a_i(t)] = \lambda_i$, where A_{max} is a finite integer. Let $s_i(t)$ be the channel state of user i in slot t. Assume that $s_i(t)$ are i.i.d. over slots, and take values in $\mathcal{S} = \{0, 1, 2, \dots, \mu_{max}\}$, where μ_{max} is a finite integer. Channel states remain the same in every slot and only change on slot boundaries. The value of $s_i(t)$ represents the maximum number of packets that can be transmitted over channel i in slot t. Channel statistics are assumed to be known and fixed.

Suppose at the beginning of every time slot, the base station decides whether or not to probe the channels. We assume that either the states of *all* channels are acquired, with power expenditure P_m (each channel measurement consumes P_m/L units of power), or none of the channels are probed (relaxation of this assumption is discussed later in Section 6.2). After possible channel acquisitions, the base station allocates service rates $\boldsymbol{\mu}(t) = (\mu_1(t), \dots, \mu_L(t)) \in \mathcal{S}^L$ chosen from a feasible set Ω, where $\mu_i(t)$ is the service rate allocated to user i. The set Ω defines any additional system restrictions. For example, in systems where at most one user can be served in every slot, each vector in Ω has at most one nonzero component.

When channel states are acquired in a slot, *channel-aware* rates $\boldsymbol{\mu}(t)$ are allocated based on the channel states. Otherwise, *channel-blind* rates $\boldsymbol{\mu}(t)$ are allocated, that is, they are allocated without the knowledge of current channel states. For each user i, if $\mu_i(t) \le s_i(t)$, which indicates that the chosen service rate is supported by the current channel state, at most $\mu_i(t)$ packets can be successfully delivered (limited by the current backlog). Otherwise, all packet transmissions over that channel fail in that slot (which may be because bad channel states and high-order modulations required for high transmission rates yield low signal-to-noise ratios and almost surely incorrect packet reception). In either case, channel i consumes a constant transmission power P_t if the chosen rate $\mu_i(t)$ is nonzero. We note that since allocating unsupportable rate $\mu_i(t) > s_i(t)$ is equivalent to allocating zero rate, we want to choose channel-supportable rates whenever possible. To be precise, for channel states $\boldsymbol{s} = (s_1, \dots, s_L)$, we define the *channel-aware feasible rate set* $\Omega(\boldsymbol{s}) \triangleq \{\boldsymbol{\mu} \in \Omega \mid \boldsymbol{\mu} \le \boldsymbol{s}\}$. Without loss of generality, we assume if channel states $\boldsymbol{s}(t)$ are acquired in a slot, we always choose transmission rates from $\Omega(\boldsymbol{s}(t))$ in that slot. We suppose at the end of each time slot, ACK/NACK feedback is received via a reliable control channel (the absence of an ACK signal is regarded as an NACK). This feedback is used for retransmission control purposes.

With the current channel states $\boldsymbol{s}(t)$ (possibly unknown) and allocated rates $\boldsymbol{\mu}(t)$, we define $\hat{\boldsymbol{\mu}}(t) = (\hat{\mu}_1(t), \dots, \hat{\mu}_L(t))$ as the *effective* transmission rates, where for each i,

$$\hat{\mu}_i(t) = \begin{cases} \mu_i(t) = \mu_i(t, s_i(t)), & \text{if channel } i \text{ is measured,} \\ \mu_i(t) 1_{[\mu_i(t) \le s_i(t)]}, & \text{otherwise.} \end{cases} \quad (1)$$

We use the notation $\mu_i(t, s_i(t))$ (or $\boldsymbol{\mu}(t, \boldsymbol{s}(t))$ in vector form) to emphasize that the transmission rates are aware of and supported by the current channel states $\boldsymbol{s}(t)$. The indicator function $1_{[\mu_i(t) \le s_i(t)]}$ is required because of the possible blind scheduling mode. Let $P_i(t)$ be the sum of measurement and transmission power consumed by user i in slot t. We have

$$P_i(t) = \begin{cases} P_m/L + P_t 1_{[\mu_i(t) > 0]}, & \text{if channel } i \text{ is measured,} \\ P_t 1_{[\mu_i(t) > 0]}, & \text{otherwise.} \end{cases} \quad (2)$$

The unfinished work $U_i(t+1)$ of user i on time slot $t+1$ can thus be represented by the queuing dynamics:

$$U_i(t+1) = \max(U_i(t) - \hat{\mu}_i(t), 0) + a_i(t), \quad (3)$$

subject to the feasibility constraint $\boldsymbol{\mu}(t) \in \Omega$. We say that the wireless downlink is *stabilized* (by some scheduling policy) if the following inequality holds [2]:

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{i=1}^{L} \mathbb{E}[U_i(\tau)] < \infty.$$

We note that while our problem description focuses on a wireless downlink, the same system model can be directly applied to an uplink system, where a base station receives transmissions from L energy-aware wireless users. This can be seen as follows: A single queue is kept by each user to hold data to be transmitted (rather than keeping all L queues at the base station). On each slot, the base station coordinates by signaling different users to either measure or not measure their channels, and then, allocates uplink rates to be used by the users. The goal for the base station is to support data rates within full capacity, while the sum power expenditure due to transmission and channel acquisition by the users is minimized.

2.2 Motivating Examples

First of all, we give the following definitions:

Definition 1. *A scheduling policy is said to be* purely channel-aware *if packets are scheduled for transmission in a slot only after the channel states are acquired in that slot. A*

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scheduling policy is said to be purely channel-blind *if it never acquires channel states and can serve packets without the channel state information.*[1]

Definition 2. *Define the* blind capacity region Λ_{blind} *to be the closure of the set of data rates that can be stabilized by purely channel-blind policies. Define the* capacity region Λ *to be the closure of the set of data rates that can be stabilized by purely channel-aware policies.*

We note that purely channel-aware policies can emulate any policy that mixes channel-aware and channel-blind decisions, including purely channel-blind policies. Thus, Λ is the *exact* capacity region of the downlink, i.e., it contains data rates that can be stabilized by any scheduling policies. Further, we have $\Lambda_{blind} \subset \Lambda$.

For motivations, we compare the performance of purely channel-aware policies to that of purely channel-blind policies in the following simplified examples.

2.2.1 Single Queue

Consider a single queue served by a Bernoulli ON/OFF channel. Packets arrive independently over slots, and in every time slot, one packet arrives with probability λ and zero with probability $1-\lambda$ (and thus, the arrival rate is equal to λ). In every time slot, the channel is ON with probability q, and OFF with probability $1-q$. One packet can be served when the channel state is ON. It is easy to show that purely channel-aware and purely channel-blind scheduling share the same capacity region $\{0 \leq \lambda \leq q\}$, but the associated power consumption to support data rate λ may differ. Purely channel-aware scheduling consumes average power $(P_m/q) + P_t$ to deliver a packet (acquiring channel states, on average, $1/q$ times to see an ON state, and transmitting a packet once). Thus, the average power to support rate λ is $\lambda(P_m/q + P_t)$. Purely channel-blind scheduling consumes average power P_t/q to deliver a packet (blindly transmitting the same packet $1/q$ times for a successful transmission), and it takes average power $\lambda P_t/q$ to support λ. Therefore, to support any data rate $\lambda \in (0, q)$, we prefer purely channel-aware scheduling than purely channel-blind if $(P_m/q) + P_t \leq P_t/q$, i.e., $P_m/P_t \leq 1-q$, and we prefer purely channel-blind scheduling if $P_m/P_t > 1-q$. This simple threshold rule indicates that purely channel-blind scheduling may, depending on the power ratio and channel statistics, outperform purely channel-aware scheduling.

2.2.2 Multiple Queues

Consider the problem of allocating a server to L queues with independent Bernoulli ON/OFF channels. Channel states are i.i.d. over slots for each channel. Only the channel with the server can serve packets and at most one packet can be served in every slot. It is equivalent to setting $\mu_{max} = 1$ and $s_i(t) \in \{0, 1\}$ for all i and t. The feasible set Ω consists of L-dimensional $0/1$ vectors in each of which at most one entry is 1. Define q_i, $i = 1, 2, \ldots, L$, as the probability of channel i being ON, i.e., $\Pr\{s_i(t) = 1\} = q_i$. The following lemmas characterize capacity region Λ, blind capacity region Λ_{blind}, and the minimum power required to stabilize data rates within Λ and Λ_{blind} (proofs are given in the Appendix).

1. Note that both purely channel-aware and purely channel-blind scheduling may take advantage of queue backlog information.

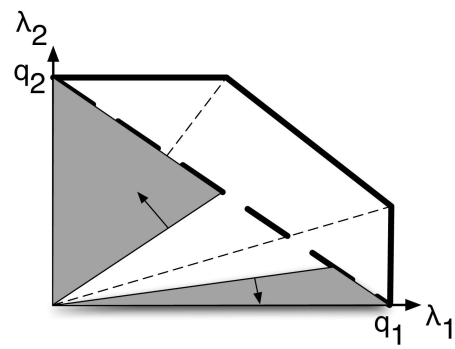

Fig. 1. For the case $L = 2$, Λ consists of data rates that are within the outer boundary (thick solid lines), while Λ_{blind} consists of data rates within the thick dotted line $\lambda_1/q_1 + \lambda_2/q_2 = 1$. Data rates in the shaded areas are those that prefer purely channel-blind scheduling. The shaded areas are decided under an additional assumption $q_1 \leq q_2$. As P_m/P_t decreases, the shaded areas shrink in the directions given in the figure.

Lemma 1. *The blind capacity region* Λ_{blind} *consists of data rates* $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_L)$ *satisfying* $\sum_{i=1}^{L} \lambda_i/q_i \leq 1$. *Further, over the class of purely channel-blind policies and for each arrival rate vector* $\boldsymbol{\lambda}$ *interior to* Λ_{blind}, *the system can be stabilized with minimum power* $(\sum_{i=1}^{L} \lambda_i/q_i)P_t$.

Lemma 2 (Theorem 1 in [17]). *The capacity region* Λ *consists of data rates* $\boldsymbol{\lambda}$ *satisfying, for each nonempty subset* I *of* $\{1, 2, \ldots, L\}$, $\sum_{i \in I} \lambda_i \leq 1 - \Pi_{i \in I}(1 - q_i)$.

Lemma 3. *Over the class of purely channel-aware policies and for each arrival rate vector* $\boldsymbol{\lambda} \neq \mathbf{0}$ *interior to* Λ, *the system can be stabilized with minimum power* $(\sum_{i=1}^{L} \lambda_i)P_t + \theta^* P_m$, *where* $\theta^* \triangleq \inf\{\theta \in (0, 1) \mid \boldsymbol{\lambda} \in \theta\Lambda\}$.

