Contention-Aware Performance Analysis of Mobility-Assisted Routing

Apoorva Jindal, *Member, IEEE*, and Konstantinos Psounis, *Member, IEEE*,

Abstract—A large body of work has theoretically analyzed the performance of mobility-assisted routing schemes for intermittently connected mobile networks. However, the vast majority of these prior studies have ignored wireless contention. Recent papers have shown through simulations that ignoring contention leads to inaccurate and misleading results, even for sparse networks.

In this paper, we analyze the performance of routing schemes under contention. First, we introduce a mathematical framework to model contention. This framework can be used to analyze any routing scheme with any mobility and channel model. Then, we use this framework to compute the expected delays for different representative mobility-assisted routing schemes under random direction, random waypoint and community-based mobility models. Finally, we use these delay expressions to optimize the design of routing schemes while demonstrating that designing and optimizing routing schemes using analytical expressions which ignore contention can lead to suboptimal or even erroneous behavior.

Index Terms—Delay Tolerant Networks, Wireless Contention, Performance Analysis, Mobility-Assisted Routing.

1 Introduction

Intermittently connected mobile networks (ICMNs) are networks where most of the time, there does not exist a complete end-to-end path from the source to the destination[1]. Even if such a path exists, it may be highly unstable because of topology changes due to mobility. Examples of such networks include sensor networks for wildlife tracking and habitat monitoring [1], military networks [2], deep-space inter-planetary networks [3], nomadic communities networks [4], networks of mobile robots [5], vehicular ad hoc networks [6] etc.

Conventional routing schemes for mobile ad-hoc networks like DSR, AODV, etc. [7] assume that a complete path exists between a source and a destination, and they try to discover these paths before any useful data is sent. Since, no end-to-end paths exist most of the times in ICMNs, these protocols will fail to deliver any data to all but the few connected nodes. To overcome this issue, researchers have proposed to exploit node mobility to carry messages around the network as part of the routing algorithm. These routing schemes are collectively referred to as *mobility-assisted or encounter-based or store-carry-and-forward routing schemes*.

A number of mobility-assisted routing schemes for intermittently connected mobile networks have been proposed in the literature [8–19]. Researchers have also tried to theoretically characterize the performance of these routing schemes [17, 20–25]. However, most of these analytical works ignore the effect of contention on the performance arguing that its effect is small in sparse, intermittently connected networks. However, recent papers [11, 17] have shown through simulations that this argument is not necessarily true. The assumption of no contention is valid only for very low traffic rates, irrespective of whether the network is sparse or not. For higher traffic rates, contention has a significant impact on the performance, especially of flooding-based routing schemes. To demonstrate the inaccuracies which arise when contention is ignored, we use simulations to compare the delay of three different routing schemes in a sparse network, both with and without contention, in Figure 1. The plot shows that ignoring contention not only grossly underestimates the delay, but also predicts incorrect trends and leads to incorrect conclusions. For example, without contention, the so called spraying scheme has the worst delay, while with contention, it has the best delay. Finally, note that a qualitatively different type of intermittently connected networks, that of non-sparse networks which are intermittently connected due to severe mobility [26], will obviously suffer from contention too.

Incorporating wireless contention complicates the analysis significantly. This is because contention manifests itself in a number of ways, including (i) finite bandwidth which limits the number of packets two nodes can exchange while they are within range, (ii) scheduling of transmissions between nearby nodes which is needed to avoid excessive interference, and (iii) interference from transmissions outside the scheduling area, which may be significant due to multipath fading [28]. So, we first propose a general framework to incorporate contention in the performance analysis of mobility-assisted routing schemes for ICMNs while keeping the analysis tractable. This framework incorporates all the three manifestations of contention, and can be used with any mobility and channel model. The framework is based on the well-

• *A. Jindal and K. Psounis are with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA, 90089. E-mail: apoorvaj, kpsounis@usc.edu.*

1. These networks are also referred to as delay tolerant or disruption tolerant networks.

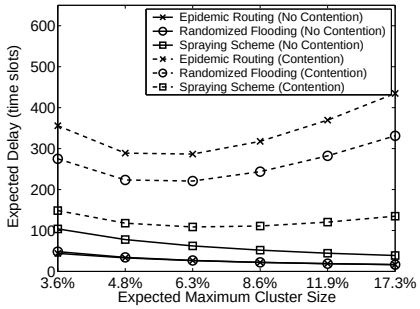


Fig. 1. Comparison of delay with and without contention for three different routing schemes in sparse networks. The simulations with contention use the scheduling mechanism and interference model described in Section 3. The expected maximum cluster size (x-axis) is defined as the percentage of total nodes in the largest connected component (cluster) and is a metric to measure connectivity in sparse networks [17]. The routing schemes compared are: epidemic routing [8], randomized flooding [27] and spraying based routing [12].

known physical layer model [29]. Prior work has used the physical layer model to derive capacity results, see, for example, [29–31], and has assumed an idealized perfect scheduler. We are interested in calculating the expected delay of various mobility-assisted routing schemes under realistic scenarios, and for this reason we assume a random access scheduler.

We then use this framework to do a contention-aware performance analysis for the following representative mobility-assisted routing schemes for ICMNs: direct transmission [16] where the source waits till it meets the destination to deliver the packet, epidemic routing [8] where the network is flooded with the packet that is routed, and different spraying-based schemes [12, 13, 23, 24] where a small number of copies per packet are injected into the network, and then each copy is routed independently towards the destination. We derive expressions for the expected end-to-end packet delay for each of these schemes.

We first derive delay expressions for the two most commonly used mobility models, the random direction and the random waypoint mobility model. However, real world mobility traces have shown that random direction/random waypoint mobility models are not realistic [32, 33]. So, we also analyze these routing schemes for the more realistic community-based mobility model proposed by Spyropoulos et al [21]. The analysis for the community-based mobility model is similar to the derivations for the random direction/random waypoint mobility models. (Note that we include the analysis for the random direction/random waypoint mobility models because it is simpler, easier to understand and naturally extends to the derivations for the more complicated community-based mobility model.)

Note that other papers have studied the performance of these routing schemes without contention in the network. For example, [11, 21] studied the performance of direct transmission, [20–22] studied epidemic routing, and [12, 17] studied different spraying based schemes. [25, 34] are preliminary efforts of ours to analyze the performance of routing schemes under contention. Specifically, [25] studies the expected delay of epidemic routing under the random walk mobility model and [34] studies randomized flooding and a spraying based scheme under the random waypoint mobility model. Here, we generalize our prior work and provide results for more efficient routing schemes under a more realistic mobility model.

Finally, we use these delay expressions to demonstrate that designing routing schemes using analytical expressions which ignore contention can lead to inaccurate and misleading results. Specifically, we choose to study how to optimally design spraying based schemes, since it has been shown that they have superior performance [17]. We compare the design decisions that result from analysis with and without contention, and highlight the scenarios where ignoring contention leads to suboptimal or even erroneous decisions.

The outline of this paper is as follows: Section 2 presents the network model used in the analysis. Section 3 presents the framework to incorporate contention in performance analysis, and then Sections 4 and 5 find the expected delay expressions for different mobility-assisted routing schemes for the random direction/random waypoint and community-based mobility models. Section 6 studies the impact of some approximations made during the analysis on its accuracy by comparing the analytical results to simulation results. Section 7 then uses the expressions derived in the previous sections to demonstrate the inaccuracies introduced by ignoring contention in the design of routing schemes. Finally, Section 8 concludes the paper.

2 Network Model

We first introduce the network model we will be assuming throughout this paper. Table 1 summarizes the notation introduced in this section. We assume that there are M nodes moving in a two dimensional torus of area N.[2] The following two sections present the physical layer, traffic and mobility models assumed in this paper.

2.1 Physical Layer and Traffic Model

2.1.1 Radio Model

An analytical model for the radio has to define the following two properties: (i) when will two nodes be within each other's range, (ii) and when is a transmission between two nodes successful. (Note that we define two nodes to be within range if the packets they send to each other are received successfully with a non-zero

2. We assume the network area to be a two-dimensional torus because the mobility properties defined in Section 2.2.2 which are needed for the analysis have been derived for the two-dimensional torus only. If these properties are known for other 2-D areas, the analysis presented in this paper can be directly applied to these areas also.

N	Area of the 2D torus
M	Number of nodes in the network
K	The transmission range
Θ	The desirable SIR ratio
s_{BW}	Bandwidth of links in units of packets per time slot

TABLE 1
Notation used throughout the paper.

probability.) If one assumes a simple distance-based attenuation model without any channel fading or interference from other nodes, then two nodes can successfully exchange packets without any loss only if the distance between them is less than a deterministic value K (also referred to as the transmission range), else they cannot exchange any packet at all. The value of K depends on the transmission power and the distance attenuation parameter. However, in presence of a fading channel and interference from other nodes, even though two nodes can potentially exchange packets if the distance between them is less than K, a transmission between them might not go through. A transmission is successful only when the signal to interference ratio (SIR) is greater than some desired threshold.

We assume the following radio model: (i) Two nodes are within each other's range if the distance between them is less than K, and (ii) any transmission between the two is successful only if the SIR is greater than a desired threshold Θ. Note that this model is not equivalent to a circular disk model because any transmission between two nodes with a distance less than K is successful with a certain probability that depends on the fading channel model and the amount of interference from other nodes.

2.1.2 Channel Model

The analysis works for any channel model.

2.1.3 Traffic Model

Each node acts as a source sending packets to a randomly selected destination.

2.2 Mobility Model

We will first present the delay analysis for the random direction/random waypoint mobility models [35] which are the most commonly used mobility models for analysis as well as for simulations. However, the real world mobility traces show that mobility models which assume that all nodes are homogeneous and move randomly all around the network, like the random direction and the random waypoint mobility models, are not realistic [32, 33]. Nodes usually have some locations where they spend a large amount of time. Additionally, node movements are not identically distributed. Different nodes visit different locations more often, and some nodes are more mobile than others. Based on this intuition, Spyropoulos et al [21] proposed a more realistic and analytically tractable community-based mobility model. Later, Hsu et al [36] showed that the statistics of real traces match with a time varying version of this community-based mobility model further proving that this model captures real world mobility properties. So, we also present the delay analysis for different mobility-assisted routing schemes for the community-based mobility model.

2.2.1 Community-based Mobility Model

We first define the family of Community-based mobility models: The model consists of two states, namely the 'local' state and the 'roaming' state. The model alternates between these two states. Each node inside the network moves as follows: (i) Each node has a local community. A node's movement consists of local and roaming epochs. (ii) A *local epoch* is a random direction movement restricted inside the node's local community. (iii) A *roaming epoch* is a random direction movement inside the entire network. (iv) If the previous epoch of the node was a local one, the next epoch is a local one with probability p_l, or a roaming epoch with probability $1 - p_l$. (v) If the previous epoch of the node was a roaming one, the next epoch is a roaming one with probability p_r, or a local one with probability $1 - p_r$.

The Community-based mobility model can be used to model a large number of scenarios by tuning its parameters. We choose a specific scenario closely resembling reality where there are r small communities. These communities are assumed to be small enough such that all nodes within a community are within each other's range. We also assume that the nodes spend most of their time within their respective communities. This scenario corresponds to the real scenario of different nodes sharing fixed communities like several office buildings on a campus or several conference rooms in a hotel, which is more realistic than a scenario where all nodes choose their community uniformly at random from the entire network.

2.2.2 Mobility Properties

We now define three properties of a mobility model. The statistics of these three properties will be used in the delay analysis of different mobility-assisted routing schemes.

(i) Meeting Time: Let nodes i and j move according to a mobility model 'mm' and start from their stationary distribution at time 0. Let $X_i(t)$ and $X_j(t)$ denote the positions of nodes i and j at time t. The meeting time (M_{mm}) between the two nodes is defined as the time it takes them to first come within range of each other, that is $M_{mm} = min_t\{t : \|X_i(t) - X_j(t)\| \leq K\}$.

(ii) Inter-Meeting Time: Let nodes i and j start from within range of each other at time 0 and then move out of range of each other at time t_1, that is $t_1 = min_t\{t : \|X_i(t) - X_j(t)\| > K\}$. The inter meeting time (M^+_{mm}) of the two nodes is defined as the time it takes them to first come within range of each other again, that is $M^+_{mm} = min_t\{t - t_1 : t > t_1, \|X_i(t) - X_j(t)\| \leq K\}$.

