Performance Analysis of Mobility-assisted Routing*

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ABSTRACT

Traditionally, ad hoc networks have been viewed as a connected graph over which end-to-end routing paths had to be established. Mobility was considered a necessary evil that invalidates paths and needs to be overcome in an intelligent way to allow for seamless communication between nodes. However, it has recently been recognized that mobility can be turned into a useful ally, by making nodes carry data around the network instead of transmitting them. This model of routing departs from the traditional paradigm and requires new theoretical tools to model its performance. A *mobility-assisted* protocol forwards data only when appropriate relays encounter each other, and thus the time between such encounters, called *hitting* or *meeting* time, is of high importance.

In this paper, we derive accurate closed form expressions for the expected encounter time between different nodes, under commonly used mobility models. We also propose a mobility model that can successfully capture some important real-world mobility characteristics, often ignored in popular mobility models, and calculate hitting times for this model as well. Finally, we integrate this results with a general theoretical framework that can be used to analyze the performance of mobility-assisted routing schemes. We demonstrate that derivative results concerning the delay of various routing schemes are very accurate, under all the mobility models examined. Hence, this work helps in better understanding the performance of various approaches in different settings, and can facilitate the design of new, improved protocols.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless Communication; C.2.2 [**Network Protocols**]: Routing Protocols

*This work is funded by NSF Nets NBD grant CNS-0520017

MobiHoc'06, May 22–25, 2006, Florence, Italy.

General Terms

Theory, Performance

Keywords

routing, hitting times, ad-hoc networks, intermittent connectivity, delay tolerant networks, mobility modelling

1. INTRODUCTION

Traditionally, ad hoc networks have been viewed as a connected graph over which end-to-end routing paths need to be established to enable data delivery. Mobility of nodes was considered a necessary evil that invalidates existing routing entries over time, and needs to be intelligently overcome to allow for seamless communication between nodes. However, it has recently been recognized that mobility can be transformed into a useful ally and be exploited to improve ad hoc network performance [1, 15].

The seminal paper by Kumar and Gupta [16] showed that the traditional model of ad hoc network routing is not scalable. Maintaining multi-hop paths and transmitting data over these paths results in excessive relay transmissions that dominate the available bandwidth. This quickly diminishes the capacity available per node for end-to-end traffic. To deal with this inherent difficulty, researchers proposed to allow only a fixed number of transmissions per message [15]. A message is handed over to the first few relays encountered, and each relay will then carry the message all the way to its destination itself [12, 15, 31]. This method guarantees a constant per node capacity, regardless of network size.

On a different note, there has been a growing interest in the past few years in applications that can operate over networks that are disconnected most of the time. Such applications often target environments where constant connectivity might be difficult, as is the case for example in deep-space interplanetary networks (IPN [10]), Vehicular Ad hoc Networks (VANETs [40]), sensor networks for wildlife tracking [23], etc. Alternatively, operation over disconnected networks may be desirable for economic reasons, as for example in the case of low-cost Internet provision in remote communities [13] or third-world countries [21, 27]. Such networks are often collectively referred to as Delay Tolerant Networks (DTN) [1]. In Delay Tolerant Networks traditional routing approaches fail, because they require the existence of complete end-to-end paths to be able to deliver any data. To overcome this issue, messages need to get carried by mobile nodes between disconnected parts of the network [30, 43]. A message gets forwarded to some other relay encountered,

carried further, gets forwarded again, and so on and so forth until it reaches its destination.

What the above two approaches share in common is that node mobility is exploited to carry messages around the network as part of the routing algorithm. We will refer to these schemes collectively as *mobility-assisted routing* (in related literature they are also referred to as *encounter-based* or *store-carry-and-forward*). Mobility-assisted routing consists of a series of *independent* forwarding decisions, that take place when two nodes meet each other, and these nodes are completely oblivious of the specific path the message will end up following. Nodes carry a set of messages, possibly for long periods of time, until other nodes are encountered, exchange messages according to the specific protocol, and continue their trip. This constitutes an important departure from the existing routing paradigm, and thus different theoretical tools and models are necessary to analyze the performance of such schemes.

Since message transmissions in this context occur only when nodes meet each other, *the time elapsed between such meetings is the basic delay component*[1]. Therefore, in order to be able to evaluate the performance of *any* mobility-assisted routing scheme, it is necessary to know the statistics of encounter times between nodes, called *hitting* or *meeting* times. These are the times until a node, which, say, just received a message, *first* encounters a *given* other node that can act as a relay. *The minimum time elapsed before the message can be forwarded any further* directly depends on these quantities. They constitute the basic component in the delay expression of any scheme, and they largely vary depending on the specific mobility model in hand.

Although a number of efforts have been made towards analyzing the delay of mobility-assisted routing [14, 26, 29, 31, 33, 36], hitting and meeting times have largely remained unstudied for most mobility models. The random walk model is, to the best of our knowledge, the only exception [30, 35, 36]. As a consequence, most existing results regarding the delay of various routing schemes are either only asymptotic [14, 26, 31] or not completely analytical, in that specific parameters of the model need to be acquired from simulations [29, 33]. However, complete analytical expressions are necessary in order to theoretically compare different schemes, and predict their performance in a large range of different settings.