Consider the case $L = 2$. Lemmas 1 and 2 show that there is a capacity region difference between Λ and Λ_{blind} (See Fig. 1). Although data rates within Λ_{blind} can be supported by both purely channel-blind and purely channel-aware policies, we note that the shaded areas in Fig. 1 illustrate areas in which purely channel-blind scheduling is more energy efficient than purely channel-aware. The shaded areas would shrink if the power ratio P_m/P_t decreased. We also note that purely channel-blind scheduling cannot stabilize the system with arrival rates located outside Λ_{blind}, and channel-aware transmissions must be enforced. We will show in the next section that a power-optimal stabilizing policy might be *neither* purely channel-blind *nor* purely channel-aware. Rather, *mixed* strategies are typically required.

3 Optimal Power for Stability

In the following, we show a theorem characterizing the minimum power to stabilize data rates $\boldsymbol{\lambda}$ when dynamic channel acquisition is allowed, using similar techniques of proving [2, Theorem 1]. We show that the minimum average power required for stabilizing $\boldsymbol{\lambda}$ can be obtained by minimizing the average power expenditure over the class of stationary randomized policies that achieve time average transmission rate μ_i greater than or equal to λ_i for each user i. Each such stationary randomized policy makes decisions independently of queue backlog, and has the following structure: On every time slot, the controller probes the channels with some probability γ $(0 \leq \gamma \leq 1)$.

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If channel states $\boldsymbol{s}$ are acquired, the controller allocates channel-aware rates $\boldsymbol{\omega} \in \Omega(\boldsymbol{s})$ (that is, $\boldsymbol{\omega} \leq \boldsymbol{s}$) with some probability $\alpha(\boldsymbol{\omega}, \boldsymbol{s})$. If channels are not probed, the controller *blindly* allocates rates $\boldsymbol{\omega} \in \Omega$ with some probability $\beta(\boldsymbol{\omega})$.

Theorem 1. *For i.i.d. channel state processes and i.i.d. arrival processes with data rates $\boldsymbol{\lambda}$ interior to Λ, the minimum power consumption to stabilize the system is the optimal objective of the following problem $\mathcal{PROB}(\boldsymbol{\lambda})$ (defined in terms of auxiliary variables γ, $\alpha(\boldsymbol{\omega}, \boldsymbol{s})$ for each channel state vector $\boldsymbol{s} \in \mathcal{S}^L$ and each channel-aware rates $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_L) \in \Omega(\boldsymbol{s})$, and $\beta(\boldsymbol{\omega})$ for each $\boldsymbol{\omega} \in \Omega$):*

$$\begin{aligned}
\text{min.} \quad & \gamma \sum_{\boldsymbol{s} \in \mathcal{S}^L} \pi_{\boldsymbol{s}} \left[\sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega}, \boldsymbol{s}) \left(P_m + \sum_{i=1}^{L} 1_{[\omega_i > 0]} P_t \right) \right] \\
& + (1-\gamma) \sum_{\boldsymbol{\omega} \in \Omega} \beta(\boldsymbol{\omega}) \left(\sum_{i=1}^{L} 1_{[\omega_i > 0]} P_t \right)
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } \boldsymbol{\lambda} \leq & \ \gamma \sum_{\boldsymbol{s} \in \mathcal{S}^L} \pi_{\boldsymbol{s}} \left(\sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega}, \boldsymbol{s}) \boldsymbol{\omega} \right) \\
& + (1-\gamma) \sum_{\boldsymbol{\omega} \in \Omega} \beta(\boldsymbol{\omega}) (\boldsymbol{\omega} \otimes \boldsymbol{Pr}(\boldsymbol{S} \geq \boldsymbol{\omega})), \\
& 0 \leq \gamma \leq 1, \quad \beta(\boldsymbol{\omega}) \geq 0 \ \forall \boldsymbol{\omega} \in \Omega, \quad \sum_{\boldsymbol{\omega} \in \Omega} \beta(\boldsymbol{\omega}) = 1, \\
& \alpha(\boldsymbol{\omega}, \boldsymbol{s}) \geq 0 \ \forall \boldsymbol{s} \in \mathcal{S}^L \quad \text{and} \quad \forall \boldsymbol{\omega} \in \Omega(\boldsymbol{s}), \\
& \sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega}, \boldsymbol{s}) = 1 \ \forall \boldsymbol{s} \in \mathcal{S}^L,
\end{aligned}$$

where $\pi_{\boldsymbol{s}}$ is the steady-state probability of channel states $\boldsymbol{s}$, and the capital $\boldsymbol{S} = (S_1, \ldots, S_L)$ denotes a random vector of channel states. The vector $\boldsymbol{\omega} \otimes \boldsymbol{Pr}(\boldsymbol{S} \geq \boldsymbol{\omega})$ is defined according to notations given at the end of Section 1.

Proof of Theorem 1. Given in the Appendix. □

We denote by $P_{opt}(\boldsymbol{\lambda})$ the optimal objective of the above optimization problem $\mathcal{PROB}(\boldsymbol{\lambda})$, that is, $P_{opt}(\boldsymbol{\lambda})$ is the minimum average power to stabilize $\boldsymbol{\lambda}$. The following corollary of Theorem 1 will be used later in the performance analysis of our proposed control algorithm:

Corollary 1. *For i.i.d. arrival and channel state processes and an interior point $\boldsymbol{\lambda}$ of Λ, the optimal stationary randomized policy that supports $\boldsymbol{\lambda}$ allocates service rates $\boldsymbol{\mu}(t)$ and consumes power $(P_1(t), \ldots, P_L(t))$ that satisfy in every slot t,*

$$\sum_{i=1}^{L} \mathbb{E}[P_i(t)] = P_{opt}(\boldsymbol{\lambda}), \quad \mathbb{E}[\hat{\boldsymbol{\mu}}(t)] \geq \boldsymbol{\lambda},$$

where $\hat{\boldsymbol{\mu}}(t)$ are the effective transmission rates (see (1)).

We note that the capacity region Λ and the blind capacity region Λ_{blind} (see Definition 2) can also be characterized as corollaries of Theorem 1: If we restrict the policy space to the class of purely channel-aware policies and neglect the power, the proof of Theorem 1 characterizes the capacity region Λ. Restricting the policy space to the class of purely channel-blind policies and neglecting the power give us the blind capacity region Λ_{blind}.

Corollary 2. *For i.i.d. arrival and channel state processes, the capacity region Λ of the L-queue downlink consists of data rates $\boldsymbol{\lambda}$ for which there exists a probability distribution $\{\alpha(\boldsymbol{\omega}, \boldsymbol{s})\}_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})}$, where, for each channel state vector $\boldsymbol{s}$, we have $\alpha(\boldsymbol{\omega}, \boldsymbol{s}) \geq 0$ for all $\boldsymbol{\omega} \in \Omega(\boldsymbol{s})$ and $\sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega}, \boldsymbol{s}) = 1$, such that*

$$\boldsymbol{\lambda} \leq \sum_{\boldsymbol{s} \in \mathcal{S}^L} \pi_{\boldsymbol{s}} \left(\sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega}, \boldsymbol{s}) \boldsymbol{\omega} \right),$$

where $\pi_{\boldsymbol{s}}$ is the steady-state probability of states $\boldsymbol{s}$.

Corollary 3. *For i.i.d. arrival and channel state processes, the blind capacity region Λ_{blind} of the L-queue downlink consists of data rates $\boldsymbol{\lambda}$ for which there exists a probability distribution $\{\beta(\boldsymbol{\omega})\}_{\boldsymbol{\omega} \in \Omega}$, where $\beta(\boldsymbol{\omega}) \geq 0$ for all $\boldsymbol{\omega} \in \Omega$ and $\sum_{\boldsymbol{\omega} \in \Omega} \beta(\boldsymbol{\omega}) = 1$, such that*

$$\boldsymbol{\lambda} \leq \sum_{\boldsymbol{\omega} \in \Omega} \beta(\boldsymbol{\omega}) (\boldsymbol{\omega} \otimes \boldsymbol{Pr}(\boldsymbol{S} \geq \boldsymbol{\omega})).$$

4 The Unified Algorithm and Performance Analysis

4.1 Dynamic Channel Acquisition Algorithm

In the previous section, we established the minimum average power consumption required for stability. Here, we develop a unified DCA algorithm that provides stability for the full capacity region and meanwhile consumes average power that is arbitrarily close to minimum, with a trade-off in average delay. The algorithm is stated as follows (in terms of a positive control parameter V, chosen as desired to affect the trade-off): On each time slot, we observe the current queue backlog $\boldsymbol{U}(t) = (U_1(t), \ldots, U_L(t))$ and decide whether or not to probe the channels. We then allocate transmission rates based on the potentially known channel states. The associated decision variables $m(t)$, $\boldsymbol{\mu}^{(c)}(t)$, and $\boldsymbol{\mu}^{(b)}(t)$ are defined as follows: $m(t)$ is equal to 1 if channels are probed in slot t, and 0 otherwise. Variables $\boldsymbol{\mu}^{(c)}(t)$ represent feasible channel-aware transmission rates allocated when channels are probed, and $\boldsymbol{\mu}^{(b)}(t)$ represent blind transmission rates allocated when channels are not probed. Define $\boldsymbol{\chi}(t) \triangleq [m(t), \boldsymbol{\mu}^{(c)}(t), \boldsymbol{\mu}^{(b)}(t)]$ as the collection of control decision variables on slot t. The DCA algorithm observes the current queue backlog $\boldsymbol{U}(t)$ and chooses $\boldsymbol{\chi}(t)$ in every slot to maximize the function $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$, defined as follows:

$$\begin{aligned}
f(\boldsymbol{U}(t), \boldsymbol{\chi}(t)) \triangleq & \ m(t) \Bigg\{ -VP_m \\
& + \mathbb{E}_{\boldsymbol{s}} \left[\sum_{i=1}^{L} \left(2U_i(t)\mu_i^{(c)}(t) - VP_t 1_{[\mu_i^{(c)}(t) > 0]} \right) \Big| \ \boldsymbol{U}(t) \right] \Bigg\} \\
& + \overline{m}(t) \sum_{i=1}^{L} \left(2U_i(t)\mu_i^{(b)}(t) \Pr\left\{ S_i \geq \mu_i^{(b)}(t) \right\} - VP_t 1_{[\mu_i^{(b)}(t) > 0]} \right),
\end{aligned} \tag{4}$$

where $\overline{m}(t) \triangleq 1 - m(t)$.

The function $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$ comes from a performance Lyapunov analysis argument [2], [18]. In essence, for the sake of stability, the design principle here is that we would

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like to create a *negative drift* of queue backlogs in the wireless network whenever the queue backlogs are sufficiently large. Such negative drift keeps the queue backlogs bounded, and thus, ensures stability in the system. We show later in Theorem 2 and its proof that maximizing $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$ in every slot is a way to generate such negative drift. The structure of $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$ comes naturally in the underlying analysis. The constant factor of 2 in $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$ is also a by-product of the analysis. It can be dropped by using a new constant $\widetilde{V} = V/2$, because maximizing a function is equivalent to maximizing the function scaled by a positive constant. In addition, we note that blending the power consumption into the drift analysis guarantees that we can create the negative drift with average power that can be made arbitrarily close to optimal. See Theorem 2 and its proof later in the paper for more technical details.