(iii) Contact Time: Assume that nodes i and j come within range of each other at time 0. The contact time τ_{mm} is defined as the time they remain in contact with each other before moving out of the range of each other, that is $\tau_{mm} = min_t\{t-1 : \|X_i(t) - X_j(t)\| > K\}$.

The statistics of the meeting time, inter-meeting time and contact time for the random direction/random waypoint mobility models are studied by us in [37]. The two important properties satisfied by both these mobility models, which we use during the course of the analysis are as follows: (i) the tail of the distribution of the meeting and the inter-meeting times is exponential, and (ii) the expected inter-meeting time is approximately equal to the expected meeting time. (Note that the latter is true only if the corresponding stochastic process regenerates itself after each moving "epoch". For random waypoint this is true by construction. For random direction we ensure this is the case by forcing epoch lengths to be large, see [37] for details.)

These statistics for the community-based mobility model are studied by us in [37, 38]. Nodes which share the same community have different statistics than nodes which belong to different communities. (Its easy to see that nodes which share the same community meet faster and stay in contact for a longer duration.) The two important properties which we use during the course of the analysis are as follows: (i) The expected meeting time for nodes belonging to different communities is equal to the inter-meeting time for these nodes. However, note that the expected meeting and inter-meeting times for nodes belonging to the same community are not equal. (ii) Even though the overall statistics of the meeting and inter-meeting times for a community-based mobility model is not exponential, after conditioning on whether the two nodes under consideration share the same community or not, the tail of the distribution of the meeting and inter-meeting times becomes exponential.

3 Contention Analysis

This section introduces a framework to analyze any routing scheme for ICMNs with contention in the network. We first identify the three manifestations of contention in Section 3.1 and then describe the framework. The proposed framework will work for any mobility model in which the process governing the mobility of nodes is stationary and the movement of each node is independent of each other. However, for ease of presentation, we first present it for a mobility model with a uniform node location distribution in Section 3.2 (commonly used mobility models like random direction and random waypoint on a torus satisfy this assumption [21, 39]). We then show how to extend it to mobility models with a non-uniform node location distribution by presenting the framework for the community-based mobility model in Section 3.3.1.

3.1 Three Manifestations of Contention

Finite Bandwidth: When two nodes meet, they might have more than one packet to exchange. Say two nodes can exchange s_{BW} packets during a unit of time. If they move out of the range of each other, they will have to wait until they meet again to transfer more packets. The number of packets which can be exchanged in a unit of time is a function of the packet size and the bandwidth of the links. We assume the packet size and the bandwidth of the links to be given, hence s_{BW} is assumed to be a given network parameter. We also assume that the s_{BW} packets to be exchanged are randomly selected from amongst the packets the two nodes want to exchange[3].
Scheduling: We assume an ideal CSMA-CA scheduling mechanism is in place which avoids any simultaneous transmission within one hop from the transmitter and the receiver. Nodes within range of each other and having at least one packet to exchange are assumed to contend for the channel. For ease of analysis, we also assume that time is slotted. At the start of the time slot, all node pairs contend for the channel and once a node pair captures the medium, it retains the medium for the entire time slot[4].
Interference: Even though the scheduling mechanism is ensuring that no simultaneous transmissions are taking place within one hop from the transmitter and the receiver, there is no restriction on simultaneous transmissions taking place outside the scheduling area. These transmissions act as noise for each other and hence can lead to packet corruption.

In the absence of contention, two nodes would exchange all the packets they want to exchange whenever they come within range of each other. Contention will result in a loss of such transmission opportunities. This loss can be caused by either of the three manifestations of contention. In general, these three manifestations are not independent of each other. We now propose a framework which uses conditioning to separate their effect and analyze each of them independently.

3. Note that assuming a random queueing discipline yields the same results as FIFO in our setting (yet simplifies analysis). This is so because a work conserving queue yields the same queueing delay for constant size packets irrespective of whether the queue service discipline is FIFO or random queueing. In addition, due to packet homogeneity (all packets are treated the same) the expected end-to-end delay will also be the same. Of course, if packet homogeneity is lost, for example by assigning higher priority to packets that are closer to their destination, the expected end-to-end delay will decrease as packets with a smaller end-to-end service requirement will be serviced first.

4. We do not describe the exact implementation of this CSMA-CA algorithm here as we ignore implementation imperfections. This allows us to focus on the fundamental performance properties of mobility-assisted routing schemes in ICMNs without worrying about implementation details. Note that any such implementation would use control messages like RTS/CTS and random backoffs. For example, the transmitters of all contending node pairs could use a backoff timer to arbitrate channel access. And, once a backoff timer expires, the corresponding node pair could exchange RTS-CTS messages to inform the rest of the contending pairs to stay silent. Contrary to our analysis, such control packets may also collide or get lost but as we said we are ignoring such imperfections.

3.2 The Framework

Lets look at a particular packet, label it packet A. Suppose two nodes i and j are within range of each other at the start of a time slot and they want to exchange this packet. Let p_{txS} denote the probability that they will successfully exchange the packet during that time slot. First, we look at how the three manifestations of contention can cause the loss of this transmission opportunity. Table 2 summarizes the extra notation used in this section.

Finite Bandwidth: Let E_{bw} denote the event that the finite link bandwidth allows nodes i and j to exchange packet A. The probability of this event depends on the total number of packets which nodes i and j want to exchange. Let there be a total of S distinct packets in the system at the given time (label this event E_S). Let there be s, $0 \leq s \leq S-1$, other packets (other than packet A) which nodes i and j want to exchange (label this event E_s^S). If $s \geq s_{BW}$, then the s_{BW} packets exchanged are randomly selected from amongst these $s+1$ packets. Thus, $P(E_{bw}) = \sum_S P(E_S)\left(\sum_{s=0}^{s_{BW}-1} P(E_s^S) + \sum_{s=s_{BW}}^{S-1} \frac{s_{BW}}{s+1} P(E_s^S)\right)$. To simplify the analysis, we make our first approximation here by replacing the random variable S by its expected value in the expression for $P(E_{bw})$[5] (see Equation (1) for the final expression for $P(E_{bw})$). Note that simulations results presented in Section 6 verify that this approximation does not have a drastic effect on the accuracy of the analysis.

Scheduling: Let E_{sch} denote the event that the scheduling mechanism allows nodes i and j to exchange packets. The scheduling mechanism prohibits any other transmission within one hop from the transmitter and the receiver. Hence, to find $P(E_{sch})$, we have to determine the number of transmitter-receiver pairs which have at least one packet to exchange and are contending with the i-j pair. Let there be a nodes within one hop from the transmitter and the receiver (label it event E_a) and let there be c nodes within two hops but not within one hop from the transmitter and the receiver (label it event E_c). These c nodes have to be accounted for because a node at the edge of the scheduling area can be within the transmission range of one of these c nodes and will contend with the desired transmitter/receiver pair. For example, transmission between nodes P and Q in Figure 2 is not allowed even though node Q is not within the scheduling area. Let $t(a,c)$ denote the expected number of possible transmissions contending with the i-j pair. By symmetry, all the contending nodes are equally likely to capture the channel. So, $P(E_{sch} \mid E_a, E_c) = 1/t(a,c)$.

Interference: Let E_{inter} denote the event that the transmission of packet A is not corrupted due to interference given that nodes i and j exchanged this packet. Let there be $M-a$ nodes outside the transmitter's scheduling area (this is equivalent to event E_a). If two of these nodes are within the transmission range of each other, then they can exchange packets which will increase the interference for the transmission between i and j. Lets label the event that packet A is successfully exchanged inspite of the interference caused by these $M-a$ nodes as I_{M-a}. Then, $P(E_{inter} \mid E_a) = P(I_{M-a})$.

5. We incorporate the arrival process through $E[S]$ in the analysis. $E[S]$ depends on the arrival rate through Little's Theorem. Thus, after deriving the expected end-to-end delay for a routing scheme in terms of $E[S]$, Little's Theorem can be used to express the delay in terms of only the arrival rate.

(i)	**Finite Bandwidth**
E_{bw}	Event that finite link bandwidth allows exchange of packet A
E_s^S	Event that i and j want to exchange s other packets given there are S distinct packets in the system
p_{ex}^R	Probability that nodes i and j want to exchange a particular packet for routing scheme R
(ii)	**Scheduling**
E_{sch}	Event that scheduling mechanism allows i and j to exchange packets
E_a	Event that there are a nodes within one hop from the transmitter and the receiver
E_c	Event that there are c nodes within two hops but not within one hop from the transmitter and the receiver
$t(a,c)$	Expected number of possible transmissions whose transmitter is within $2K$ distance from the transmitter
p_{pkt}	Probability that two nodes have at least one packet to exchange
(iii)	**Interference**
E_{inter}	Event that transmission of packet A is not corrupted due to interference
I_{M-a}	Event that packet A is successfully exchanged inspite of the interference from $M-a$ nodes outside the scheduling area
$E[x]$	Average number of interfering transmissions

TABLE 2
Notation used in Section 3.2

Packet A will be successfully exchanged by nodes i and j only if the following three events occur: (i) the scheduling mechanism allows these nodes to exchange packets, (ii) nodes i and j decide to exchange packet A from amongst the other packets they want to exchange, and (iii) this transmission does not get corrupted due to interference from transmissions outside the scheduling area. Thus,

$$\begin{aligned} p_{txS} &= P(E_{bw}) \sum_{a,c} P(E_a, E_c) P(E_{sch} \mid E_a, E_c) P(E_{inter} \mid E_a) \\ &= \left(\sum_{s=0}^{s_{BW}-1} P(E_s^{E[S]}) + \sum_{s=s_{BW}}^{E[S]-1} \frac{s_{BW} P(E_s^{E[S]})}{s+1} \right) \\ &\times \sum_{a,c} \frac{P(E_a) P(E_c \mid E_a) P(I_{M-a})}{t(a,c)}. \end{aligned} \tag{1}$$

Remark: Note that even though the proposed framework models the main factors contributing to contention like bandwidth restrictions, random access scheduling, fading effects and interference from multiple nodes, it does not model everything. Specifically, it does not incorporate capture effects, losses due to packet collisions, losses due to finite queue buffers and auto-rate adaptation at the physical layer. Even though it would be

doable to incorporate these effects in the framework, this would complicate the analysis quite a bit. Given that our end-goal is to analytically study the fundamental delay properties of mobility-assisted routing schemes under contention, we choose not to further complicate the contention model in an effort to strike the right balance between simplicity, analytical tractability, and realism. Instead, we only offer a qualitative discussion on how one could incorporate these effects in the framework in Section 3.3.2.

Next, we find expressions for the unknown values in Equation (1).

3.2.1 Finite Bandwidth

To account for finite bandwidth, we have to find $P(E_s^{E[S]})$ (the probability that nodes i and j have s other packets to exchange given there are $E[S]$ distinct packets in the system). Let p_{ex}^R be the probability that nodes i and j want to exchange a particular packet for the routing scheme R. Now, since there are $E[S]-1$ packets other than packet A in the network, $P(E_s^{E[S]}) = \begin{pmatrix} E[S]-1 \\ s \end{pmatrix} \left(p_{ex}^R\right)^s \left(1-p_{ex}^R\right)^{E[S]-s-1}$. The value of p_{ex}^R depends on the routing mechanism at hand because which packets should the two nodes exchange is dictated by the routing policy. Note that this is the only term affected by the routing mechanism in the analysis. We will derive its value for different routing mechanisms in Section 4.

3.2.2 Scheduling

To account for scheduling, we have to figure out $P(E_a)$ (the probability that there are a nodes within the scheduling area), $P(E_c \mid E_a)$ (the probability that out of the remaining $M-a$ nodes, there are c nodes within two hops from the transmitter or the receiver but not in the scheduling area) and $t(a,c)$ (the expected number of possible transmissions competing with the i-j pair).

Each of the other $M-2$ nodes (other than i and j) are equally likely to be anywhere in the two dimensional space because the mobility model has a uniform stationary distribution. So, we use geometric arguments to figure out how many transmissions contend with the transmission between i and j.