Additionally, the majority of mobility models commonly used in theoretical analysis or simulation are not realistic. Mobility models like Random Waypoint [9], Random Direction [5], Random Walk, [2] etc., assume that each node may move equally frequently to every network location. Furthermore, such models usually assume that all nodes have the same mobility characteristics, that is, every node's mobility process is identical and independently distributed from all others (IID). However, numerous recent studies based on mobility traces from real networks (e.g. university campuses, conferences, etc.) have demonstrated that these two assumptions rarely hold in real-life situations [4, 18–20, 25]. Although there exist some efforts to create more accurate mobility models [3, 22, 37], a lot of these models are quite complicated and/or tailored to specific scenarios, making them difficult to be used in theoretical studies.

[1]It is reasonable to assume that message transmission will be significantly faster than node movement for modern wireless devices.

In this paper, we analyze the hitting times for a number of popular mobility models like the Random Waypoint and the Random Direction models, and derive accurate closed form expressions for their expected value. We show how these expressions can be used in a general analytical framework to calculate the expected delay of various mobility-assisted routing schemes in a "plug n' calculate" manner. Specifically, we demonstrate that derivative results based on these expressions and concerning the delay of various routing schemes under various mobility models are also very accurate. Additionally, we propose the *Heterogeneous Community based Mobility Model*, a novel mobility model that, unlike the previous models, successfully captures important insight regarding real-world mobility, and still is analytically tractable. To demonstrate the latter, we derive encounter-based statistics for this model, and show how it can be incorporated in the general analytical framework, as well.

In the next section we go over some related work, and summarize our contributions. In Section 3 we derive theoretical closed form expressions for the hitting times under the Random Waypoint and the Random Direction mobility models. Then, in Section 4 we introduce our own Heterogeneous Community-based model and analyze its encounter based statistics. Section 5 incorporates the hitting times into the general analytical framework that can be used to predict the performance of mobility-assisted routing under various mobility models. Finally, Section 6 concludes the paper.

2. RELATED WORK

The first paper that proposed the use of mobility to overcome the limited capacity problem of ad hoc networks was the "2-hop relay" scheme of Grossglauser et al. [15]. In this scheme, at most one relay is allowed to carry a copy of the message, and thus the message path is at most two hops long. Although this scheme guarantees a $\Theta(1)$ capacity per node, it may result in long delivery delays because the relay has to wait until it moves itself all the way within range of the destination.

A significant research thread has spawned thereafter exploring the fundamental trade-offs between the capacity and the delay of the 2-hop and other similar schemes (e.g. [12, 14, 26, 28, 31]). For example, it has been shown that by handing over copies to more than one (yet fixed number of) relays, delay is reduced for most mobility models, while asymptotic capacity (i.e. per node capacity as the number of nodes grows to infinity) remains constant [14].

Despite the amount of existing work, most results are of asymptotic nature. Although asymptotic results provide valuable insight on the scalability of a given family of protocols, explicit results are often necessary to design and compare practical schemes. Furthermore, the majority of these works are concerned with delay in *connected* networks, where the transmission range of each node has to scale with the number of nodes, in order to ensure connectivity [17]. This makes all such analytical results strictly a function of the number of nodes (e.g. [14, 31]). Here, we're interested in a much wider range of connectivity scenarios, where transmission range, number of nodes, and network size are *independent* parameters, whose individual effect on performance we would like to quantify.

A significant amount of theoretical work has also recently emerged in the context of intermittently connected or Delay

Tolerant Networks [29, 31, 33, 35, 36, 39, 42]. In contrast to connected networks, where mobility is only used to reduce the number of transmissions, there, mobility is a *necessary component* of the routing functionality. These papers try to analyze the delay of flooding or other schemes that route one or more message copies according to some algorithm, in networks that are not connected for the majority of time. However, in doing so, they often assume that the expected time between encounters is just a parameter of the mobility model that can be acquired from simulations or curve fitting [29, 31, 33]. Although this makes these results quite generic, at the same time it also significantly reduces the usefulness of analytical expressions. Running a simulation every time for the specific network configuration in hand, to estimate a parameter of the theoretical model, comes short of being able to predict and compare the performance of different schemes under a wide range of settings. Explicit expressions for encounter times under a given mobility model are necessary, to derive explicit expressions for the delay of *any* mobility-assisted routing scheme.

To fill this important gap, in this paper we analyze the hitting times for some popular mobility models, like the Random Waypoint [9] and the Random Direction model [5]. Although various statistical properties of the Random Waypoint and the Random Direction model (e.g. node distribution [6], convergence [8, 41], etc.) have been studied, hitting and meeting times under such models are not available. To the best of our knowledge, hitting and meeting times have only been analyzed in the past for Random Walk mobility [2, 30, 35]. The work that is closest to ours is probably that of [32]. There, the authors use a similar methodology to the one we'll use, but only derive upper and lower bounds on the meeting time between two nodes performing Random Waypoint movement, and use it to calculate an asymptotic result. Finally, we also show how hitting times can be integrated with generic routing delay expressions like the ones in [33, 35, 36, 39] into a powerful framework. A framework along these lines was proposed in [29].