Although $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$ looks complex at the first glance, maximizing it can be achieved as follows: First, we separately maximize the multiplicands of $m(t)$ and $\overline{m}(t)$ in (4). Then we compare them. We let $m(t) = 1$ if its optimal multiplicand is greater than that of $\overline{m}(t)$, and $m(t) = 0$, otherwise. If $m(t) = 1$, we measure the current channel states $\boldsymbol{s}(t)$ and allocate feasible rates $\boldsymbol{\mu}^{(c)}(t) \leq \boldsymbol{s}(t)$ as the maximizer of the sum

$$\sum_{i=1}^{L} \left(2U_i(t)\mu_i^{(c)}(t) - VP_t 1_{[\mu_i^{(c)}(t)>0]}\right). \tag{5}$$

Otherwise, $m(t) = 0$, and we blindly allocate feasible rates $\boldsymbol{\mu}^{(b)}(t) \in \Omega$ that maximize the multiplicand of $\overline{m}(t)$. We note that the option of idling the system is included in taking $\overline{m}(t) = 1$ and $\boldsymbol{\mu}^{(b)}(t) = \mathbf{0}$.

Intuitively, the DCA algorithm *estimates the expected gain* of channel-aware and channel-blind transmissions, and picks the one with the *better gain*. We observe that these estimations are the most complicated part of the algorithm, and they require joint and marginal distributions of channel states. In particular, in (4), the multiplicand of $m(t)$ is a conditional expectation taken over channel states $\boldsymbol{s}$. To maximize it, we need to optimize the quantity (5) over channel-aware rates $\boldsymbol{\mu}^{(c)}(t)$ supported by each channel state vector $\boldsymbol{s}$, and then, take an expectation of the optimized (5) using joint channel state distributions. Optimizing the multiplicand of $\overline{m}(t)$ needs the marginal distribution of channel states. In practice, the channel statistics can be estimated by taking samples of channel states and taking time averages of them. In Section 4.2, we show that the DCA algorithm can be done in polynomial time for the special case where at most one packet can be served in every slot, provided that channel statistics are known (or well estimated).

To prove the performance of the DCA algorithm, it is important to note that the function $f(\boldsymbol{U}(t), \boldsymbol{\chi}(t))$ can be rewritten as

$$f(\boldsymbol{U}(t), \boldsymbol{\chi}(t)) = \left(\sum_{i=1}^{L} 2U_i(t)\mathbb{E}_{\boldsymbol{s}}[\hat{\mu}_i(t) \mid \boldsymbol{U}(t)]\right) - V\mathbb{E}_{\boldsymbol{s}}\left[\sum_{i=1}^{L} P_i(t) \mid \boldsymbol{U}(t)\right],$$

where $\hat{\mu}_i(t)$ and $P_i(t)$ are, respectively, the effective service rate and the associated power consumption for user i, which can be represented as

$$\hat{\mu}_i(t) \triangleq m(t)\mu_i^{(c)}(t) + \overline{m}(t)\mu_i^{(b)}(t)1_{[\mu_i^{(b)}(t)\leq s_i(t)]},$$

$$P_i(t) \triangleq m(t)\left(\frac{P_m}{L} + P_t 1_{[\mu_i^{(c)}(t)>0]}\right) + \overline{m}(t)P_t 1_{[\mu_i^{(b)}(t)>0]}.$$

The following theorem characterizes the performance of the DCA algorithm:

Theorem 2. *For arrival rates $\boldsymbol{\lambda}$ interior to Λ, the DCA algorithm implemented with any control parameter $V > 0$ stabilizes the system with time average queue backlog and time average power expenditure satisfying:*

$$\limsup_{\tau\to\infty} \frac{1}{\tau}\sum_{t=0}^{\tau-1}\sum_{i=1}^{L} \mathbb{E}[U_i(t)] \leq \frac{B + V(P_m + LP_t)}{2\epsilon_{\max}}, \tag{6}$$

$$\limsup_{\tau\to\infty} \frac{1}{\tau}\sum_{t=0}^{\tau-1}\sum_{i=1}^{L} \mathbb{E}[P_i(t)] \leq \frac{B}{V} + P_{opt}(\boldsymbol{\lambda}), \tag{7}$$

where $B \triangleq (\mu_{\max}^2 + A_{\max}^2)L$, $\epsilon_{\max} > 0$ is the largest value such that $(\boldsymbol{\lambda} + \boldsymbol{\epsilon}_{\max}) \in \Lambda$, where $\boldsymbol{\epsilon}_{\max}$ is an all-$\epsilon_{\max}$ vector.

Proof of Theorem 2. Given in the Appendix. □

The two upper bounds in (6) and (7) are parameterized by the positive scalar V, where V can be chosen as large as desired to push the average power consumption arbitrarily close (within B/V) to the optimal $P_{opt}(\boldsymbol{\lambda})$, at the expense of the linearly increasing average congestion bound (which, by Little's theorem, yields an average delay bound).

As an aside, we note that the i.i.d. channel assumption is crucial to Theorem 1, and here we only show that the DCA algorithm is power and throughput optimal for i.i.d. channels. For general ergodic channels, the DCA algorithm may not be optimal. For example, in time-correlated channels, the history of observed channel states provides partial information of future channel states. Thus, an optimal policy for such channels should take advantage of the time correlations. The DCA algorithm, however, makes decisions only based on current queue backlogs, not on the history of the system (which includes the history of observed channel states, queue backlogs, and control actions). Thus, the DCA algorithm may not be optimal for general ergodic channels.

Despite the above, we note that DCA may still be a good suboptimal policy for general ergodic channels. In particular, it can be shown that the optimal power level $P_{opt}(\boldsymbol{\lambda})$ given in Theorem 1 for i.i.d. channels can also be achieved in the case of ergodic channels with the same steady-state distribution as the i.i.d. channels. Further, it can be shown that DCA achieves within $O(1/V)$ of this average power in the general ergodic case. While $P_{opt}(\boldsymbol{\lambda})$ is no longer optimal in the case of channels with memory, the DCA algorithm, in this case, can be viewed as optimizing over the restricted class of policies that neglect past channel history in making decisions. This simplifies algorithm design when channel correlations are too complex to track.

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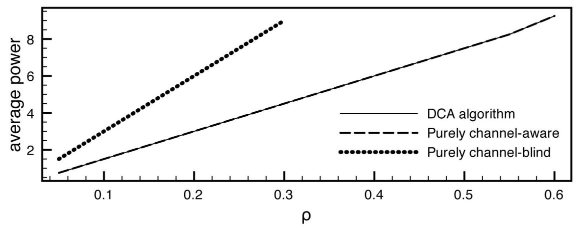

Fig. 2. Average power of the DCA algorithm and optimal pure policies for $P_m = 0$. Note that the curves of purely channel-aware and DCA overlap each other.

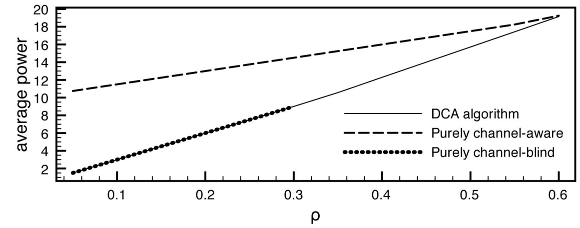

Fig. 3. Average power of the DCA algorithm and optimal pure policies for $P_m = 10$. Note that the curves of purely channel-blind and DCA overlap each other for $\rho \in [0.05, 0.3]$.

4.2 Server Allocation Problem and Algorithm Implementation

Consider the simplified example of the L-queue downlink given in Section 2.2.2, where at most one queue can be served in every slot, and this queue can transmit at most one packet. We show that the DCA algorithm, following the explanation in Section 4.1, can be simply implemented as follows: In each slot, if channel states are acquired, among all users with ON channel state, we allocate the server to the user with the largest positive $f_i^{(c)}(t) \triangleq 2U_i(t) - VP_t$. If $f_i^{(c)}(t)$ is nonpositive for all users with ON state, we idle the server. Otherwise, channels are not probed, and we allocate the server to the user with the largest positive $f_i^{(b)}(t) \triangleq 2U_i(t)\Pr\{S_i = ON\} - VP_t$. If $f_i^{(b)}(t)$ is nonpositive for all users, we idle the server.

To decide whether or not to acquire channel states, we compare the optimal multiplicands of $m(t)$ and $\overline{m}(t)$ in (4). Choosing the largest positive $f_i^{(b)}(t)$ gives us the optimal multiplicand of $\overline{m}(t)$. The optimal multiplicand of $m(t)$ can be computed as

$$-VP_m + \sum_{i=1}^{L}(2U_i(t) - VP_t)1_{[2U_i(t)>VP_t]} \Pr\Big\{s_i(t) = 1, \; s_j(t) < \frac{U_i(t)}{U_j(t)}, \forall j < i, \; s_k(t) \le \frac{U_i(t)}{U_k(t)}, \forall k > i\Big\}. \quad (8)$$

It is because we assign the server to user i when channel i in ON ($s_i(t) = 1$), $f_i^{(c)}(t) > 0$, and $U_i(t) \ge s_j(t)U_j(t)$ for all $j \ne i$ (we break ties by choosing the smallest index), which occurs with probability

$$\Pr\Big\{s_i(t) = 1, s_j(t) < \frac{U_i(t)}{U_j(t)}, \forall j < i, \; s_k(t) \le \frac{U_i(t)}{U_k(t)}, \forall k > i\Big\}.$$

Then we acquire channel states if the optimal multiplicand of $m(t)$ is greater than that of $\overline{m}(t)$.

Again, unlike the EECA algorithm in [2] which does not require channel statistics, the DCA algorithm indeed requires the joint channel statistics to make channel acquisition decisions. If channels are independent, only the marginal distribution of each channel is required. For example, in the case of independent i.i.d. Bernoulli ON/OFF channels, (8) can be easily computed in polynomial time and requires storage of only the known marginals. When channels have spatial correlations, (8) can also be easily computed in polynomial time, provided that the joint cumulative distribution function is known or estimated.

5 Simulations

5.1 Multirate Channels

We simulate the DCA algorithm for server allocation in a symmetric three-user downlink, defined as follows: Three users have independent Poisson arrivals with equal rates $\boldsymbol{\lambda} = \rho(1, 1, 1)$, where ρ is a scaling factor. Each user is served over an independent i.i.d. channel, which has three states $\{G, M, B\}$. In state G, M, and B, the channel can serve at most 2, 1, and 0 packets, respectively. For each channel i in slot t, define probability $q_G \triangleq \Pr\{s_i(t) = G\} = 0.5$. Probabilities q_M and q_B are defined similarly with $q_M = 0.3$ and $q_B = 0.2$. In every slot, the controller picks at most one user to serve.