Lemma 3.1: $P(E_a) = \binom{M-2}{a-2}(p_1)^{a-2}(1-p_1)^{M-a}$ where $p_1 = \frac{A_1}{N}$ is the probability that a particular node lies within the scheduling area, which has an average value equal to $A_1 = \left(2\pi + \frac{4\sqrt{2}}{9} - 2cos^{-1}\left(\frac{1}{3}\right)\right)K^2$.

Proof: The node is equally likely to be anywhere in the two dimensional space. Consequently, p_1 = Pr[a particular node is within one hop of either the transmitter or the receiver] = Pr[a particular node is within one hop of the transmitter] + Pr[a particular node is within one hop of the receiver] - Pr[a particular node is within one hop of both the transmitter and the receiver]. Replacing the distance between the transmitter and the receiver by its expected value yields $p_1 = \frac{\left(2\pi+\frac{4\sqrt{2}}{9}-2cos^{-1}\left(\frac{1}{3}\right)\right)K^2}{N}$. Recall that nodes i and j are already within the scheduling area. So, $P(E_a) = \begin{pmatrix} M-2 \\ a-2 \end{pmatrix}(p_1)^{a-2}(1-p_1)^{M-a}$. □

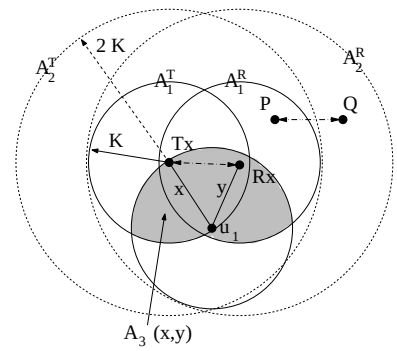


Fig. 2. Tx and Rx denote the transmitter and the receiver. A_1^T and A_1^R (A_2^T and A_2^R) denotes the circular area within one hop (two hops) from the transmitter and receiver respectively. Thus A_1 is the area within $A_1^T \cup A_1^R$ and A_2 is the area within $\left(A_2^T \cup A_2^R\right) - \left(A_1^T \cup A_1^R\right)$. Node-pair P and Q also contend with the desired transmitter and receiver even though Q is not within one hop from either Tx or Rx. Finally, node u_1 is a node within the scheduling area and at a distance x from the transmitter and at a distance y from the receiver. The shaded area represents $A_3(x,y)$.

Corollary 3.1: $P(E_c \mid E_a) = \binom{M-a}{c}(p_2)^c(1-p_2)^{M-a-c}$ where $p_2 = \frac{A_2-A_1}{N}$ is the probability that a particular node lies within two hops from either the transmitter or the receiver but not within the scheduling area, and $A_2 = \left(8\pi + \frac{2\sqrt{35}}{9} - 8cos^{-1}\left(\frac{1}{6}\right)\right)K^2$ is the average value of the area within two hops from either the transmitter or the receiver.

Lemma 3.2: $t(a,c) = \left(1 + p_a p_{pkt}\left(\binom{a}{2}-1\right)\right) + acp_c p_{pkt}$ where $p_a = \int\int_{x,y} \frac{A_3(x,y)}{A_1} f(x,y)dxdy$ is the probability that two nodes are within range of each other given that both of them are in the scheduling area, $p_c = \int\int_{x,y} \frac{\pi K^2 - A_3(x,y)}{A_2} f(x,y)dxdy$ is the probability that two nodes are within range of each other given that one of them is within the scheduling area and the other node is outside the scheduling area but within two hops from either the transmitter or the receiver, $p_{pkt} = 1-\left(1-p_{ex}^R\right)^{E[S]}$ is the probability that two nodes have at least one packet to exchange, $f(x,y)$is the probability that a node within the scheduling area is at a distance x and y from the transmitter and the receiver respectively, and $A_3(x,y)$ is the average value of the area within the scheduling area and within one hop from a node at a distance x and y from the transmitter and the receiver (see Figure 2). We state the value of $f(x,y)$ and $A_3(x,y)$ in the proof.

Proof: See Appendix. □

3.2.3 Interference

The interference caused by other nodes depends on the number of simultaneous transmissions and the distance between the transmitters of these simultaneous transmissions and the desired receiver. Given that there are $M-a$ nodes outside the scheduling area (event E_a), let there be

x interfering transmissions at a distance of $r_1, r_2, \ldots, r_x$ from the desired receiver. Then, using the law of total probability, we get

$$P(I_{M-a}) = \sum_{x} \sum_{r_1, r_2, \ldots, r_x} P(I_{M-a} \mid x, r_1, r_2, \ldots r_x) \times P(r_1, r_2, \ldots, r_x \mid x) P(x). \quad (2)$$

While it is possible to calculate $P(x)$ to substitute in the expression of $P(I_{M-a})$, the resulting expression will be very complicated. Motivated by this, we replace x by its expected value. (Simulations results presented in Section 6 verify that this approximation does not have a drastic effect on the accuracy of the analysis.)

First, we compute $E[x]$. There are $\binom{M-a}{2}$ possible pairs of nodes, and for a particular pair of nodes to interfere with the transmission between i and j, they should be within range of each other, have at least one packet to exchange, and the scheduling mechanism should allow them to exchange packets. Hence, the expected number of interfering transmissions equals $\frac{\pi K^2}{N} p_{pkt} \left(\sum_{a,c} \frac{1}{t(a,c)} P(E_a) P(E_c \mid E_a) \right) \binom{M-a}{2}$.

Now we compute $f(r)$ which denotes the probability density function of the distance between any two nodes. (Since each node is moving independently of each other, $f(r)$ is the same for all the nodes.) The following lemma derives the expression for $f(r)$ for a torus of area N.

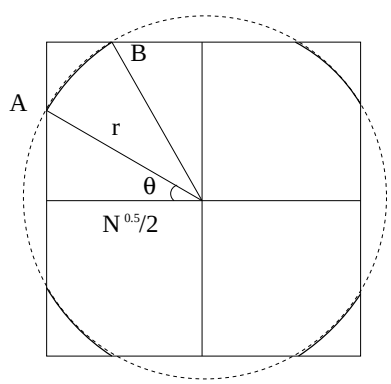


Fig. 3. Arc containing a node at a distance $r > \frac{\sqrt{N}}{2}$ from the origin.

Lemma 3.3:
$$f(r) = \begin{cases} \frac{2\pi r}{N} & r \leq \frac{\sqrt{N}}{2} \\ \frac{4r}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2r}\right)\right) & \frac{\sqrt{N}}{2} < r < \frac{\sqrt{N}}{\sqrt{2}} \end{cases}.$$

Proof: Let one of the nodes be the origin. First consider the case when r is less than $\frac{\sqrt{N}}{2}$. The other node will lie on the circumference of the circle of radius r, hence $f(r) = \frac{2\pi r}{N}$. Now lets consider the case when r is greater than $\frac{\sqrt{N}}{2}$. The other node will again lie on the circumference of the circle of radius r, however the difference is now that this entire circle will not be contained in the torus. To derive the circumference of the circle of radius r, look at Figure 3. By elementary trigonometry, $\theta = cos^{-1}\left(\frac{\sqrt{N}}{2r}\right)$. Thus, the circumference of the arc AB is equal to $\frac{r}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2r}\right)\right)$. There are four such arcs, hence $f(r) = \frac{4r}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2r}\right)\right)$. □

$P(I_{M-a} \mid x, r_1, r_2, \ldots r_x)$ is the complement of the outage probability and depends on the channel model. The channel model only affects this term in the entire analysis. The outage probabilities have been calculated for several realistic channel models including the Rayleigh-Rayleigh fading channel [40] (both the desired signal and the interfering signal are Rayleigh distributed), the Rician-Rayleigh fading channel [41] (the desired signal has Rician and the interfering signal has Rayleigh distribution), and the superimposed Rayleigh fading and log normal shadowing channel [42]. The results from these papers can be directly used here to make the framework work for any of these channel models. The following lemma uses the result from [40] to derive $P(I_{M-a})$ for the Rayleigh-Rayleigh fading channel model.

Lemma 3.4: For the Rayleigh-Rayleigh fading channel model, $P(I_{M-a}) = \int_r \left(1 + \frac{\Theta\left(\frac{2K}{3}\right)^4}{r^4}\right)^{-E[x]} f(r)dr$.

Proof: Kandukuri et al [40] evaluated the outage probability for the Rayleigh-Rayleigh fading channel to be $1 - \prod_{i=1}^{x}\left(1 + \frac{\Theta P_i^R}{P_0^R}\right)^{-1}$, where P_0^R is the received power from the desired signal and P_i^R is the received power from the ith interferer. Assuming all the nodes are transmitting at the same power level and $\alpha = 4$ in the distance attenuation model, $P(I_{M-a} \mid x, r_1, r_2, \ldots r_x) = \prod_{i=1}^{x}\left(1 + \frac{\Theta r_0^4}{r_i^4}\right)^{-1}$, where r_0 is the distance between nodes i and j. Replacing r_0 and x with their expected values and removing the condition on r_i's by using the law of total probability yields the result. □

Note that the transmitters of the simultaneous transmissions as well as the desired receiver will move during the message exchange. Thus, the amount of interference at the receiver will vary. We ignore the effect of this variation as it will cause negligible change in the value of $P(I_{M-a})$ because of the following two reasons. (i) The density function of the distance between the desired receiver and the transmitters of simultaneous transmissions will still be dictated by Lemma 3.3 if these transmitters remain outside the scheduling area during the entire message exchange. (ii) In a sparse network, the probability that a significant number of interfering transmitters move within the scheduling area during the message exchange is negligible.

Now, we have all the components to put together to find p_{txS} in Equation (1). In Sections 4 and 5, we present case studies to demonstrate how the framework is used for performance analysis of routing schemes.

3.3 Extensions

Now we discuss how to remove certain simplifying assumptions we have used so far.

3.3.1 Non-Uniform Node Location Distribution: Community-based Mobility Model

Equation (1) is independent of the mobility model, and hence still holds. However, to extend the framework to a mobility model with a non-uniform node location distribution, the values of $P(E_a)$, $P(E_c \mid E_a)$, $t(a,c)$ and $P(I_{M-a})$ will have to be re-derived. In general, these expressions will be evaluated after conditioning on the current transmitter and receiver location, and then, the law of total probability would be used to remove the condition.

For the community-based mobility model described in Section 2.2, we will condition over whether the current transmitter and receiver belong to the same community or to different communities and whether they meet within a community or outside. For nodes belonging to the same community who meet within their common community, let p^R_{txS1} denote the probability that these two nodes are able to successfully exchange a particular packet inspite of contention. The probability of nodes belonging to different communities meeting within a community is negligible as the communities are very small. (Similarly, the probability of nodes belonging to the same community meeting within a community not their own is also negligible.) If two nodes do not meet within any community, let p^R_{txS2} denote the probability that these two nodes are able to successfully exchange a particular packet inspite of contention (irrespective of whether the two nodes belong to the same community or different communities). The following lemma derives the value of p^R_{txS1}. For ease of presentation, the following lemma assumes that the number of nodes sharing a community is equal to $\frac{M}{r}$, and the position of each community is chosen uniformly at random from the entire network.

Lemma 3.5: $p^R_{txS1} = \left(\sum_{s=0}^{s_{BW}-1} P(E_s^{E[S]}) + \sum_{s=s_{BW}}^{E[S]-1} \frac{s_{BW}}{s+1} P(E_s^{E[S]})\right) \times \left(\sum_{k=2}^{\frac{M}{r}} \sum_{a,c} Pr(E_k) \frac{1}{t(a,c,k)} P(E_a \mid E_k) P(E_c \mid E_a, E_k) P(E_{M-a-k})\right)$, where:

(a) E_k is the event that there are k nodes in the community. $P(E_k) = \binom{\frac{M}{r}-2}{k-2} \pi_l^{k-2} \pi_r^{\frac{M}{r}-k}$ where $\pi_l = \frac{1-p_r}{2-p_l-p_r}$ is the probability that a particular node is in the local state and $\pi_r = \frac{1-p_l}{2-p_l-p_r}$ is the probability that a particular node is in the roaming state.