The maximum sum throughput of the downlink is

$$\begin{aligned} &2\Pr\{\text{at least one channel is } G\} \\ &\quad + 1\Pr\{\text{none of the channels is } G, \text{at least one is } M\} \\ &= 2[1 - (1 - q_G)^3] + (1 - q_G)^3 - (1 - q_G - q_M)^3 = 1.867. \end{aligned}$$

Thus, one face of the capacity region boundary satisfies $\lambda_1 + \lambda_2 + \lambda_3 \le 1.867$, and the scaled vector $\rho(1,1,1)$ intersects this face at $\rho \approx 0.622$. In blind transmission mode, the maximum sum throughput of the downlink is equal to $2 \cdot q_G = 1$. One face of the boundary of Λ_{blind} is $\lambda_1 + \lambda_2 + \lambda_3 = 1$, and $\rho(1,1,1)$ intersects it at $\rho \approx 0.33$. According to these boundary information, we simulate the DCA algorithm for ρ from 0.05 to 0.6 with step size 0.05. Transmission power P_t is set to be 10 units, and each simulation is run for 10 million time slots.

Figs. 2 and 3 compare the power consumption of DCA with the theoretical minimum of purely channel-aware and purely channel-blind policies for $P_m = 0$ and 10, respectively. Parameter V is set to 100. The theoretical power minimum of pure policies is computed by solving the optimization problem in Theorem 1. The curve of purely channel-blind is drawn up to $\rho = 0.3$, a point close to the boundary of Λ_{blind}.

When $P_m = 0$, channel states can be acquired with no cost, and it is always better to probe the channels before allocating rates. Therefore, purely channel-aware is no worse than any mixed strategies, and thus, is optimal. Fig. 2 shows that DCA consumes the same average power as the optimal purely channel-aware policy for all values of data rates. For $P_m = 10$, it is sufficiently large so that channel-blind transmissions are more energy efficient than channel-aware ones for $\boldsymbol{\lambda} \in \Lambda_{blind}$. In this case, Fig. 3 shows that DCA performs as good as the optimal purely channel-blind policy for $\boldsymbol{\lambda} \in \Lambda_{blind}$ (i.e., $0 < \rho < 0.33$). When data

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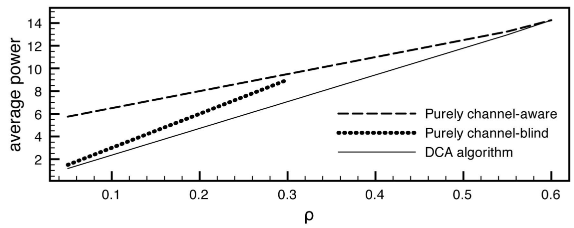

Fig. 4. Average power of the DCA algorithm and optimal pure policies for $P_m = 5$.

rates go beyond Λ_{blind} ($\rho \geq 0.33$), DCA starts to incorporate channel-aware transmissions for stability concerns, but still yields a significant power gain over purely channel-aware policies. These two cases show that at extreme values of P_m, the DCA algorithm is adaptive and optimal.

Fig. 4 shows the performance of DCA for $P_m = 5$. For a mediate value of P_m, the DCA algorithm outperforms both types of pure policies. To take a closer look, for a fixed arrival rate vector $\boldsymbol{\lambda} = (0.3, 0.3, 0.3)$, Fig. 5 shows the power gain of DCA over pure policies as a function of P_m values. One important observation here is that DCA has the largest power gain when purely channel-aware and purely channel-blind have the same performance (around $P_m = 4.5$). It is counter intuitive because when the two types of pure policies perform the same, we expect that mixing channel-aware and channel-blind actions will not help. Nevertheless, the simulation shows that we benefit more from mixing strategies especially when one type of pure policy does not significantly outperform the other. In this example, DCA has as much as a 30 percent power gain over purely channel-aware and purely channel-blind policies.

5.2 ON/OFF Channels

To have more insights on how DCA works, we perform another set of simulations. The simulation setup is the same as the previous one except for the channel model. Here, we suppose that each user is served by an independent i.i.d. Bernoulli ON/OFF channel. In every slot, channels 1, 2, and 3 are ON with probability 0.8, 0.5, and 0.2, respectively. When a channel is ON, one packet can be served, and zero otherwise. We simulate on arrival rates of the form $\rho(3, 2, 1)$.

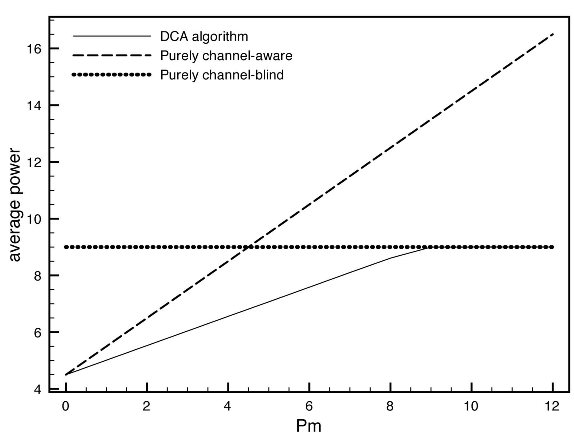

Fig. 5. Average power of the DCA algorithm and optimal pure policies for different values of P_m. Note that the curve of the DCA algorithm overlaps with other curves at both ends of P_m values.

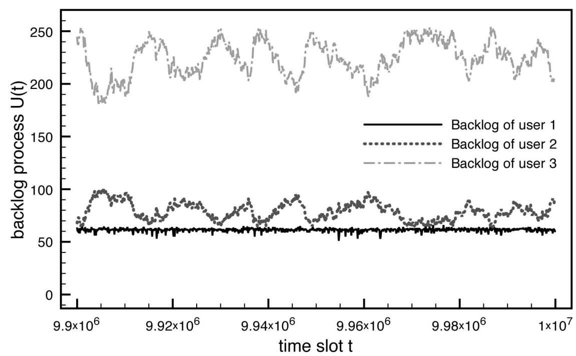

Fig. 6. Sample backlog processes in the last 10^5 slots of the simulation.

It is easy to show that $\rho \approx 0.1533$ and 0.0784 correspond to the boundary of the associated Λ and Λ_{blind}, respectively. We set $P_t = 10$, and each simulation is again run for 10 million time slots.

5.2.1 User Backlogs

We first simulate on $V = P_m = 10$ and $\rho = 0.07$. Fig. 6 shows sample backlog processes of the three users in the last 10^5 time slots of the simulation. We observe in (4) that we blindly serve user 3 only if $U_3(t) \geq VP_t/(2q_3) = 250$. We serve user 3 in channel-aware mode if $U_3(t) \geq VP_t/2 = 50$. Fig. 6 shows that most of the time, user 3 maintains its backlog under 250 (but above 50). It is consistent with an observation that user 3 serves packets mostly in channel-aware mode. As mentioned in Section 4, the DCA algorithm generates a negative drift pushing the backlog of user 3 back under 250 whenever it is above 250. It explains why the user 3 backlog is maintained around 250. Similar arguments can be made for the other two users, where the reference backlog levels for negative drifts for users 1 and 2 are 62.5 and 100, respectively. We note that much of this backlog can be eliminated by using the *place holder packet* technique from [19]. Indeed, from (4), we see that no packet is ever transmitted from queue i if $2U_i(t) < VP_t$, and so place holder packets reduce average backlog by roughly $VP_t/2$ in each queue, with no loss of energy optimality.

For different values of ρ, we show in Fig. 7 the average backlog of each user and the sum average backlog of the system. The DCA algorithm maintains roughly constant average backlogs (around the reference point $VP_t/(2q_i)$ mentioned earlier) for all users, except when data rates are close to the capacity region boundary. When data rates

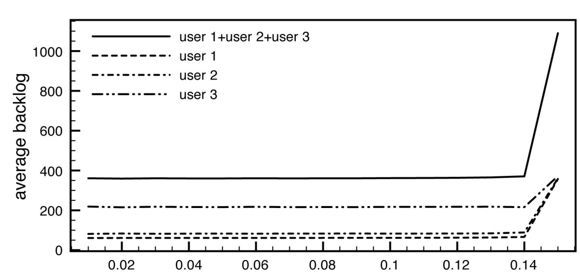

Fig. 7. Average backlogs of three users and the average sum backlog of the system.

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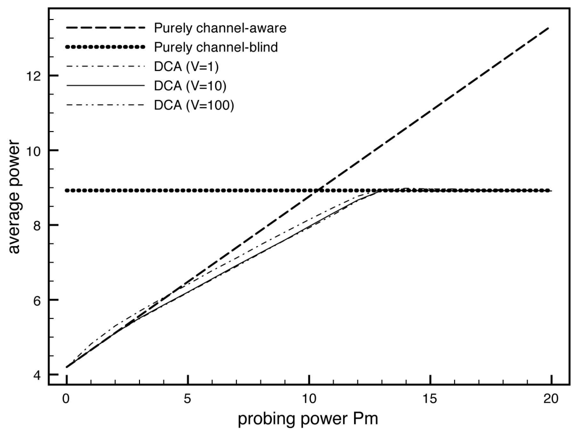

Fig. 8. Average power consumption of the DCA algorithm and optimal pure policies under different values of V and $\rho = 0.07$.

approach the capacity region boundary up to some point, the negative drift cannot withhold the rapid increase of average backlog.

5.2.2 How Control Parameter V Affects the Performance

For control parameter $V \in \{1, 10, 100\}$, Figs. 8 and 9 show the average power consumption of DCA (with different P_m values) and the average sum backlogs. We see that as V increases, the power consumption improves (but the improvement gets small) at the expense of increasing network delays, which are suggested by (6) and (7).[2] Figs. 8 and 9 also demonstrate that, in practice, a moderate V value should be chosen to maintain reasonable network delays without sacrificing much power consumption.[3]

6 Application to More General Models

6.1 Channel Acquisition-Induced Timing Overhead

We generalize the system model in Section 2 by assuming that channel acquisition indeed consumes time. Specifically, suppose in every slot, channel acquisition consumes $(1 - \beta)$ of a slot for some $0 < \beta < 1$. Therefore, if channel states are acquired and rate vector $\boldsymbol{\mu}(t)$ is allocated, the true allocated service rates are $\beta\boldsymbol{\mu}(t)$.

This timing overhead affects the system dramatically. For example, the class of purely channel-aware policies can no longer support the original capacity region Λ because the new capacity region for purely channel-aware policies is $\beta\Lambda$. As a result, there may be some data rates that cannot be supported by purely channel-aware policies, but can be supportable by purely channel-blind policies. So, for the sake of stability, blind data transmissions must be incorporated. In fact, it can be shown that the capacity region of the new system with timing overhead, denoted by Λ_{new}, is the convex hull of the union of $\beta\Lambda$ and Λ_{blind} (see Fig. 10).[4]

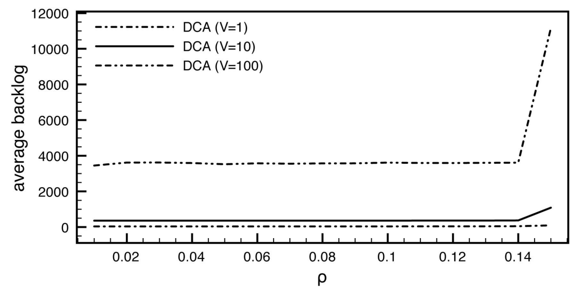

Fig. 9. The average backlog of the DCA algorithm for different values of V. P_m is set to 10.

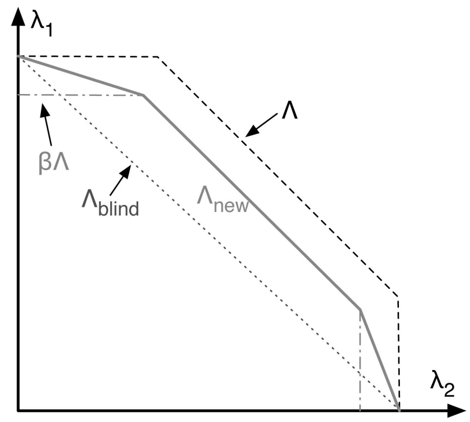

Fig. 10. The new capacity regions of a two-queue wireless downlink with independent i.i.d. ON/OFF channels, server allocation constraint, and timing overhead assumption.

For example, consider the simplified two-queue downlink with the server allocation constraint in Section 2.2.2. In Fig. 10, Λ is the original capacity region without the timing overhead assumption. $\beta\Lambda$ is the new capacity region for purely channel-aware policies with timing overhead. Λ_{blind} is the capacity region of purely channel-blind policies, not affected by the timing overhead. Note that data rates within the set $\Lambda_{blind} - \beta\Lambda$ are supported by purely channel-blind policies, but not by purely channel-aware policies.

Although the timing overhead assumption changes the system characteristics dramatically, extending the DCA algorithm to the new system is easy. We only need to substitute every channel-aware rate allocation $\boldsymbol{\mu}(t)$ with the vector $\beta\boldsymbol{\mu}(t)$. As an example, the new DCA algorithm applied to the simplified two-queue downlink in Section 2.2.2 can be shown as follows:

Example DCA algorithm with timing overhead.

1. Channel acquisition: At the beginning of every slot, define

$$f_1 \triangleq -VP_m + q_1 q_2 [2\beta \max(U_1(t), U_2(t)) - VP_t]^+ + q_1 \overline{q}_2 [2\beta U_1(t) - VP_t]^+ + \overline{q}_1 q_2 [2\beta U_2(t) - VP_t]^+,$$

2. We note that performance bounds (6) and (7) are loose for Poisson arrivals because the associated $A_{max} = \infty$ (and thus, $B = \infty$). It is a minor concern because A_{max} is bounded with probability arbitrarily close to one, and bound (6) is well known to be loose.

3. We note that in Fig. 8, DCA does not beat purely channel-aware when $V = 1$ and P_m close to zero. It may be because smaller V values do not allow DCA to maintain reasonable backlog levels in all queues, and as a result, opportunistic scheduling gain cannot be fully enjoyed in channel-aware mode. It is a minor concern, but still suggests that a moderate V value is needed.

4. It can be proved by following the proof of Theorem 1, where we neglect the power part and substitute every channel-aware rates $\boldsymbol{\mu}(t)$ with the scaled rate vector $\beta\boldsymbol{\mu}(t)$. We can see that together with Corollaries 2 and 3, the new capacity region Λ_{new} consists of data rates $\boldsymbol{\lambda}$ for which there exists a scalar γ, $0 \leq \gamma \leq 1$, such that $\boldsymbol{\lambda} \in \gamma(\beta\Lambda) + (1-\gamma)\Lambda_{blind}$.

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where $\overline{q_i} \triangleq 1 - q_i$, and $[x]^+ \triangleq \max(x, 0)$. Define

$$f_2 \triangleq [\max(2q_1U_1(t), 2q_2U_2(t)) - VP_t]^+.$$

Then, we probe the channels if and only if $f_1 > f_2$.

2. Server allocation: If channel states are acquired, serve the ON channel with the largest positive $(2\beta U_i(t) - VP_t)$. Do not serve any channel if all channels are OFF or $(2\beta U_i(t) - VP_t) \le 0$ for all ON channels. If channel states are not acquired, blindly serve the channel with the largest positive $(2q_iU_i(t) - VP_t)$. Do not serve any channel if $(2q_iU_i(t) - VP_t) \le 0$ for all i.

6.2 Partial Channel State Acquisition

In the system model in Section 2, we restrict possible actions on channel acquisition to measuring either all channels or none of the channels. To relax this assumption, suppose now we allow acquiring the states of *any subset of channels* in every slot (such partial channel probing model is first introduced in [10]). Specifically, we denote by $J(t) \subset \{1, 2, \dots, L\}$ the subset of channels that are measured on slot t. For example, $J(t) = \{3, 5\}$ indicates that the third and the fifth channels are measured on slot t. After channels in $J(t)$ are measured, rates $\boldsymbol{\mu}(t) = (\mu_1(t), \dots, \mu_L(t))$ are allocated in the way that $\mu_i(t)$ is chosen aware of the current channel state if $i \in J(t)$, and is chosen blindly if $i \notin J(t)$. The effective service rate $\hat{\mu}_i(t)$ and the associated power consumption $P_i(t)$ for user i can thus be represented as

$$\hat{\mu}_i(t) = 1_{[i\in J(t)]}\mu_i(t) + 1_{[i\notin J(t)]}\mu_i(t)1_{[\mu_i(t)\le s_i(t)]}, \tag{9}$$

$$P_i(t) = 1_{[i\in J(t)]}\left(\frac{P_m}{L} + P_t1_{[\mu_i(t)>0]}\right) + 1_{[i\notin J(t)]}P_t1_{[\mu_i(t)>0]}. \tag{10}$$

To derive the corresponding DCA algorithm, we note that the same drift analysis in the proof of Theorem 2 can be directly applied here by substituting (9) and (10) into the drift analysis. In particular, the drift inequality (35) still holds. Again, the DCA algorithm is designed to minimize the right side of (35) over all feasible decision options. The difference is that now our decisions allow for measuring only a subset of the channels. We describe the corresponding DCA algorithm as follows:

Generalized DCA algorithm. In every slot t,

1. Channel acquisition: Suppose the current backlog is $\boldsymbol{U}(t)$. For each $J(t) \subset \{1, 2, \dots, L\}$, we define $H_{J(t)} \subset \mathcal{S}^{|J(t)|}$ as a set of the probed channel states, i.e., $H_{J(t)} = \{s_i(t), i \in J(t)\}$. We define $H_{J(t)} = \emptyset$ if $J(t) = \emptyset$. Then, for each $J(t)$ and for each possible set $H_{J(t)}$ of observed partial channel states associated with $J(t)$, we define $g^*(J(t), \boldsymbol{U}(t), H_{J(t)})$ as the maximum of the quantity

$$\begin{aligned}&\sum_{i\in J(t)}\left(2U_i(t)\mu_i(t) - V\left(\frac{P_m}{L} + P_t1_{[\mu_i(t)>0]}\right)\right)\\ &\quad + \sum_{i\notin J(t)}\big(2U_i(t)\mu_i(t)\Pr\{S_i \ge \mu_i(t) \mid H_{J(t)}\}\\ &\quad - VP_t1_{[\mu_i(t)>0]}\big)\end{aligned} \tag{11}$$

over $(\mu_1(t), \dots, \mu_L(t)) \in \Omega$ and $\mu_i(t) \le s_i(t)$ if $i \in J(t)$. We denote by $\boldsymbol{\mu}^*(J(t), \boldsymbol{U}(t), H_{J(t)})$ the maximizer of (11). Then we compute the quantity $\underline{g}^*(J(t), \boldsymbol{U}(t))$ defined by

$$\underline{g}^*(J(t), \boldsymbol{U}(t)) \triangleq \mathbb{E}\{g^*(J(t), \boldsymbol{U}(t), H_{J(t)})\},$$

where the expectation is taken over $H_{J(t)}$. Define

$$J^*(t) \triangleq \arg\max_{J(t)} \underline{g}^*(J(t), \boldsymbol{U}(t)).$$

If $\max_{J(t)}\underline{g}^*(J(t), \boldsymbol{U}(t))$ is positive, we probe the channels in $J^*(t)$ (no channel is probed if $J^*(t) = \emptyset$). Otherwise, it is easy to show that $\max_{J(t)}\underline{g}^*(J(t), \boldsymbol{U}(t)) = 0$ and we idle the system (including skipping the Rate Allocation step described next).

2. Rate allocation: Suppose the set of acquired channel states in $J^*(t)$ is $H_{J^*(t)}$. We allocate to the system the rates $\boldsymbol{\mu}^*(J^*(t), \boldsymbol{U}(t), H_{J^*(t)})$ (computed in the previous step).

Essentially, in our original system model, the decision space of channel measurement has only two elements: measure all or none of the channels. Allowing to measure any subset of channels expands the measurement decision space from cardinality 2 to 2^L, where L is the number of channels. It is not hard to show that the corresponding Theorem 1 to the generalized system demonstrates that an optimal policy that stabilizes rates $\boldsymbol{\lambda}$ with minimum power consumption is a convex combination of 2^L stationary randomized policies, each of which corresponds to a partial channel acquisition decision $J(t)$. The generalized DCA algorithm evaluates a metric for each of the 2^L measurement decisions and executes one with the best metric. Note that the associated computational complexity is exponential. We may restrict the measurement space to a predefined collection of subsets of channels, from which the set of channels to probe must be chosen. In this way, we incorporate a more general system model than our original one, but the complexity of the DCA algorithm does not grow exponentially in the number of channels.

7 Conclusion

Under the assumption that channel acquisition incurs power overhead, we propose a DCA algorithm that dynamically acquires channel states to stabilize a wireless downlink. DCA is a unified treatment of incorporating both channel-aware and channel-blind actions to achieve energy optimality. Through Lyapunov analysis, we prove that DCA can stabilize the system with average power arbitrarily close to optimal, at the expense of increasing network delays. Simulations show that DCA can optimally adapt to different system parameters, including input data rates and transmission and channel probing power. DCA has the largest power gain when both types of pure strategies have the same performance, which is counter intuitive. We also discussed how to extend the DCA algorithm to two generalized models of nonzero timing overhead and completely partial channel acquisition.