(b) $P(E_a \mid E_k), P(E_c \mid E_a, E_k)$ and $P(E_{M-a-k})$ are derived in a manner similar to the derivation of the corresponding probabilities in Section 3.2.2.

(c) $t(a,c,k) = 1 + p_{pkt}\left(\left(\binom{a+k}{2} - \binom{a}{2} - 1\right) + p_a\binom{a}{2} + acp_c\right)$.

Proof: The probability of loss due to finite bandwidth is derived using the same arguments made in Section 3.2.1. To derive the probability of loss due to scheduling, we have to find the number of nodes within the scheduling area, while to derive the probability of loss due to interference, we have to find the number of simultaneous transmissions within the network and the distance between the transmitters of these simultaneous transmissions and the desired receiver. The proof of this lemma is based on the following observation: All nodes within the community are within the scheduling area while the remaining nodes will be uniformly distributed over the entire network.

(a) The probability that a particular node is in the local state or the roaming state (π_l and π_r) was derived in [21]. Since each of these nodes in moving independently of each other, the number of nodes in their local states is binomially distributed. Since the transmitter and the receiver are in their local states (as stated earlier), $P(E_k) = \binom{\frac{M}{r}-2}{k-2} \pi_l^{k-2} \pi_r^{\frac{M}{r}-k}$.

(b) The remaining $M-k$ nodes are uniformly distributed over the entire network. The probability that a of these nodes are within the scheduling area ($P(E_a \mid E_k)$) and the probability that c of these nodes are within the two hops from the transmitter or the receiver but not within the scheduling area ($P(E_c \mid E_a, E_k)$) is derived using the same arguments as used in Lemma 3.1 to be equal to $P(E_a \mid E_k) = \binom{M-k}{a}(p_1)^{a-2}(1-p_1)^{M-a}$ and $P(E_c \mid E_a, E_k) = \binom{M-k-a}{c}(p_2)^c(1-p_2)^{M-a-c}$. p_1 and p_2 were defined and derived in Lemma 3.1 and Corollary 3.1 respectively. Finally, note that there are $M-k-a$ nodes outside the scheduling area, which if within range of each other, are allowed to simultaneously transmit. Since, these $M-k-a$ nodes are uniformly distributed over the entire network, the probability of loss due to interference is equal to $P(E_{M-a-k})$ whose value was derived in Section 3.2.3.

(c) The k nodes within the community are within each other's range. The a nodes within the scheduling area but not within the community, will be within the transmission range of the k nodes within the community. However, they will be within each other's transmission range with probabiity p_a (defined and derived in Lemma 3.2). The c nodes within two hops from the transmitter or the receiver but not within the scheduling area will not lie within range from the k nodes within the community, however they will be within range of the a nodes within the scheduling area with probability p_c (defined and derived in Lemma 3.2). All the node pairs within each other's range will contend with the desired transmission only if they have at least one packet to exchange (probability of this event is equal to p_{pkt} and its value was derived in Lemma 3.2). Putting everything together yields $t(a,c,k) = 1 + p_{pkt}\left(\left(\binom{a+k}{2} - \binom{a}{2} - 1\right) + p_a\binom{a}{2} + acp_c\right)$. □

The value of p^R_{txS2} is also derived in a similar fashion.

3.3.2 Other extensions: Capture, collisions, finite buffers, and auto-rate adaptation

This is a qualitative discussion on how one could further extend the contention model to account for additional real-world limitations.

Capture: Two nodes within a distance K from each other can transmit/receive simultaneously due to fading/shadowing effects. One could derive the probability that capture occurs in a manner similar to the derivation

of $P(I_{M-a})$. Then, one would incorporate this probability in the derivation of $t(a,c)$.

Collisions: The collision probability of a link depends on the local topology around this link. One could use results from the literature, e.g. [43,44], to find the collision probability for a given topology for practical CSMA-CA schemes like 802.11, then find the probability of the topology occurring in the network, and finally remove the condition on the topology by using the law of total probability.

Finite Buffers: The expected number of packets in the queue of a node can be easily calculated as a function of $E[S]$. Then well known bounds like the Chernoff bound could be used to find the probability that the number of packets in the queue exceed the buffer size.

Auto-rate Adaptation: Depending on the SINR value on the link, the transmission rate of each link may vary. A different transmission rate will obviously change the value of s_{BW}. Also, since each transmission rate corresponds to a different modulation scheme, the value of Θ will also change. To incorporate auto-rate adaptation, one could first find the probability that a particular transmission rate is employed at a link, derive the value of p_{txS} conditioned on the value of the transmission rate, and then use the law of total probability to remove the condition.

4 Delay Analysis for Popular Mobility Models

In this section, we find the expected end-to-end packet delay of four different mobility-assisted routing schemes for intermittently connected mobile networks, when nodes move according to the random direction or random waypoint mobility models. For each routing scheme R, we first define the routing algorithm, then derive the value of p_{ex}^R and finally derive the value of the expected end-to-end delay[6]. Parameters that depend on the mobility model, are denoted by using 'mm' as a super- or sub-script.

4.1 Direct Transmission

Direct transmission is one of the simplest possible routing schemes. Node A forwards a message to another node B it encounters, only if B is the message's destination. We now analyze its performance with contention.

Lemma 4.1: $p_{ex}^{dt} = \frac{2}{M(M-1)}$.

Proof: In direct transmission, each packet undergoes only one transmission, from the source to the destination. A packet has node i as its source with probability $\frac{1}{M}$. The probability that j is the destination given i is the source is $\frac{1}{M-1}$ (the destination is chosen uniformly at random from amongst the other $M-1$ nodes). Thus, the probability that i and j want to exchange a particular packet is equal to $\frac{2}{M(M-1)}$ (i is the source and j is the destination or vice versa). □

Theorem 4.1: Let $E[D_{dt}^{mm}]$ denote the expected delay of direct transmission. Then, $E[D_{dt}^{mm}] = \frac{E[M_{mm}]}{p_{success}^{dt}}$, where $E[M_{mm}]$ is the expected meeting time of the mobility model 'mm', $p_{success}^{dt} = 1 - \left(1 - p_{txS}^{dt}\right)^{E[\tau_{mm}]}$ is the probability that when two nodes come within range of each other, they successfully exchange the packet before going out of each other's range (within the contact time τ_{mm}).

Proof: The expected time it takes for the source to meet the destination for the first time is $E[M_{mm}]$ (the expected meeting time). With probability $1 - p_{txS}^{dt}$, the two nodes are unable to exchange the packet in one time slot due to contention. They are within range of each other for $E[\tau_{mm}]$ number of time slots. (We are making an approximation here by replacing τ_{mm} by its expected value.) Thus $p_{success}^{dt} = \left(1 - p_{txS}^{dt}\right)^{E[\tau_{mm}]}$ is the probability that the source and the destination fail to exchange the packet while they are within range of each other. Then they will have to wait for one inter-meeting time to come within range of each other again. If they fail yet again, they will have to wait another inter-meeting time to come within range. Thus, $E[D_{dt}^{mm}] = E[M_{mm}] + p_{success}^{dt} \left((1 - p_{success}^{dt})E[M_{mm}^{+}] + 2(1 - p_{success}^{dt})^2 \quad E[M_{mm}^{+}] + \ldots\right) = E[M_{mm}] + \frac{(1-p_{success}^{dt})E[M_{mm}^{+}]}{p_{success}^{dt}}$. Since $E[M_{mm}^{+}] = E[M_{mm}]$ for both random direction and random waypoint mobility models, $E[D_{dt}^{mm}]$ evaluates to $\frac{E[M_{mm}]}{p_{success}^{dt}}$. □

4.2 Epidemic Routing

Epidemic routing [8] extends the concept of flooding to ICMN's. It is one of the first schemes proposed to enable message delivery in such networks. Each node maintains a list of all messages it carries, whose delivery is pending. Whenever it encounters another node, the two nodes exchange all messages that they don't have in common. This way, all messages are eventually spread to all nodes. The packet is delivered when the first node carrying a copy of the packet meets the destination. The packet will keep on getting copied from one node to the other node till its Time-To-Live (TTL) expires. For ease of analysis, we assume that as soon as the packet is delivered to the destination, no further copies of the packet are spread.

To find the expected end-to-end delay for epidemic routing, we first find $E[D_{epidemic}^{mm}(m)]$ which is the expected time it takes for the number of nodes having a copy of the packet to increase from m to $m+1$.

Lemma 4.2: $E[D_{epidemic}^{mm}(m)] = \frac{E[M_{mm}]}{m(M-m)p_{success}^{epidemic}}$, where $p_{success}^{epidemic} = 1 - \left(1 - p_{txS}^{epidemic}\right)^{E[\tau_{mm}]}$.

Proof: When there are m copies of a packet in the network, if one of the m nodes having a copy meets one of the other $M-m$ nodes not having a copy, there is a transmission opportunity to increase the number of copies by one. Since ICMNs are sparse networks, we look at the tail of the distribution of the meeting time which is exponential for both the random direction and

6. Note that p_{ex}^R is the only parameter in the framework which depends on the routing scheme.

the random waypoint mobility models. The time it takes for one of the m nodes to meet one of the other $M-m$ nodes is equal to the minimum of $m(M-m)$ exponentials, which is again an exponential random variable with mean $\frac{E[M_{mm}]}{m(M-m)}$. Now when they meet, the probability that the two nodes are able to successfully exchange the packet is $p_{success}^{epidemic}$. If they fail to exchange the packet, they will have to wait one inter-meeting time to meet again. Since $E[M_{mm}] = E[M_{mm}^{+}]$ for both the random direction and the random waypoint mobility model, and both meeting and inter-meeting times have memoryless tails, the expected time it takes for one of the m nodes to meet one of the other $M-m$ nodes again is still equal to $\frac{E[M_{mm}]}{m(M-m)}$. Hence, $E[D_{epidemic}^{mm}(m)] = p_{success}^{epidemic}\frac{E[M_{mm}]}{m(M-m)} + 2p_{success}^{epidemic}(1-p_{success}^{epidemic})\frac{E[M_{mm}]}{m(M-m)} + \ldots = \frac{E[M_{mm}]}{m(M-m)p_{success}^{epidemic}}$. The value of $p_{success}^{epidemic}$ can be derived in a manner similar to the derivation of $p_{success}^{dt}$ in Theorem 4.1. □

Now, we find the value of $p_{ex}^{epidemic}$ and then find the expected end-to-end delay for epidemic routing (denoted by $E[D_{epidemic}^{mm}]$).

Lemma 4.3: $p_{ex}^{epidemic} = \sum_{m=1}^{M-1}\frac{2m(M-m)}{M(M-1)}\sum_{i=m}^{M-1}\frac{1}{M-1}\frac{\frac{1}{m(M-m)}}{\sum_{j=1}^{i}\frac{1}{j(M-j)}}$.

Proof: Let there be m copies of a particular packet in the network. Then the probability that node i has a copy is equal to $\frac{m}{M}$ and the probability that node j does not have a copy given that node i has one is equal to $\frac{(M-m)}{M-1}$. Thus, the probability that nodes i and j want to exchange the packet given that there are m copies of the packet in the network is equal to $\frac{2m(M-m)}{M(M-1)}$. Now, we find the probability that there are m copies of the packet in the network. The copies of a packet keep on increasing till the packet is delivered to the destination. The probability that the destination is the kth node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2 \leq k \leq M$. A packet will have m copies in the network only if the destination wasn't amongst the first $m-1$ nodes to receive a copy. The amount of time a packet has m copies in the network is equal to $E[D_{epidemic}^{mm}(m)]$. Hence, the probability that there are m copies of a packet in the network equals $\sum_{i=m}^{M-1}\frac{1}{M-1}\frac{E[D_{epidemic}^{mm}(m)]}{\sum_{j=1}^{i}E[D_{epidemic}^{mm}(j)]}$. Applying the law of total probability over the random variable m and substituting the value of $E[D_{epidemic}^{mm}(m)]$ from Lemma 4.2 gives $p_{ex}^{epidemic}$. □

Theorem 4.2: $E[D_{epidemic}^{mm}] = \sum_{i=1}^{M-1}\frac{1}{M-1}\sum_{m=1}^{i}\frac{E[M_{mm}]}{m(M-m)p_{success}^{epidemic}}$.