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APPENDIX

Proof of Lemma 1. First, assume that rates $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_L)$ can be stabilized by some purely channel-blind policy. Since a successful blind packet transmission through channel $i \in \{1, 2, \dots, L\}$ takes, on average, $1/q_i$ attempts, the fraction of time the single server is busy is equal to $\sum_{i=1}^{L} \lambda_i/q_i$ (by Little's theorem). This must be less than or equal to 1 for stability. The associated necessary average power consumption is equal to $(\sum_{i=1}^{L} \lambda_i/q_i)P_t$. Conversely, for each rate vector $\boldsymbol{\lambda} \neq \mathbf{0}$ satisfying $\sum_{i=1}^{L} \lambda_i/q_i < 1$, we define $\rho \triangleq \sum_{i=1}^{L} \lambda_i/q_i$, and there exists some $\epsilon > 0$ such that $\rho + \epsilon < 1$. Consider the policy that, in every slot, assigns the server to queue i with probability $(\rho + \epsilon)\alpha_i$, where $\alpha_i \triangleq \lambda_i/(q_i\rho)$, and is idle with probability $1 - \rho - \epsilon$. The associated average power consumption is $(\rho + \epsilon)P_t$. It is easy to see that this policy yields average transmission rates strictly greater than $\boldsymbol{\lambda}$ entrywise, and thus, stabilizes the system. By passing $\epsilon \to 0$, the rate vector $\boldsymbol{\lambda}$ is stabilized with average power consumption arbitrarily close to $\rho P_t = (\sum_{i=1}^{L} \lambda_i/q_i)P_t$. □

Proof of Lemma 3. Suppose the rate vector $\boldsymbol{\lambda}$ can be stabilized by a purely channel-aware policy Φ. For simplicity, we assume that Φ is ergodic with well-defined time averages (the general case can be proven similarly, as in [2]). Define θ to be the fraction of time Φ probes the channels. Then the average power consumption to stabilize $\boldsymbol{\lambda}$ is $(\sum_{i=1}^{L} \lambda_i)P_t + \theta P_m$. Suppose θ satisfies $0 < \theta < \theta^*$. A necessary condition to stabilize $\boldsymbol{\lambda}$ is that for each subset $J \subset \{1, 2, \dots, L\}$ of channels, the partial sum $\sum_{i \in J} \lambda_i$ must be less than or equal to the fraction of time at least one channel in J can serve packets. In other words, for each J, we have

$$\sum_{i \in J} \lambda_i \leq \theta \left(1 - \prod_{i \in J} (1 - q_i) \right).$$

In other words, $\boldsymbol{\lambda} \in \theta\Lambda$. But this contradicts the definition of θ^*, finishing the proof of the necessity part.

Conversely, since $\boldsymbol{\lambda} \in \theta^*\Lambda$, $\boldsymbol{\lambda}$ is an interior point of the set $(\theta^* + \epsilon)\Lambda$ for some $\epsilon > 0$ satisfying $\theta^* + \epsilon < 1$. We define a policy Φ that works as follows: On every slot, Φ probes the channels with probability $(\theta^* + \epsilon)$. When channels are probed in a slot, we serve the longest ON queue. Otherwise, we idle the system. We note that applying Φ to the original downlink is equivalent to applying Longest Connected Queue (LCQ) policy [17] to a new wireless downlink in which a channel is ON if and only if it is probed and known to be ON. It is easy to see that the capacity region of the new system is $(\theta^* + \epsilon)\Lambda$. Although channels in different links in the new system are correlated, it is well known that LCQ policy is still throughput optimal to the new system. Equivalently, if $\boldsymbol{\lambda}$ is interior to $(\theta^* + \epsilon)\Lambda$, policy Φ stabilizes $\boldsymbol{\lambda}$ with average power consumption equal to $(\sum_{i=1}^{L} \lambda_i)P_t + (\theta^* + \epsilon)P_m$. Passing $\epsilon \to 0$ finishes the sufficiency part of the proof. □

Proof of Theorem 1. Suppose rates $\boldsymbol{\lambda}$ are interior to Λ. We first show the necessity part of the proof saying that there is no policy able to stabilize $\boldsymbol{\lambda}$ with average power strictly less than $P_{opt}(\boldsymbol{\lambda})$. Then we finish the sufficiency part of the proof by showing that rates $\boldsymbol{\lambda}$ can be stabilized with average power arbitrarily close to $P_{opt}(\boldsymbol{\lambda})$.

(Necessity). Suppose rates $\boldsymbol{\lambda}$ can be stabilized by a policy Φ which decides in which slots channels are probed and allocates transmission rates $\boldsymbol{\mu}(t)$ with power consumption $(P_1(t), \dots, P_L(t))$ (see (2)) in every slot t. In an interval $[0, M)$, $M \in \mathbb{N}$, we define

$$\hat{\boldsymbol{\mu}}_{av}(M) \triangleq \frac{1}{M} \sum_{\tau=0}^{M-1} \hat{\boldsymbol{\mu}}(\tau), \tag{12}$$

as the empirical service rates of policy Φ (rates $\hat{\boldsymbol{\mu}}(\tau)$ are defined in (1)). We define

$$P_{av}(M) \triangleq \frac{1}{M} \sum_{\tau=0}^{M-1} \sum_{i=1}^{L} P_i(\tau), \tag{13}$$

as the empirical average power consumption of Φ. Let

$$\overline{P}_{av} \triangleq \liminf_{M \to \infty} P_{av}(M). \tag{14}$$

$\overline{P}_{av}$ is a lower bound on the average power consumption of Φ.[5] We show in the following that for any $\boldsymbol{\lambda}$-stabilizing policy Φ (ergodic or nonergodic), there exists a stationary randomized policy $\hat{\Phi}$ that consumes average power $\overline{P}_{av}$ and yields average service rates $\boldsymbol{\mu} \geq \boldsymbol{\lambda}$. In particular, policy $\hat{\Phi}$ is *feasible* to the optimization problem $\mathcal{PROB}(\boldsymbol{\lambda})$ (defined in Theorem 1) in the sense that the associated parameters of $\hat{\Phi}$ constitute a feasible point of $\mathcal{PROB}(\boldsymbol{\lambda})$. Consequently, we have $\overline{P}_{av} \geq P_{opt}(\boldsymbol{\lambda})$. By (14), policy Φ consumes average power at least $\overline{P}_{av} \geq P_{opt}(\boldsymbol{\lambda})$. This is true for any $\boldsymbol{\lambda}$-stabilizing policy Φ. Thus, the necessary power to stabilize $\boldsymbol{\lambda}$ is at least $P_{opt}(\boldsymbol{\lambda})$.

Let $T_M^{(c)}$ and $T_M^{(b)}$ be the sets of slots in $[0, M)$ in which channels are probed and not probed, respectively. Note that $|T_M^{(c)}| + |T_M^{(b)}| = M$ for all M. Without loss of generality, we assume that $T_M^{(c)}$ and $T_M^{(b)}$ are nonempty. According to (1), we define

$$\hat{\boldsymbol{\mu}}_{av}^{(c)}(M) \triangleq \frac{1}{M} \sum_{\tau \in T_M^{(c)}} \hat{\boldsymbol{\mu}}(\tau) = \frac{1}{M} \sum_{\tau \in T_M^{(c)}} \boldsymbol{\mu}(\tau, \boldsymbol{s}(\tau)), \tag{15}$$

$$\hat{\boldsymbol{\mu}}_{av}^{(b)}(M) \triangleq \frac{1}{M} \sum_{\tau \in T_M^{(b)}} \hat{\boldsymbol{\mu}}(\tau) = \frac{1}{M} \sum_{\tau \in T_M^{(b)}} \boldsymbol{\mu}(\tau) \otimes \mathbf{1}_{[\boldsymbol{\mu}(\tau) \leq \boldsymbol{s}(\tau)]}, \tag{16}$$

where $\mathbf{1}_{[\boldsymbol{\mu}(\tau) \leq \boldsymbol{s}(\tau)]}$ is defined at the end of Section 1. It is easy to see that $\hat{\boldsymbol{\mu}}_{av}(M)$ (defined in (12)) satisfies

$$\hat{\boldsymbol{\mu}}_{av}(M) = \hat{\boldsymbol{\mu}}_{av}^{(c)}(M) + \hat{\boldsymbol{\mu}}_{av}^{(b)}(M). \tag{17}$$

We next define

$$P_{av}^{(c)}(M) \triangleq \frac{1}{M} \sum_{\tau \in T_M^{(c)}} \sum_{i=1}^{L} P_i(\tau), \tag{18}$$

5. When Φ is ergodic, limiting time averages are well defined and $\overline{P}_{av}$ is the exact limiting time average power consumption of Φ. If Φ is nonergodic, limiting time averages may not exist and $\overline{P}_{av}$ is a lower bound on the average power consumption of Φ over an infinite horizon.

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$$P_{av}^{(b)}(M) \triangleq \frac{1}{M} \sum_{\tau \in T_M^{(b)}} \sum_{i=1}^{L} P_i(\tau), \tag{19}$$

and $P_{av}(M)$ (defined in (13)) satisfies

$$P_{av}(M) = P_{av}^{(c)}(M) + P_{av}^{(b)}(M). \tag{20}$$

Consider the rate-power vector $(\hat{\boldsymbol{\mu}}_{av}^{(c)}(M); P_{av}^{(c)}(M))$ associated with channel-aware transmissions. By (15), (18), and simple arithmetics, we have

$$\begin{aligned}
\left(\hat{\boldsymbol{\mu}}_{av}^{(c)}(M); P_{av}^{(c)}(M)\right) &= \frac{1}{M} \sum_{\tau \in T_M^{(c)}} \left(\boldsymbol{\mu}(\tau, \boldsymbol{s}(\tau)); \sum_{i=1}^{L} P_i(\tau) \right) \\
&= \frac{|T_M^{(c)}|}{M} \sum_{\boldsymbol{s} \in \mathcal{S}^L} \frac{|T_M^{(c)}(\boldsymbol{s})|}{|T_M^{(c)}|} \frac{1}{|T_M^{(c)}(\boldsymbol{s})|} \sum_{\tau \in T_M^{(c)}(\boldsymbol{s})} \left(\boldsymbol{\mu}(\tau, \boldsymbol{s}); \sum_{i=1}^{L} P_i(\tau) \right) \\
&= \gamma_M \sum_{\boldsymbol{s} \in \mathcal{S}^L} \sigma_M(\boldsymbol{s}) \, \boldsymbol{x}_M(\boldsymbol{s}),
\end{aligned} \tag{21}$$

where we define

$$\gamma_M \triangleq \frac{|T_M^{(c)}|}{M}, \quad \sigma_M(\boldsymbol{s}) \triangleq \frac{|T_M^{(c)}(\boldsymbol{s})|}{|T_M^{(c)}|}, \tag{22}$$