Proof: The probability that the destination is the ith node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2 \leq i \leq M$. The amount of time it takes for the ith copy to be delivered is equal to $\sum_{m=1}^{i}E[D_{epidemic}^{mm}(m)]$. Applying the law of total probability over the random variable i and substituting the value of $E[D_{epidemic}^{mm}(m)]$ from Lemma 4.2 yields $E[D_{epidemic}^{mm}]$. □

4.3 Spraying a small fixed number of copies

Another approach to route packets in sparse networks is that of controlled replication or *spraying* [12, 13, 23, 24]. A small, fixed number of copies are distributed to a number of distinct relays. Then, each relay routes its copy independently towards the destination. By having multiple relays routing a copy independently and in parallel towards the destination, these protocols create enough diversity to explore the sparse network more efficiently while keeping the resource usage per message low.

Different spraying schemes may differ in how they distribute the copies and/or how they route each copy. We study two different spraying based routing schemes here. These two differ in the way they distribute their copies.

4.3.1 Source Spray and Wait

Source spray and wait is one of the simplest spraying schemes proposed in the literature [12]. For this scheme, the source node forwards all the copies (lets label the number of copies being sprayed as L) to the first L distinct nodes it encounters. (In other words, no other node except the source node can forward a copy of the packet.) And, once these copies get distributed, each copy performs direct transmission.

First, we find the value $E[D_{ssw}^{mm}(m)]$, then we find p_{ex}^{ssw} and finally, we derive the expected end-to-end delay for source spray and wait (denoted by $E[D_{ssw}^{mm}]$).

Lemma 4.4: $E[D_{ssw}^{mm}(m)] = \begin{cases} \frac{E[M_{mm}]}{(M-1)p_{success}^{ssw}} & 1 \leq m < L \\ \frac{E[M_{mm}]}{Lp_{success}^{ssw}} & m = L \end{cases}$

where $p_{success}^{ssw} = 1-(1-p_{txS}^{ssw})^{E[\tau_{mm}]}$.

Proof: See Appendix. □

Lemma 4.5: $p_{ex}^{ssw} = \left(\frac{2Lp_{dest}^{ssw}(L)}{M(M-1)}\frac{E[D_{ssw}^{mm}(L)]}{\sum_{k=1}^{L}E[D_{ssw}^{mm}(k)]}\right) + \left(\frac{2}{M-1}\sum_{m=1}^{L-1}\sum_{i=m}^{L}p_{dest}^{ssw}(i)\frac{E[D_{ssw}^{mm}(m)]}{\sum_{k=1}^{i}E[D_{ssw}^{mm}(k)]}\right)$, where $p_{dest}^{ssw}(i) = \begin{cases} \left(\prod_{j=1}^{i-1}\frac{M-j-1}{M-1}\right)\frac{i}{M-1} & 1 \leq i < L \\ \left(\prod_{j=1}^{i-1}\frac{M-j-1}{M-1}\right) & i = L \end{cases}$ is the probability that the destination is the $(i+1)^{th}$ node to receive a copy of the packet.

Proof: The proof runs along the same lines as the proof of Lemma 4.3. □

Theorem 4.3: $E[D_{ssw}^{mm}] = \sum_{i=1}^{L}p_{dest}^{ssw}(i)\sum_{m=1}^{i}E[D_{ssw}^{mm}(m)]$.

Proof: The proof runs along similar lines as the proof of Theorem 4.2. □

4.3.2 Fast Spray and Wait

In Fast Spray and Wait, every relay node can forward a copy of the packet to a non-destination node which it encounters in the spray phase. (Recall that in source spray and wait, only the source node can forward copies to non-destination nodes.) There is a centralized mechanism which ensures that after L copies of the packet have been spread, no more copies get transmitted to non-destination nodes. Note that this is not a practical way to distribute copies, however we include it in the analysis because it spreads copies whenever there is any opportunity to do so and hence has the minimum spraying time when there is no contention in the network. Once these copies get distributed, each copy performs direct

transmission. We now derive the expected delay of fast spray and wait with contention in the network.

First, we find the value $E[D_{fsw}^{mm}(m)]$, then we find p_{ex}^{fsw} and finally, we derive the expected end-to-end delay for fast spray and wait (denoted by $E[D_{fsw}^{mm}]$). All the derivations are very similar to the corresponding derivations for epidemic routing. The only difference is that when $m = L$ nodes have a copy of the packet, a transmission opportunity will arise only when one of these $m = L$ nodes meet the destination.

Lemma 4.6: $E[D_{fsw}^{mm}(m)] = \begin{cases} \frac{E[M_{mm}]}{m(M-m)p_{success}^{fsw}} & 1 \leq m < L \\ \frac{E[M_{mm}]}{Lp_{success}^{fsw}} & m = L \end{cases}$

where $p_{success}^{fsw} = 1 - \left(1 - p_{txS}^{fsw}\right)^{E[\tau_{mm}]}$.

Proof: The proof runs along the same lines as the proof of Lemma 4.4. □

Lemma 4.7: $p_{ex}^{fsw} = \left(\frac{2Lp_{dest}^{fsw}(L)}{M(M-1)} \frac{E[D_{fsw}^{mm}(L)]}{\sum_{k=1}^{L} E[D_{fsw}^{mm}(k)]}\right) + \left(\sum_{m=1}^{L-1} \frac{2m(M-m)}{M(M-1)} \sum_{i=m}^{L} p_{dest}^{fsw}(i) \frac{E[D_{fsw}^{mm}(m)]}{\sum_{k=1}^{i} E[D_{fsw}^{mm}(k)]}\right)$,

where $p_{dest}^{fsw}(i) = \begin{cases} \frac{1}{M-1} & 1 \leq i < L \\ \frac{M-L}{M-1} & i = L \end{cases}$ is the probability that the destination is the $(i+1)^{th}$ node to receive a copy of the packet.

Proof: The proof runs along the same lines as the proof of Lemma 4.3. □

Theorem 4.4: $E[D_{fsw}^{mm}] = \sum_{i=1}^{L} p_{dest}^{fsw}(i) \sum_{m=1}^{i} E[D_{fsw}^{mm}(m)]$.

Proof: The proof runs along similar lines as the proof of Theorem 4.2. □

5 Delay Analysis of Routing Schemes with the Community-based Mobility Model

In this section, we derive the expected delay values for four different mobility-assisted routing schemes with the community-based mobility model. We first analyze direct transmission and epidemic routing as these two form the basic building block for all routing schemes. Then, we analyze two different spraying based schemes: fast spray and wait and fast spray and focus which differ in the way they route each individual copy towards the destination after the spray phase. Note that the value of p_{ex}^{R} for each routing scheme remains the same as derived in Section 4. The derivation of the expected delay for the community-based mobility model uses arguments similar to the ones used in the derivation of the expected delay for the random direction / random waypoint mobility model. The proofs which are very similar are not discussed to keep the exposition interesting. To simplify the presentation in this section, we assume that the number of nodes sharing a community is equal across all r communities, that is the number of nodes sharing a community is equal to $\frac{M}{r}$. Finally, we define the notation related to the statistics of the mobility properties for the community-based mobility model. Let $E[M_{comm,same}]$ ($E[M_{comm,diff}]$), $E[M_{comm,same}^{+}]$ ($E[M_{comm,diff}^{+}]$) and $E[\tau_{comm,same}]$ ($E[\tau_{comm,diff}]$) denote the expected meeting time, inter-meeting and contact time for nodes which belong to the same community (belong to different communities) respectively. Please refer to [37,38] for their exact values.

5.1 Direct Transmission

Let $E[D_{dt}^{comm}]$ denote the expected delay of direct transmission for the community-based mobility model. Further, let $p_{success1}^{dt}$ be the probability that when two nodes belonging to the same community come within each other's range, they successfully exchange the packet before going out of each other's range and let $p_{success2}^{dt}$ be the probability that when two nodes belonging to different communities come within each other's range, they successfully exchange the packet before going out of each other's range.

Theorem 5.1: $E[D_{dt}^{comm}] = \frac{(r-1)m}{r(m-1)} \frac{E[M_{comm,diff}]}{p_{success2}^{dt}} + \frac{m-r}{r(m-1)} \left(E[M_{comm,same}] + \frac{(1-p_{success1}^{dt})E[M_{comm,same}^{+}]}{p_{success1}^{dt}}\right)$,

where $p_{success1}^{dt} = 1 - (1 - p_{txS1}^{dt})^{E[\tau_{comm,diff}]}$ and $p_{success2}^{dt} = 1 - (1 - p_{txS2}^{dt})^{E[\tau_{comm,same}]}$.

Proof: The probability that the destination belongs to a different community than the source is equal to $\frac{(r-1)m}{r(m-1)}$. The derivation of the expected delay after conditioning on whether the source and the destination belong to the same community or not is similar to the derivation of $E[D_{dt}^{mm}]$ in Theorem 4.1. Finally, using the law of total probability to remove the conditioning yields $E[D_{dt}^{comm}]$. □

5.2 Epidemic Routing

This section derives the expected delay of epidemic routing for the community-based mobility model. Since each node spends most of its time within its community (which implies $E[M_{comm,diff}] >> E[M_{comm,same}]$), we make an approximation to simplify the exposition by assuming that with high probability, a node starting from its stationary location distribution will first meet a node within its own community than a node belonging to a different community. This implies that once a node gets a copy of a packet, with high probability, all members of its community will get the copy before any node outside its community. A simple outcome of this is that the first $\frac{M}{r} - 1$ nodes to get a copy of the packet belong to the source's community.

We first study how much time it takes for all nodes within the source's community to get a copy of the packet. This derivation is different from all the derivations in Section 4 because $E[M_{comm,same}] \neq E[M_{comm,same}^{+}]$. Thus, we need to keep track of which pair of nodes have met in the past but were unable to successfully exchange the packet. We model the system using the following state space: (m, m_p) where $1 \leq m \leq \frac{M}{r}$ is the number of nodes which have a copy of the packet and $0 \leq m_p \leq m\left(\frac{M}{r} - m\right)$ is the number of node pairs such that only one node of the pair has a copy of the packet, they have met at least once after the node (which has the copy) received its copy, and they were unable to

successfully exchange this packet in their past meetings. Let $E[D_{in}(m)]$ denote the expected time it takes for the number of nodes having a copy of the packet to increase from m to $m+1$ given $m < \frac{M}{r}$ (which implies that all nodes within the source's community have not yet received a copy of the packet).

Lemma 5.1: $E[D_{in}(m)] = \sum_{m_p=0}^{m\left(\frac{M}{r}-m\right)} p_{m,m_p} \frac{E[T_{m,m_p}]}{1-p^{self}_{m,m_p}}$, where $E[T_{m,m_p}]$ is the expected time elapsed till one of the nodes not having a copy meets a node having a copy of the packet given that the system is in state (m, m_p), p^{self}_{m,m_p} is the probability that the system remains in the state (m, m_p) after these nodes (which met after $E[T_{m,m_p}]$) are unable to successfully exchange the packet, and p_{m,m_p} is the probability that the system visits state (m, m_p).

Proof: Let the system be in state (m, m_p). We first derive the expected time duration after which the system moves to another state. A transmission opportunity will arise only when one of the m nodes carrying a copy of the packet meet one of the $\frac{M}{r} - m$ not having a copy of the packet. There are a total of $m\left(\frac{M}{r} - m\right)$ such node pairs of which m_p have already met before. Since, both the meeting and inter-meeting times have exponential tails, the expected time elapsed till one of these $m\left(\frac{M}{r} - m\right)$ node pairs come within range is $E[T_{m,m_p}] = \left(\frac{m\left(\frac{M}{r}-m\right)-m_p}{E[M_{comm,same}]} + \frac{m_p}{E[M^+_{comm,same}]}\right)^{-1}$. If the two nodes which met are not able to successfully exchange the packet, then the system will remain in the same state if these two nodes were one of the m_p node pairs which have already met at least once in the past, otherwise the system will move to (m, m_p+1). Thus, the probability that the system remains in the same state is $p^{self}_{m,m_p} = (1 - p^{epidemic}_{success1})\frac{m_p E[T_{m,m_p}]}{E[M^+_{comm,same}]}$, where $p^{epidemic}_{success1} = 1 - \left(1 - p^{epidemic}_{txS1}\right)^{E[\tau_{comm,same}]}$. If the system remains in the same state, then it will take yet another time duration equal to $E[T_{m,m_p}]$ for a transmission possibility. Again, with p^{self}_{m,m_p} the system will remain in the same state. Thus, the expected amount of time the system remains in state (m, m_p) is equal to $\frac{E[T_{m,m_p}]}{1-p^{self}_{m,m_p}}$.

In a manner similar to the derivation of p^{self}_{m,m_p}, the probability that the system moves to (m, m_p+1) is derived to be $(1 - p^{epidemic}_{success1})\left(1 - \frac{m_p E[T_{m,m_p}]}{E[M^+_{comm,same}]}\right)$. The transmission is successful with probability $p^{epidemic}_{success1}$, in which case the system moves to the state $(m+1, m_p - \frac{m_p}{\frac{M}{r}-m})$. Since each node not having a copy of the packet has met on an average $\left(\frac{m_p}{\frac{M}{r}-m}\right)$ nodes which have a copy of the packet, when a new node receives the packet, this number has to be subtracted from m_p.

Now, we find the probability that the system will visit the state (m, m_p) (denoted by p_{m,m_p}). The system can move to state (m, m_p) from states $(m-1, m_p + \frac{m_p}{\frac{M}{r}-m-1})$(with probability $p^{epidemic}_{success1}$) and $(m, m_p - 1)$ (with probability $(1 - p^{epidemic}_{success1})\left(1 - \frac{(m_p-1)E[T_{m,m_p-1}]}{E[M^+_{comm,same}]}\right)$). Thus,

$$p_{m,m_p} = \begin{cases} p^{epidemic}_{success1} p_{m-1,m_p+\frac{m_p}{\frac{M}{r}-m-1}} \\ + \left(1 - \frac{(m_p-1)E[T_{m,m_p-1}]}{E[M^+_{comm,same}]}\right) & \text{if } m > 1 \\ (1 - p^{epidemic}_{success1}) p_{m,m_p-1} \\ \left(1 - \frac{(m_p-1)E[T_{m,m_p-1}]}{E[M^+_{comm,same}]}\right) & \text{if } m = 1, m_p > 0 \\ (1 - p^{epidemic}_{success1}) p_{m,m_p-1} \\ 1 & \text{if } m = 1, m_p = 0. \end{cases}$$

Solving this set of linear equations yields p_{m,m_p}. □

Now, we find $E[D^{comm}_{epidemic}(m)]$ which is the expected time it takes for the number of nodes having a copy of the packet to increase from m to $m+1$.

Lemma 5.2:

$$E[D^{comm}_{epidemic}(m)] = \begin{cases} E[D_{in}\left(\text{rem}\left(m, \frac{M}{r}\right)\right) & \text{if rem}\left(m, \frac{M}{r}\right) \neq 0 \\ \frac{E[M_{comm,diff}]}{m(M-m)p^{epidemic}_{success2}} & \text{if rem}\left(m, \frac{M}{r}\right) = 0 \end{cases}$$

where $p^{epidemic}_{success2} = 1 - \left(1 - p^{epidemic}_{txS2}\right)^{E[\tau_{comm,diff}]}$ and $\text{rem}(x, y)$ is the remainder left after dividing x by y.

Proof: As previously discussed, the first $\frac{M}{r} - 1$ nodes to receive a copy of the packet are the nodes belonging to the source's community. Then, a node belonging to another community (lets label it community Y) will receive a copy from one of the nodes belonging to the source's community. After that, the next $\frac{M}{r} - 1$ nodes to get a copy of the packet are the ones which belong to community Y. Even though there are other nodes which have a copy of the packet (belonging to the source's community), with high probability, the nodes in community Y will receive a copy of the packet from a node belonging to its own community. Thus, the expected time for the copies to spread within community Y is equal to the expected time for the copies to spread within the source's community. Similarly, the expected time for the copies to spread within any community after a node belonging to that community obtains a copy, is equal to the expected time for the copies to spread within the source's community (irrespective of how many nodes outside the community have copies of the packet). Finally, for the scenario when for all communities, either all or no nodes in a community have a copy of the packet, the expected time for the copies to increase can be found in a manner similar to the derivation of $E[D^{mm}_{epidemic}(m)]$ in Lemma 4.2. □

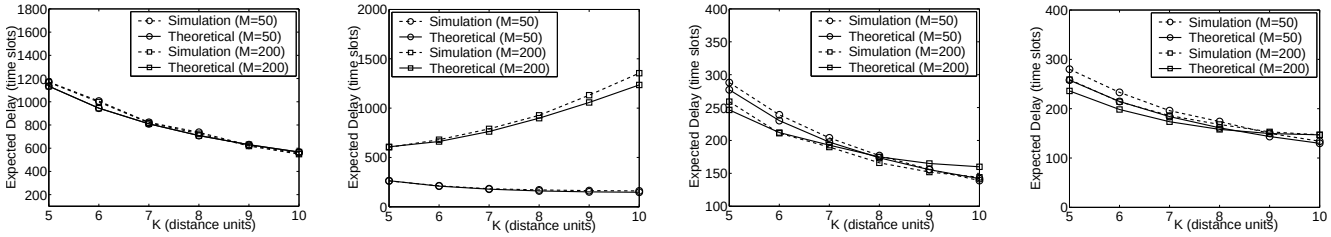
Finally, we derive the expected delay of epidemic routing for the community based mobility model (denoted by $E[D^{comm}_{epidemic}]$) in terms of $E[D^{comm}_{epidemic}(m)]$ using the same argument used to derive $E[D^{mm}_{epidemic}]$ in Theorem 4.2.

Theorem 5.2:
$E[D^{comm}_{epidemic}] = \sum_{i=1}^{M-1} \frac{1}{M-1} \sum_{m=1}^{i} E[D^{comm}_{epidemic}(m)]$.

5.3 Spraying a small fixed number of copies

5.3.1 Fast Spray and Wait

This section derives the expected delay of fast spray and wait routing scheme for the community-based mo-



(a) Direct Transmission. (b) Epidemic Routing. (c) Source Spray and Wait with $L = 5$. (d) Fast Spray and Wait with $L = 5$.

Fig. 4. Simulation and analytical results for the expected delay for the random waypoint mobility model. Network parameters: $N = 120 \times 120$ square units, $\Theta = 5, s_{BW} = 1$ packet/time slot. The expected maximum cluster size varies from $5\%(K = 5, M = 50)$ to $23\%(K = 10, M = 200)$. (Note that the number of instances for which each Monte Carlo simulation is run is chosen so as to ensure that the 90% confidence interval is within 5% of the simulation value.)

bility model. As before, first we derive the value of $E[D^{comm}_{fsw}(m)]$. For $m < L$ (in the spray phase), the value of $E[D^{comm}_{fsw}(m)]$ is derived in a manner similar to the derivation of $E[D^{comm}_{epidemic}(m)]$ as flooding is used to spread the L copies in the spray phase. Now, we derive the value of $E[D^{comm}_{fsw}(L)]$ which is the expected time to find the destination in the wait phase.

Lemma 5.3: $E[D^{comm}_{fsw}(L)] = \frac{\frac{M}{r}-\hat{l}}{M-L}\left(\sum_{m_p=0}^{\hat{l}(\frac{M}{r}-\hat{l})} p_{\hat{l},m_p} E[T_{\frac{m_p}{\frac{M}{r}-\hat{l}}}]\right) + \left(1-\frac{\frac{M}{r}-\hat{l}}{M-L}\right)\frac{E[M_{comm,diff}]}{Lp^{fsw}_{success2}}$, where $\hat{l} = \text{rem}\left(L, \frac{M}{r}\right)$, $E[T_s]$ is the expected time till the destination receives a copy of the packet given there are s nodes belonging to the destination's community which were unable to successfully exchange the packet with the destination in the past, and $p^{fsw}_{success2} = 1 - \left(1 - p^{fsw}_{txS2}\right)^{E[\tau_{comm,diff}]}$.

Proof: After the spray phase (after L copies have been spread), there is a community which has only $\hat{l} = \text{rem}\left(L, \frac{M}{r}\right)$ nodes carrying a copy of the packet. The probability that the destination is one of the remaining $\frac{M}{r} - \hat{l}$ nodes belonging to this community is equal to $\frac{\frac{M}{r}-\hat{l}}{M-L}$. First we derive the expected delay in the wait phase when the destination belongs to this community. Then, we derive the expected delay when the destination does not belong to this community. Finally we use the law of total probability to combine everything together and get the result. Please see the Appendix for proof details. □

Finally, we derive the expected delay of fast spray and wait for the community based mobility model (denoted by $E[D^{comm}_{fsw}]$) in terms of $E[D^{comm}_{fsw}(m)]$ using the same argument used to derive $E[D^{nm}_{fsw}]$.

Theorem 5.3:
$E[D^{comm}_{fsw}] = \sum_{i=1}^{L} p^{fsw}_{dest}(i) \sum_{m=1}^{i} E[D^{comm}_{fsw}(m)]$.

5.3.2 Fast Spray and Focus

Spray and Focus schemes [15] differ from spray and wait schemes in how each relay routes the copy towards the destination. Instead of doing direct transmission, each relay does a utility-based forwarding towards the destination, that is, whenever a relay carrying a copy of the packet meets another node (label it node B) which has a higher utility, the relay gives its copy to node B. Node B now does a utility based forwarding towards the destination and the relay drops the packet from its queue. [15] showed that spray and focus has huge performance gains over spray and wait for heterogeneous networks (networks where each node is not the same). Community-based mobility model introduces an inherent heterogeneity in the network as nodes differ depending on which community they belong to. So, we study a spray and focus scheme for the community-based mobility model, and later we compare it to the corresponding spray and wait scheme.

Fast spray and focus performs fast spraying in the spray phase. To be able to do utility-based forwarding in the focus phase, [15] maintained last encounter timers to build the utility function. For community-based mobility models, [18] proposed the use of a simpler function as a utility function for their 'Label' scheme: If a relay meets a node which belongs to the same community as the destination, the relay hands over its copy to the new node. We use this simple utility function to route copies of the packet in the focus phase.

This section derives the expected delay of fast spray and focus for the community-based mobility model. p^{fsf}_{ex} can be derived in a manner similar to the derivation of p^{fsw}_{ex}. To avoid repetition, we skip the derivation of p^{fsf}_{ex} here.

As before, first we derive $E[D^{comm}_{fsf}(m)]$. Since flooding is used to spread the copies in the spray phase, $E[D^{comm}_{fsf}(m)]$ for $m < L$ can be derived in a manner similar to the derivation of $E[D^{comm}_{epidemic}(m)]$. The next lemma derives the value of $E[D^{comm}_{fsf}(L)]$ which is the expected time it takes for the packet to get delivered to the destination in the focus phase.

Lemma 5.4: $E[D^{comm}_{fsf}(L)] = \frac{\frac{M}{r}-\hat{l}}{M-L}\left(\sum_{m_p=0}^{\hat{l}(\frac{M}{r}-\hat{l})} p_{\hat{l},m_p} E[T_{\frac{m_p}{\frac{M}{r}-\hat{l}}}]\right) + \left(1-\frac{\frac{M}{r}-\hat{l}}{M-L}\right)\left(\frac{E[M_{comm,diff}]}{L\frac{M}{r}p^{fsf}_{success2}} + \frac{\frac{M}{r}-1}{\frac{M}{r}}\left(E[M_{comm,same}] + \frac{(1-p^{fsf}_{successs1})E[M^{+}_{comm,same}]}{p^{fsf}_{success1}}\right)\right)$, where $\hat{l} = \text{rem}\left(L, \frac{M}{r}\right)$, $p^{fsf}_{success1} = 1 - \left(1 - p^{fsw}_{txS1}\right)^{E[\tau_{comm,diff}]}$ and $p^{fsf}_{success2} = 1 - \left(1 - p^{fsw}_{txS2}\right)^{E[\tau_{comm,same}]}$.