$T_M^{(c)}(\boldsymbol{s}) \subset T_M^{(c)}$ is defined as the subset of slots in $T_M^{(c)}$ in which channel states are $\boldsymbol{s}$, and

$$\boldsymbol{x}_M(\boldsymbol{s}) \triangleq \frac{1}{|T_M^{(c)}(\boldsymbol{s})|} \sum_{\tau \in T_M^{(c)}(\boldsymbol{s})} \left(\boldsymbol{\mu}(\tau, \boldsymbol{s}); \sum_{i=1}^{L} P_i(\tau) \right). \tag{23}$$

Observe that $\boldsymbol{x}_M(\boldsymbol{s})$ is a convex combination of vectors of the form $(\boldsymbol{\mu}(\tau, \boldsymbol{s}); \sum_{i=1}^{L} P_i(\tau))$. By regrouping terms in (23), there exists a real sequence $\{\alpha_M(\boldsymbol{\omega}, \boldsymbol{s})\}_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})}$, $\alpha_M(\boldsymbol{\omega}, \boldsymbol{s}) \geq 0$ for each $\boldsymbol{\omega} \in \Omega(\boldsymbol{s})$ and $\sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha_M(\boldsymbol{\omega}, \boldsymbol{s}) = 1$, such that $\boldsymbol{x}_M(\boldsymbol{s})$ can be rewritten as

$$\boldsymbol{x}_M(\boldsymbol{s}) = \sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha_M(\boldsymbol{\omega}, \boldsymbol{s}) \left(\boldsymbol{\omega}; P_m + \sum_{i=1}^{L} 1_{[\omega_i > 0]} P_t \right). \tag{24}$$

We note that the sequence $\{\alpha_M(\boldsymbol{\omega}, \boldsymbol{s})\}_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})}$ can be viewed as a probability distribution.

Next, the rate-power vector $(\hat{\boldsymbol{\mu}}_{av}^{(b)}(M); P_{av}^{(b)}(M))$ associated with channel-blind transmissions satisfies

$$\begin{aligned}
&(\hat{\boldsymbol{\mu}}_{av}^{(b)}(M); P_{av}^{(b)}(M)) \\
&\quad = \frac{1}{M} \sum_{\tau \in T_M^{(b)}} \left(\boldsymbol{\mu}(\tau) \otimes \mathbf{1}_{[\boldsymbol{\mu}(\tau) \leq \boldsymbol{s}(\tau)]}; \sum_{i=1}^{L} P_i(\tau) \right) \\
&\quad = \frac{1}{M} \sum_{\boldsymbol{\omega} \in \Omega} \sum_{\tau \in T_M^{(b)}(\boldsymbol{\omega})} \left(\boldsymbol{\omega} \otimes \mathbf{1}_{[\boldsymbol{\omega} \leq \boldsymbol{s}(\tau)]}; \sum_{i=1}^{L} P_i(\tau) \right) \\
&\quad = (1 - \gamma_M) \sum_{\boldsymbol{\omega} \in \Omega} \beta_M(\boldsymbol{\omega}) \, \boldsymbol{y}_M(\boldsymbol{\omega}),
\end{aligned} \tag{25}$$

where in (25), the first equality is by (16) and (19). The second equality is by regrouping terms and defining the set $T_M^{(b)}(\boldsymbol{\omega}) \subset T_M^{(b)}$ as the subset of slots in which transmission rates $\boldsymbol{\mu}(\tau) = \boldsymbol{\omega}$ are allocated. The third equality is by defining $\beta_M(\boldsymbol{\omega}) \triangleq |T_M^{(b)}(\boldsymbol{\omega})| / |T_M^{(b)}|$ and

$$\boldsymbol{y}_M(\boldsymbol{\omega}) \triangleq \frac{1}{|T_M^{(b)}(\boldsymbol{\omega})|} \sum_{\tau \in T_M^{(b)}(\boldsymbol{\omega})} \left(\boldsymbol{\omega} \otimes \mathbf{1}_{[\boldsymbol{\omega} \leq \boldsymbol{s}(\tau)]}; \sum_{i=1}^{L} P_i(\tau) \right), \tag{26}$$

and seeing that $1 - \gamma_M = |T_M^{(b)}|/M$. Note that, for every M, we have $\sum_{\boldsymbol{\omega} \in \Omega} \beta_M(\boldsymbol{\omega}) = 1$, and thus, $\{\beta_M(\boldsymbol{\omega})\}_{\boldsymbol{\omega} \in \Omega}$ can be viewed as a probability distribution as well.

Next, observe that the M-indexed sequence $\{P_{av}(M))\}$ is bounded because power consumption in a slot is nonnegative and at most $(P_m + LP_t)$. Together with Weierstrass's theorem [20, Theorem 2.42], limit points of $\{P_{av}(M))\}$ exist and are all finite. By [20, Theorem 3.17], $\liminf$ of $\{P_{av}(M))\}$ is a limit point of $\{P_{av}(M))\}$ and there exists an integer subsequence $\{M_n\}$ such that

$$\lim_{n \to \infty} P_{av}(M_n) = \liminf_{M \to \infty} P_{av}(M) = \overline{P}_{av}. \tag{27}$$

By (17), (20), (21), and (25), we have

$$\begin{aligned}
&(\hat{\boldsymbol{\mu}}_{av}(M); P_{av}(M)) \\
&= \gamma_M \sum_{\boldsymbol{s} \in \mathcal{S}^L} \sigma_M(\boldsymbol{s}) \, \boldsymbol{x}_M(\boldsymbol{s}) + (1 - \gamma_M) \sum_{\boldsymbol{\omega} \in \Omega} \beta_M(\boldsymbol{\omega}) \, \boldsymbol{y}_M(\boldsymbol{\omega}),
\end{aligned} \tag{28}$$

where $\boldsymbol{x}_M(\boldsymbol{s})$ and $\boldsymbol{y}_M(\boldsymbol{\omega})$ can be expanded by (24) and (26), respectively. For terms in (28) where $\boldsymbol{x}_M(\boldsymbol{s})$ is expanded by (24), we observe the M_n-indexed subsequences $\{\sigma_{M_n}(\boldsymbol{s})\}$ and $\{\alpha_{M_n}(\boldsymbol{\omega}, \boldsymbol{s})\}$ for all $\boldsymbol{s}$ and $\boldsymbol{\omega} \in \Omega(\boldsymbol{s})$, $\{\beta_{M_n}(\boldsymbol{\omega})\}$ and $\{\boldsymbol{y}_{M_n}(\boldsymbol{\omega})\}$ for all $\boldsymbol{\omega} \in \Omega$, and $\{\gamma_{M_n}\}$ are all bounded as well. By iteratively applying Weierstrass's theorem to the above subsequences, there exists a subsequence $\{M_k\}$ of $\{M_n\}$ such that the above subsequences all have converging sub-subsequences indexed by $\{M_k\}$. Consequently, there exists a scalar γ, $0 \leq \gamma \leq 1$, a probability distribution $\{\alpha(\boldsymbol{\omega}, \boldsymbol{s})\}_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})}$ for each $\boldsymbol{s}$, and a probability distribution $\{\beta(\boldsymbol{\omega})\}_{\boldsymbol{\omega} \in \Omega}$ such that as k goes to infinity, we have $\gamma_{M_k} \to \gamma$ and

$$\begin{aligned}
&\alpha_{M_k}(\boldsymbol{\omega}, \boldsymbol{s}) \to \alpha(\boldsymbol{\omega}, \boldsymbol{s}), \quad \text{for all } \boldsymbol{s} \in \mathcal{S}^L \text{ and } \boldsymbol{\omega} \in \Omega(\boldsymbol{s}), \\
&\beta_{M_k}(\boldsymbol{\omega}) \to \beta(\boldsymbol{\omega}), \quad \text{for all } \boldsymbol{\omega} \in \Omega.
\end{aligned}$$

Further, let $\pi_{\boldsymbol{s}}$ be the steady-state probability of channel states $\boldsymbol{s}$. Because channel acquisition in a slot is independent of channel states in that slot, by the Law of Large Numbers (LLNs), $\sigma_{M_k}(\boldsymbol{s}) \to \pi_{\boldsymbol{s}}$ as $k \to \infty$ (see (22)). We also observe that for each $\boldsymbol{\omega} \in \Omega$, the vectors $\boldsymbol{\omega} \otimes \mathbf{1}_{[\boldsymbol{\omega} \leq \boldsymbol{s}(\tau)]}$ are i.i.d. over time slots $\tau \in T_{M_k}^{(b)}(\boldsymbol{\omega})$. Again by LLN,

$$\boldsymbol{y}_{M_k}(\boldsymbol{\omega}) \to \left(\boldsymbol{\omega} \otimes \boldsymbol{Pr}(\boldsymbol{S} \geq \boldsymbol{\omega}); \sum_{i=1}^{L} 1_{[\omega_i > 0]} P_t \right), \quad \text{as } k \to \infty.$$

By the above discussion, as k goes to infinity, the sub-subsequence $\{(\hat{\boldsymbol{\mu}}_{av}(M_k); P_{av}(M_k))\}$ converges to

$$\begin{aligned}
&\gamma \sum_{\boldsymbol{s} \in \mathcal{S}^L} \pi_{\boldsymbol{s}} \left[\sum_{\boldsymbol{\omega} \in \Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega}, \boldsymbol{s}) \left(\boldsymbol{\omega}; P_m + \sum_{i=1}^{L} 1_{[\omega_i > 0]} P_t \right) \right] \\
&\quad + (1 - \gamma) \sum_{\boldsymbol{\omega} \in \Omega} \beta(\boldsymbol{\omega}) \left(\boldsymbol{\omega} \otimes \boldsymbol{Pr}(\boldsymbol{S} \geq \boldsymbol{\omega}); \sum_{i=1}^{L} 1_{[\omega_i > 0]} P_t \right).
\end{aligned} \tag{29}$$

Using the stability definition and [5, Lemma 1], the following necessary condition holds with probability 1:

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$$\boldsymbol{\lambda} \leq \liminf_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \hat{\boldsymbol{\mu}}(\tau). \tag{30}$$