Proof: See Appendix. □

Now we derive the expected delay of fast spray and focus for the community based mobility model (denoted

by $E[D_{fsf}^{comm}]$) in terms of $E[D_{fsf}^{comm}(m)]$.

Theorem 5.4:
$E[D_{fsf}^{comm}] = \sum_{i=1}^{L} p_{dest}^{fsf}(i) \sum_{m=1}^{i} E[D_{fsf}^{comm}(m)]$, where $p_{dest}^{fsf}(i) = \begin{cases} \frac{1}{M-1} & i < L \\ \frac{M-L}{M-1} & i = L \end{cases}$.

Proof: The proof runs along similar lines as the proof of Theorem 4.2. □

6 Accuracy of Analysis

In the previous sections, we made a number of approximations to keep the analysis tractable. Here, we assess to which extent these approximations create inaccuracies. We focus on the following approximations: (i) replacing S by $E[S]$ in the expression of $P(E_{bw})$ in Section 3.2, (ii) replacing the random variable representing the number of interfering transmissions (x) by its expected value in Section 3.2.3. (iii) replacing the contact time by its expected value in the expression of $p_{success}^{R}$ in the delay analysis of all routing schemes, (iv) assuming the entire meeting and inter-meeting time distribution to be exponential in the delay analysis of flooding-based routing schemes, and (v) assuming that a node starting from its stationary distribution will meet a node belonging to its own community before a node from some other community with high probability in the delay analysis of routing schemes for the community-based mobility model. We use simulations to verify that these approximations do not have a significant impact on the accuracy of the analysis.

We use a custom simulator written in C++ for simulations. The simulator avoids excessive interference by implementing the scheduling scheme described in Section 3.1. At the start of a time slot, all contending node pairs initialize a backoff timer to a random value choosen from a uniform distribution. When the backoff timer expires, the node pair exchanges control messages to silence all other interfering node pairs. To incorporate channel fading, the received signal strength is derived from the distribution corresponding to the fading model. For example, to model Rayleigh fading, the received signal strength at the receiver is drawn from an exponential distribution. Interference is incorporated by adding the received signal from other simultaneous transmissions (outside the scheduling area) and comparing the signal to interference ratio to the desired threshold. The simulator allows the user to choose from different physical layer, mobility and traffic models. We choose the Rayleigh-Rayleigh fading model for the channel and Poisson arrivals in our simulations. There is no limit on the buffer size. The simulator has been made publicly available for researchers at the author's website.

We study the robustness of all the approximations by varying the level of connectivity in the network (which in turn is achieved by altering the transmission range K, the number of nodes in the network M and the number of communities in the network r for the community-based mobility model). As a connectivity metric we use the expected maximum cluster size, which is defined as the percentile of nodes that belong to the largest connected cluster, and denote its value in the figures' captions. Figures 4(a)-4(d) and 5(a)-5(d) compare the expected end-to-end delay for different routing schemes obtained through analysis and simulations for different values of K and M for the random waypoint mobility model and for different values of K, M and r for the community based mobility model. (Note that K is expressed in the same distance units as $\sqrt{N}$.) Since both the simulation and the analytical curves are close to each other in all the scenarios, we conclude that the analysis is fairly accurate.

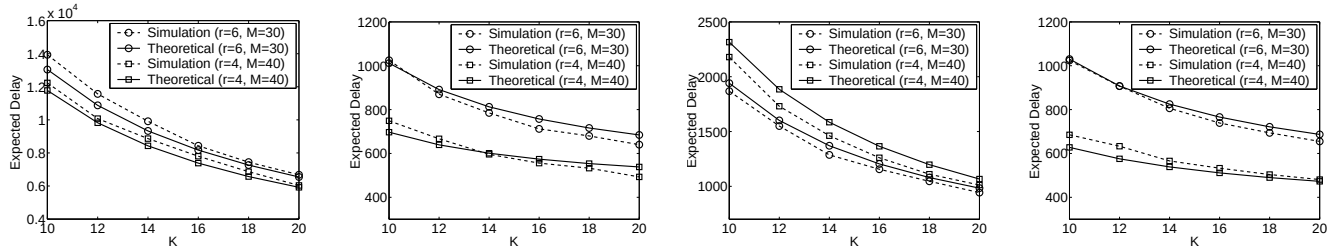
Now we comment on which approximations create small yet noticeable errors. For the random waypoint mobility model, the approximation of assuming the entire meeting and inter-meeting time distribution to be exponential creates a noticeable error. (Replacing the values derived based on this approximation by actual values derived from simulations makes the simulation and analytical curves indistinguishable.) The effect of this approximation worsens as the node density increases (either K or M increases). For the community-based mobility model, the assumption of exponential distribution for the inter-meeting time for nodes belonging to different communities results in underestimating the expected delay. This effect of this approximation is significant for smaller values of K. Also, the following additional approximation plays a noticeable role: assuming that starting from its stationary distribution, a node will meet a node belonging to its own community before a node from some other community. This approximation results in overestimating the expected delay and worsens as the number of nodes in other communities increases (r increases). The first approximation dominates for lower values of K and r, and the second approximation becomes more dominant as K and r increases.

7 Application: Design of Spraying-based Routing Schemes

The design of spraying-based routing schemes poses the following three fundamental questions: (i) How many copies to spray? (ii) How to spray these copies in the spraying phase? (iii) How to route each individual copy towards the destination after the spraying phase? [12, 15, 17] answered these questions assuming there is no contention in the network. In this section, we use the expressions derived in the previous sections to study if incorporating contention introduces significant differences in the answers to these questions.

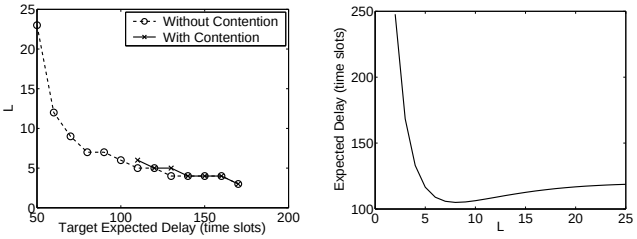
7.1 How Many Copies to Spray

This section studies the error introduced by ignoring contention when one has to find the minimum value of L (the number of copies sprayed) in order for a spraying-based scheme to achieve a specific expected delay. (Note



(a) Direct Transmission. (b) Epidemic Routing. (c) Fast spray and wait with $L = 10$. (d) Fast spray and focus with $L = 10$.

Fig. 5. Simulation and analytical results for the expected delay for the community-based mobility model. Network parameters: $N = 500 \times 500$ square units, $\Theta = 5, p_l = 0.8, p_r = 0.2, s_{BW} = 1$ packet/time slot. The expected maximum cluster size varies from $15\%(K = 10, r = 6, M = 30)$ to $24\%(K = 20, r = 4, M = 40)$. (Note that the number of instances for which each Monte Carlo simulation is run is chosen so as to ensure that the 90% confidence interval is within 5% of the simulation value.)



(a) (b)

Fig. 6. (a) Minimum value of L which achieves the target expected delay for source spray and wait. (b) L against expected delay (with contention). Network parameters: $N = 100 \times 100, K = 8, M = 150, \Theta = 5, E[S] = 70, \overline{T}_{stop} = 0, \overline{v} = 1, s_{BW} = 1$.

that we want the minimum value of L which achieves the target delay as bigger values of L consume more resources.) We choose the source spray and wait scheme with the random waypoint mobility model as the case study in this section. We numerically solve the expression for $E[D_{ssw}^{rwp}]$ in Theorem 4.3 to find the minimum value of L which achieves a target delay and plot it in Figure 6(a) both with and without contention for a sparse network. (For the expected delay of source spray and wait without contention, we use the expression derived in [12].) This figure shows that an analysis without contention would be accurate for smaller values of L (smaller values of L generate lower contention in the network), however it would predict that one can use a large number of copies to achieve a target expected delay which actually will not be achievable in practice due to contention. For example, the analysis without contention indicates that a delay of 50 time units is achievable with $L = 23$ while the contention-aware analysis indicates that it is not achievable. Figure 6(b) shows that $L = 23$ results in an expected delay of more than 118 time units, which is also achievable by $L = 5$. Thus choosing a value of L based on predictions from a contention-ignorant analysis led to a value of delay which is not only much higher than expected but also would have been achieved by nearly four times fewer copies.

7.2 How to Spray Multiple Copies

Intuitively, spraying copies as fast as possible is the best way to spread copies if all the relay nodes are equal/homogeneous. (One might want to bank copies for future encounters with 'super nodes' when relay nodes are heterogeneous, see our prior work [45].). To answer whether spraying the copies as fast as possible is optimal under a homogeneous relays scenario, we compare the two different spraying schemes introduced in Section 4.3, source spray and wait and fast spray and wait for the random waypoint mobility model. Since fast spray and wait spreads copies whenever there is any opportunity to do so, it has the minimum spraying time when there is no contention in the network [17]. On the other hand, since source spray and wait does not use relays to forward copies, it is one of the slower spraying mechanisms when there is no contention in the network.

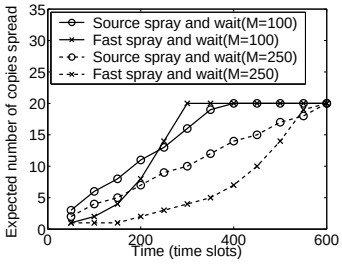


Fig. 7. Comparison of fast spray and wait and source spray and wait: Expected number of copies spread vs time elapsed since the packet was generated. Network parameters: $N = 100 \times 100$ square units, $K = 5, \Theta = 5, s_{BW} = 1$ packet/time slot, $L = 20$. Expected maximum cluster size (metric to measure connectivity) for these network parameters is equal to 4.6% for $M = 100$ and 5.2% for $M = 250$.

Now we study how fast the two schemes spread copies of a packet when there is contention in the network. Figure 7 plots the number of copies spread as a function of the time elapsed since the packet was generated. Somewhat surprisingly, depending on the density of the network, source spray and wait can spray copies faster than fast spray and wait. This occurs because fast spray and wait generates more contention around the source as it tries to transmit at every possible transmis-

sion opportunity. Such a behavior is expected for dense networks, but these results show that increased contention can deteriorate fast spray and wait's performance even in sparse networks. In general, unless the network is very sparse, strategies which spray copies slower yield better performance than more aggressive schemes thanks to reducing contention. In ongoing work, we are trying to find the optimal spraying algorithm and design practical and implementable heuristics which achieve performance very close to the optimal. [45] is a first step in this direction. It derives the optimal spraying scheme and a simple heuristic which performs very close to the optimal, but it assumes that there is no contention in the network. Currently, we are merging this work with the contention framework proposed in this paper to find the optimal spraying scheme with contention in the network.

7.3 How to Route Individual Copies

Without contention, performing utility-based forwarding on each individual copy outperforms spray and wait schemes because it identifies appropriate forwarding opportunities that could deliver the message faster [15]. However, utility-based forwarding requires more transmissions and hence, increases the contention in the network. So we study how much performance gains are achieved by spray and focus over spray and wait (for the community-based mobility model) both with and without contention in the network by plotting the minimum value of the average number of transmissions it takes to achieve a given target expected delay for both the schemes in Figure 8. We first find the minimum value of L which achieves the given target expected delay for both the schemes and then find the average number of transmissions which is equal to $\sum_{i=1}^{L} i p_{dest}^{R}(i)$. (The minimum value of L is computed using the analytical expressions derived in Section 5.3. The value of $p_{dest}^{R}(i)$ for both the schemes was derived in Theorems 5.3 and 5.4.) We observe that fast spray and focus outperforms fast spray and wait even with contention in the network, with gains being larger with contention. Since $E[M_{comm,diff}] >> E[M_{comm,same}]$, forwarding a copy to any node in the destination's community in the focus phase significantly reduces the delay for the same L without significantly increasing the contention as it requires only one extra message per copy. Hence, fast spray and focus shows more performance gains over fast spray and wait after incorporating contention.

8 Conclusions

In this paper, we first propose an analytical framework to model contention to analyze the performance of any given mobility-assisted routing scheme for any given mobility and channel model. Then we find the expected delay for representative mobility-assisted routing schemes for intermittently connected mobile networks (direct transmission, epidemic routing and different spraying based schemes) with contention in the network for the random direction, random waypoint and the more realistic community-based mobility model. Finally, we use these delay expressions to demonstrate that designing routing schemes using analytical expressions which ignore contention can lead to suboptimal or even erroneous decisions.

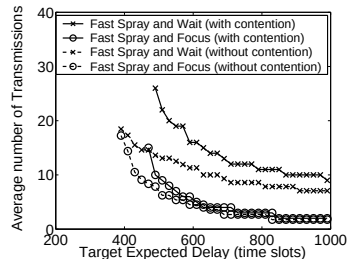


Fig. 8. Comparison of fast spray and wait and fast spray and focus. Average number of transmissions required to deliver the packet to the destination vs target expected delay. Network parameters: $N = 500 \times 500$ square units, $M = 40, K = 20, \Theta = 5, s_{BW} = 1$ packet/time slot, $p_l = 0.8, p_r = 0.15, r = 4$.

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APPENDIX A

Proof: (**Lemma 3.2**) It is given that there are a nodes within scheduling area. Hence, there are $\begin{pmatrix} a \\ 2 \end{pmatrix}$ pairs of these nodes. Lets choose one such pair and let $p_a =$ Pr[the nodes of this pair are within a distance K of each other] and let $p_{pkt} =$ Pr[the nodes of this pair have at least one packet to exchange]. Out of these a nodes, i and j are within K distance of each other and have at least one packet to exchange. The rest are within K distance of each other and have at least one packet to exchange with probability $p_a p_{pkt}$. Hence, the expected number of possible transmissions amongst these a nodes is $1 + p_a p_{pkt}\left(\begin{pmatrix} a \\ 2 \end{pmatrix} - 1\right)$. To figure out the value of p_a, lets choose a pair of nodes amongst these a nodes and label the nodes u_1 and u_2. Let $f(x, y)$ denote the pdf that a node u_1 is a distance x from from the transmitter and at a distance y from the receiver. Then, using simple combinatorics, we derive $f(x, y)$ to be equal to

$$\begin{cases} \dfrac{2cos^{-1}\left(\frac{\frac{4K^2}{9}+2Kd+d^2}{\frac{4K}{3}(K+d)}\right)}{A_1\left(\frac{2K}{3}-d\right)} & \text{if } K < x = K + d < K + \frac{2K}{3}, \\ & d + \frac{K}{3} < y < K \\ \frac{1}{A_1} & \text{if } 0 < x \leq K, 0 \leq \theta < 2\pi, \\ & y = \sqrt{x^2 + \frac{4K^2}{9} - \frac{4Kxcos(\theta)}{3}} \end{cases}.$$

Now, conditioned over the fact that node u_1 is at a distance x from the transmitter and y from the receiver, we determine the probability that node u_2 is within range from u_1. By assumption, u_2 is within one hop from the transmitter and the receiver. If u_2 is also within a distance K from u_1, that is, u_2 lies in the area marked by the intersection of the following three circles: (i) centered at the transmitter with radius equal to K, (ii) centered at the receiver with radius equal to K, and (iii) centered at u_1 with radius equal to K, then u_2 is within range of u_1. Thus, the probability that u_1 and u_2 are within range of each other given that u_1 is at a distance x and a distance y from the transmitter and the receiver respectively, is equal to $\frac{A_3(x,y)}{A_1}$ where $A_3(x, y) = A_4(x, y) + A_5(x, y) - A_6(x, y)$, $A_4(x, y) = 2K^2cos^{-1}\left(\frac{x}{2K}\right) - \frac{x}{2}\sqrt{4K^2 - x^2}$, $A_5(x, y) = 2K^2cos^{-1}\left(\frac{y}{2K}\right) - \frac{y}{2}\sqrt{4K^2 - y^2}$ and $A_6(x, y) = K^2\left(sin^{-1}\left(\frac{x}{2K}\right) + sin^{-1}\left(\frac{y}{2K}\right) + sin^{-1}\left(\frac{1}{3}\right)\right)$ $+\frac{1}{4}\sqrt{\left((x+y)^2 - \frac{4K^2}{9}\right)\left(x - y + \frac{2K}{3}\right)\left(y - x + \frac{2K}{3}\right)}$ $-\frac{x\sqrt{4K^2-x^2}}{4} - \frac{y\sqrt{4K^2-y^2}}{4} - \frac{2\sqrt{2}K^2}{9}$. The value of A_1 was derived in Lemma 3.1. Removing the condition

on the location of u_1 using the law of total probability yields the value of p_a. The value of p_{pkt} can be derived from simple combinatorics to be $1-\left(1-p_{ex}^{R}\right)^{E[S]}$.

Now, we quantify the contention due to the c nodes within two hops from the either the transmitter or the receiver but not in the scheduling area. Contention arises when one of the a nodes is within range of one of the c nodes. There are ac such pairs. Lets choose one such pair and label the corresponding nodes u_1 and u_3, where u_1 lies in the scheduling area while u_3 is within two hops from the either the transmitter or the receiver but not in the scheduling area. Define p_c = Pr[u_1 and u_3 are within range of each other]. Then, the expected number of transmissions contending are acp_cp_{pkt}. p_c is derived in a manner similar to the derivation of p_a using the following two observations: (i) u_3 can lie anywhere within two hops from either the transmitter or the receiver, and (ii) Conditioned over the fact that node u_1 is at a distance x from the transmitter and y from the receiver, u_3 will be within a distance K from u_1 only if it lies in the circle of radius K centered at u_3 but not in the scheduling area. □

Proof: (**Lemma 4.4**) The proof runs along the same lines as the proof of Lemma 4.2. When there are $1 \leq m < L$ copies of a packet in the network, there are m nodes which can deliver a copy to the destination only, and there is one source node which can deliver a copy to any of the $M-m-1$ other nodes which do not have a copy of the packet. Hence, there are a total of $m+M-m-1 = M-1$ node pairs, which when meet, have an opportunity to increase the number of copies from m to $m+1$. The expected time it takes for one of these $M-1$ node pairs to meet is $\frac{E[M_{mm}]}{M-1}$. Using the same argument as in the proof of Lemma 4.2, $E[D_{ssw}^{mm}(m)]$ can be derived to be $\frac{E[M_{mm}]}{(M-1)p_{success}^{ssw}}$.

When there are L copies of a packet in the network, there are L nodes which can deliver a copy to the destination but even if the source meets some other node which does not have a copy, it cannot attempt to transmit a copy to the other node. The expression for $E[D_{ssw}^{mm}(L)]$ is derived in a manner similar to the derivation of Lemma 4.2 to be $\frac{E[M_{mm}]}{Lp_{success}^{ssw}}$. □

Proof: (**Lemma 5.3**) After the spray phase (after L copies have been spread), there is a community which has only $\hat{l} = \text{rem}\left(L, \frac{M}{r}\right)$ nodes carrying a copy of the packet. The probability that the destination is one of the remaining $\frac{M}{r} - \hat{l}$ nodes belonging to this community is equal to $\frac{\frac{M}{r}-\hat{l}}{M-L}$. First we will derive the expected delay in the wait phase when the destination belongs to this community. The probability that the system state is $(\hat{l}, m_p)$ (where m_p denotes the number of node pairs in the community which want to exchange this packet, and had an opportunity in the past to exchange this packet but were unable to do so due to contention) is equal to $p_{\hat{l},m_p}$. (The value of $p_{\hat{l},m_p}$ was derived in Lemma 5.1.) Given the system state in which the spray phase ended is $(\hat{l}, m_p)$, the number of nodes which had an opportunity to deliver the packet to the destination but were unable to do so is equal to $\frac{m_p}{\frac{M}{r}-\hat{l}}$. (As discussed in the proof of Lemma 5.1, each node not having a copy of the packet has met on an average $\frac{m_p}{\frac{M}{r}-\hat{l}}$ nodes which have a copy of the packet.) To derive the delay associated with the wait phase, we define a new system state: (s) where s is the number of nodes in the destination's community which had an opportunity to deliver the packet to the destination but were unable to do so due to contention. Let T_s denote the additional time it will take to deliver the packet to the destination given the current system state is (s). Then, given that nodes in the destination's community have a copy of the packet, $E[D_{fsw}^{comm}(L)]$ is equal to $\left(\sum_{m_p=0}^{\hat{l}\left(\frac{M}{r}-\hat{l}\right)} p_{\hat{l},m_p} E[T_{\frac{m_p}{\frac{M}{r}-\hat{l}}}]\right)$.

To complete the previous proof, we now describe how to derive the value of $E[T_s]$. One of the nodes carrying the packet meets the destination after an expected time duration of $\left(\frac{\hat{l}-s}{E[M_{comm,same}]} + \frac{s}{E[M_{comm,same}^{+}]}\right)^{-1}$. With probability $p_{success1}^{fsw}$, this node is able to deliver the packet to the destination (where $p_{success1}^{fsw} = 1-\left(1-p_{txS1}^{fsw}\right)^{E[\tau_{comm,same}]}$). With probability $p_s = \left(\frac{\hat{l}-s}{E[M_{comm,same}]} + \frac{s}{E[M_{comm,same}^{+}]}\right)\left(\frac{E[M_{comm,same}^{+}]}{s}\right)^{-1}$, the node which meets the destination is one of the s nodes which have missed an opportunity to deliver the packet to the destination in the past. Hence, with probability $p_s\left(1-p_{success1}^{fsw}\right)$ the packet does not get delivered to the destination and the system remains in state s and will take an additional $E[T_s]$ time to deliver the packet to the destination. On the other hand, with probability $(1-p_s)\left(1-p_{success1}^{fsw}\right)$, the packet does not get delivered to the destination and the system moves to state $s+1$ (as one more node belonging to the destination's community has missed an opportunity to deliver the packet to the destination) and will take an additional $E[T_{s+1}]$ time to deliver the packet to the destination. Thus, $E[T_s] = \left(\frac{\hat{l}-s}{E[M_{comm,same}]} + \frac{s}{E[M_{comm,same}^{+}]}\right)^{-1} + p_s\left(1-p_{success1}^{fsw}\right)E[T_s] + (1-p_s)\left(1-p_{success1}^{fsw}\right)E[T_{s+1}]$. This set of linear equations can be solved to find $E[T_s]$.

Now, with probability $1-\left(\frac{\frac{M}{r}-\hat{l}}{M-L}\right)$, none of the nodes belonging to the destination's community have a copy of the packet and the expected time it takes for the L nodes to deliver the packet to destination can be derived in manner similar to the derivation of Lemma 4.2 to be equal to $\frac{E[M_{comm,diff}]}{Lp_{success1}^{fsw}}$.

Finally combining everything together by using the law of total probability to remove the condition on whether a node belonging to the destination's community had a copy of the packet after the spray phase or not, yields the result. □

Proof: (**Lemma 5.4**) After the spray phase (after L copies have been spread), there is a community which

has only $\hat{l} = \text{rem}\left(L, \frac{M}{r}\right)$ nodes carrying a copy of the packet. The probability that the destination is one of the remaining $\frac{M}{r} - \hat{l}$ nodes belonging to this community is equal to $\frac{\frac{M}{r}-\hat{l}}{M-L}$. The expected delivery delay to the destination for this scenario is derived in a manner similar to the derivation of $E[D_{fsw}^{comm}(L)]$ in Lemma 5.3.

Now we derive the delivery delay for the scenario when the nodes in the destination's community do not have a copy of the packet. The expected time it takes for the L nodes carrying a copy to deliver a copy to one of the $\frac{M}{r}$ in the destination's community is equal to $\frac{E[M_{comm,diff}]}{L\frac{M}{r}p_{success2}^{fsf}}$. (This is derived in a manner similar to the derivation of Lemma 4.2). With probability $\frac{\frac{M}{r}-1}{\frac{M}{r}}$, the packet copy is received by a node which itself is not the destination but belongs to the destination's community. This node does a direct transmission to the destination which takes an additional time whose expected value is equal to $E[M_{comm,same}]+\frac{(1-p_{successs1}^{fsf})E[M_{comm,same}^{+}]}{p_{successs1}^{fsf}}$. (This is derived in a manner similar to the derivation of Lemma 5.1.) $\square$