It yields that

$$\begin{aligned} \boldsymbol{\lambda} &\leq \lim_{k\to\infty} \frac{1}{M_k}\sum_{\tau=0}^{M_k-1} \hat{\boldsymbol{\mu}}(\tau) = \lim_{k\to\infty} \hat{\boldsymbol{\mu}}_{av}(M_k) \\ &= \gamma \sum_{\boldsymbol{s}\in\mathcal{S}^L} \pi_{\boldsymbol{s}} \left(\sum_{\boldsymbol{\omega}\in\Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega},\boldsymbol{s})\boldsymbol{\omega} \right) \\ &\quad + (1-\gamma)\sum_{\boldsymbol{\omega}\in\Omega} \beta(\boldsymbol{\omega})(\boldsymbol{\omega}\otimes \boldsymbol{Pr}(\boldsymbol{S}\geq\boldsymbol{\omega})), \end{aligned} \tag{31}$$

where the first inequality of (31) is due to the fact that $\liminf$ of a sequence is a lower bound of all limit points of the sequence. Further, (27) and (29) yield that

$$\begin{aligned} \overline{P}_{av} &= \gamma \sum_{\boldsymbol{s}\in\mathcal{S}^L} \pi_{\boldsymbol{s}} \left[\sum_{\boldsymbol{\omega}\in\Omega(\boldsymbol{s})} \alpha(\boldsymbol{\omega},\boldsymbol{s}) \left(P_m + \sum_{i=1}^{L} 1_{[\omega_i>0]} P_t \right) \right] \\ &\quad + (1-\gamma)\sum_{\boldsymbol{\omega}\in\Omega} \beta(\boldsymbol{\omega}) \left(\sum_{i=1}^{L} 1_{[\omega_i>0]} P_t \right). \end{aligned} \tag{32}$$

From (31) and (32), if there is a $\boldsymbol{\lambda}$-stabilizing policy Φ, there is a stationary randomized policy $\hat{\Phi}$ operating as follows: In every slot, $\hat{\Phi}$ probes the channels with probability γ. If channel states $\boldsymbol{s}$ are acquired, $\hat{\Phi}$ allocates channel-aware rates $\boldsymbol{\omega}\in\Omega(\boldsymbol{s})$ with probability $\alpha(\boldsymbol{\omega},\boldsymbol{s})$. Otherwise, $\hat{\Phi}$ blindly allocates rates $\boldsymbol{\omega}\in\Omega$ with probability $\beta(\boldsymbol{\omega})$. Further, policy $\hat{\Phi}$ has service rates greater than or equal to $\boldsymbol{\lambda}$ entrywise, and consumes average power equal to $\overline{P}_{av}$. This finishes the necessity part of the proof.

(Sufficiency) Conversely, for each rate vector $\boldsymbol{\lambda}$ interior to Λ, there exists a positive scalar ϵ such that $\boldsymbol{\lambda}+\boldsymbol{\epsilon}$ is interior to Λ, where $\boldsymbol{\epsilon}$ is a vector of which every component is ϵ. The optimal solution to $\mathcal{PROB}(\boldsymbol{\lambda}+\boldsymbol{\epsilon})$ yields a stationary randomized policy $\hat{\Phi}$ whose average service rates are greater than or equal to $\boldsymbol{\lambda}+\boldsymbol{\epsilon}$ entrywise. By [18, Lemma 3.6], policy $\hat{\Phi}$ stabilizes $\boldsymbol{\lambda}$ with average power consumption equal to $P_{opt}(\boldsymbol{\lambda}+\boldsymbol{\epsilon})$. By pushing ϵ to zero, there exists a stationary randomized policy which stabilizes $\boldsymbol{\lambda}$ with average power consumption arbitrarily close to $P_{opt}(\boldsymbol{\lambda})$. □

Proof of Theorem 2. First, by squaring (3) for each i and the facts that

$$\begin{gathered} (\max(U_i(t)-\hat{\mu}_i(t),0))^2 \leq (U_i(t)-\hat{\mu}_i(t))^2, \\ U_i(t)-\hat{\mu}_i(t) \leq U_i(t),\ \hat{\mu}_i(t)\leq \mu_{max},\ a_i(t)\leq A_{max}, \end{gathered}$$

we have

$$\sum_{i=1}^{L} \left(U_i^2(t+1) - U_i^2(t)\right) \leq B - 2\sum_{i=1}^{L} U_i(t)(\hat{\mu}_i(t) - a_i(t)), \tag{33}$$

where $B \triangleq (\mu_{max}^2 + A_{max}^2)L$. We define the Lyapunov function

$$L(t) \triangleq \sum_{i=1}^{L} U_i^2(t)$$

and the one-step Lyapunov drift

$$\Delta(\boldsymbol{U}(t)) \triangleq \mathbb{E}[L(t+1) - L(t)|\boldsymbol{U}(t)].$$

By taking expectation of (33) conditioning on current backlog $\boldsymbol{U}(t)$ and noting that arrival processes are i.i.d. over slots, it is easy to show that

$$\Delta(\boldsymbol{U}(t)) \leq B + 2\sum_{i=1}^{L} U_i(t)\lambda_i - \sum_{i=1}^{L} 2U_i(t)\mathbb{E}[\hat{\mu}_i(t) \mid \boldsymbol{U}(t)]. \tag{34}$$

Motivated by the performance optimal Lyapunov optimization technique developed in [2] and [18], we add the cost metric $V\mathbb{E}[\sum_{i=1}^{L} P_i(t) \mid \boldsymbol{U}(t)]$ which is weighted by V to both sides of (34), yielding

$$\begin{aligned} \Delta(\boldsymbol{U}(t)) + V\mathbb{E}\left[\sum_{i=1}^{L} P_i(t) \mid \boldsymbol{U}(t)\right] &\leq B + 2\sum_{i=1}^{L} U_i(t)\lambda_i \\ &- \left(\sum_{i=1}^{L} 2U_i(t)\mathbb{E}[\hat{\mu}_i(t) \mid \boldsymbol{U}(t)] - V\mathbb{E}\left[\sum_{i=1}^{L} P_i(t) \mid \boldsymbol{U}(t)\right]\right). \end{aligned} \tag{35}$$

The DCA algorithm is designed to minimize the right side of (35) over all possible control options $\boldsymbol{\chi}(t)$. Equivalently, the DCA algorithm maximizes $f(\boldsymbol{U}(t),\boldsymbol{\chi}(t))$ (see (4)) over all feasible $\boldsymbol{\chi}(t)$, which can be done by the procedures described in Section 4.1

For performance analysis, we note that the resulting right side of (35) under the DCA algorithm is less than or equal to a right side associated with the use of *some other policy*. In particular, we choose the *some other policy* to be the optimal stationary randomized policy, denoted by Φ^r, associated with the optimal solution to the problem $\mathcal{PROB}(\boldsymbol{\lambda}+\boldsymbol{\epsilon})$ in Theorem 1, where the vector $\boldsymbol{\epsilon}>\boldsymbol{0}$ is an all-ϵ vector such that $\boldsymbol{\lambda}+\boldsymbol{\epsilon}$ is interior to Λ. Let $\hat{\boldsymbol{\mu}}^r(t)$ and $(P_1^r(t),\ldots,P_L^r(t))$ be the effective service rates and power consumption associated with Φ^r in slot t. Then, from (35), we have for each slot t

$$\begin{aligned} \Delta(\boldsymbol{U}(t)) + V\mathbb{E}\left[\sum_{i=1}^{L} P_i(t) \mid \boldsymbol{U}(t)\right] &\leq B + 2\sum_{i=1}^{L} U_i(t)\lambda_i \\ &- \left(\sum_{i=1}^{L} 2U_i(t)\mathbb{E}\left[\hat{\mu}_i^r(t) \mid \boldsymbol{U}(t)\right] - V\mathbb{E}\left[\sum_{i=1}^{L} P_i^r(t) \mid \boldsymbol{U}(t)\right]\right). \end{aligned} \tag{36}$$

By Corollary 1, the policy Φ^r makes control decisions $\boldsymbol{\chi}^r(t)$, independent of current queue backlog $\boldsymbol{U}(t)$, resulting in effective service rates $\hat{\boldsymbol{\mu}}^r(t)$ and power consumption $(P_1^r(t),\ldots,P_L^r(t))$ that satisfy

$$\mathbb{E}[\hat{\boldsymbol{\mu}}^r(t) \mid \boldsymbol{U}(t)] \geq \boldsymbol{\lambda}+\boldsymbol{\epsilon}, \tag{37}$$

$$\mathbb{E}\left[\sum_{i=1}^{L} P_i^r(t) \mid \boldsymbol{U}(t)\right] = P_{opt}(\boldsymbol{\lambda}+\boldsymbol{\epsilon}), \tag{38}$$

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in every slot t. Plugging (37) and (38) into (36) yields

$$\begin{aligned}&\Delta(\boldsymbol{U}(t)) + V\mathbb{E}\left[\sum_{i=1}^{L} P_i(t) \mid \boldsymbol{U}(t)\right] \\ &\leq B - 2\epsilon \sum_{i=1}^{L} U_i(t) + V P_{opt}(\boldsymbol{\lambda} + \boldsymbol{\epsilon}).\end{aligned} \tag{39}$$

Taking expectation of (39) over $\boldsymbol{U}(t)$, summing it from $t = 0$ to $\tau - 1$, and dividing the sum by τ yield

$$\begin{aligned}&\frac{2\epsilon}{\tau} \sum_{t=0}^{\tau-1} \sum_{i=1}^{L} \mathbb{E}[U_i(t)] \leq B + \frac{\mathbb{E}[L(\boldsymbol{U}(0))] - \mathbb{E}[L(\boldsymbol{U}(\tau))]}{\tau} \\ &+ V P_{opt}(\boldsymbol{\lambda} + \boldsymbol{\epsilon}) - \frac{V}{\tau} \mathbb{E}\left[\sum_{t=0}^{\tau-1} \sum_{i=1}^{L} P_i(t)\right].\end{aligned} \tag{40}$$

Suppose the initial backlog $\boldsymbol{U}(0)$ is finite. By taking $\limsup$ of (40) as $\tau \to \infty$ and the fact that $P_{opt}(\boldsymbol{\lambda} + \boldsymbol{\epsilon}) \leq P_m + LP_t$, we have

$$\limsup_{\tau\to\infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} \sum_{i=1}^{L} \mathbb{E}[U_i(t)] \leq \frac{B + V(P_m + LP_t)}{2\epsilon}, \tag{41}$$

$$\limsup_{\tau\to\infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} \sum_{i=1}^{L} \mathbb{E}[P_i(t)] \leq \frac{B}{V} + P_{opt}(\boldsymbol{\lambda} + \boldsymbol{\epsilon}). \tag{42}$$

Equations (41) and (42) hold for any $\epsilon > 0$ that satisfies the condition that $\boldsymbol{\lambda} + \boldsymbol{\epsilon}$ is interior to the capacity region Λ. Thus, we can tighten the bounds by setting $\epsilon = \epsilon_{max}$ in (41), where $\epsilon_{max} > 0$ is the largest real number satisfying $\boldsymbol{\lambda} + \boldsymbol{\epsilon}_{max} \in \Lambda$, and by setting $\epsilon = 0$ in (42). It yields

$$\limsup_{\tau\to\infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} \sum_{i=1}^{L} \mathbb{E}[U_i(t)] \leq \frac{B + V(P_m + LP_t)}{2\epsilon_{max}},$$

$$\limsup_{\tau\to\infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} \sum_{i=1}^{L} \mathbb{E}[P_i(t)] \leq \frac{B}{V} + P_{opt}(\boldsymbol{\lambda}).$$

□

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