A lot of work exists also in the arena of mobility modelling. It has been widely recognized that mobility models commonly used in simulation studies or theoretical analysis, like Random Waypoint, Brownian motion, etc. [11], although easy to use, do not closely resemble mobility involving humans [19, 20, 22]. A large amount of mobility traces from university or conference environments have recently become available and have been extensively analyzed to extract basic characteristics of real-world mobility [4, 18–20, 25]. These traces indicate that, in most of these scenarios, nodes do not move randomly around the whole network area. Instead, nodes usually have some locations where they spend a large amount of time (e.g. home, office building, etc.). A few times in the day, they might have to commute back and forth from these locations, or might roam around the network for some time (e.g. running errands). Additionally, contrary to the common assumption, node movement is not IID. Different nodes visit different location more often, and some nodes may be more mobile than others.

A number of mobility models have been proposed that better capture specific scenarios, like freeway movement [3], human mobility in campus environments [22, 37], group mobility models [11], etc. The work closest to ours in terms of mobility modelling is probably that of [24]. Although this work manages to capture the locality of node movement, it does not account for heterogeneity in different nodes' mobility processes. Finally, we believe that a model that would capture most of the details of real-life mobility, a very challenging endeavor itself, would be extremely difficult to allow any theoretical analysis. For this reason, we propose here a novel mobility model that: (i) successfully captures the basic intuition derived from traces regarding real-world mobility, (ii) is highly tunable to be able to model a large range of scenarios, and (iii) is analytically tractable. We calculate encounter-based statistics under this more realistic mobility model as well, and show how it can be incorporated into the general framework.

3. HITTING AND MEETING TIMES UNDER POPULAR MOBILITY MODELS

In this section we will look into the encounter-based statistics of two commonly used mobility models, namely Random Waypoint (RWP) and Random Direction (RD) mobility. However, before we do so we first introduce some useful definitions and notation and state the assumptions we'll be making throughout the remaining of the paper.

(a) All nodes exist in area U of size $\|U\| = N$, and have a transmission range equal to K. The position of node i at time t is denoted as $X_i(t)$ or X_i if it is static.

(b) All the mobility models we deal with are *epoch-based*; An *epoch* is a given period of time during which a node moves towards the same direction and with the same speed; Each node's trajectory is a sequence of epochs.

(c) The length L of an epoch, measured as the distance between the starting and finishing points of it, is a random variable with expected value $\overline{L}$.

(d) The speed of a node during an epoch is randomly chosen from $[v_{min}, v_{max}]$, with $v_{min} > 0$, $v_{max} < \infty$ and average speed $\overline{v}$.

(e) At the end of each epoch a node pauses for a random amount of time chosen from $[0, T_{max}]$, with average pause time $\overline{T}_{stop}$.

(f) The expected duration of an epoch (without the pause time) is denoted as $\overline{T}$.

(g) Let $\vec{v}_i$ denote the velocity of node i and $\overline{v}_{mm} = \|\vec{v_i} - \vec{v_j}\|$ be the mean relative speed between two nodes i and j when both are moving according to mobility model *MM*. Then we define the *normalized relative speed* $\hat{v}_{mm}$ as $\hat{v}_{mm} = \frac{\overline{v}_{mm}}{\overline{v}}$.

The following formally defines hitting and meeting times.

Definition 3.1 (Hitting and Meeting Time). *Let a node i move according to mobility process "MM", and starting from its stationary distribution at time* 0. *Then,*

i. If j is a static node with uniformly chosen X_j, then the expected hitting time *under mobility model* MM *is $ET_{mm} = \min_t\{t : \|X_i(t) - X_j\| \leq K\}$.*

ii. If j is a mobile node also starting from its stationary distribution, then the expected meeting time *between the two nodes is $EM_{mm} = \min_t\{t : \|X_i(t) - X_j(t)\| \leq K\}$.*

Table 1: Notation

ET_{mm}	expected hitting time under "MM"
EM_{mm}	expected meeting time under "MM"
N	size of network area
K	transmission range
$\overline{L}$	expected epoch length
$\overline{v}$	average node speed
$\overline{T}_{stop}$	average pause time after an epoch
$\overline{T}$	expected epoch duration
$\hat{v}_{mm}$	normalized relative speed under "MM"

Table 1 summarizes our notation.

We first analyze the hitting and meeting time for the Random Direction model. Although the Random Waypoint model was proposed first, it was quickly recognized that it results in a non-uniform stationary node distribution. This not only complicates analysis, but is also in discord with the common assumption of uniformity made in many studies. To overcome this, the Random Direction model was proposed, which induces a uniform node distribution [5].

3.1 Random Direction Mobility Model

Definition 3.2 (Random Direction). *In the Random Direction (RD) model each node moves as follows: (i) choose a direction θ uniformly in $[0, 2\pi)$; (ii) choose a speed according to assumption (d); (iii) choose a duration T of movement from an exponential distribution with average $\frac{\overline{L}}{\overline{v}}$; (Note that we assume $\overline{L} = O(\sqrt{N})$.)[2] (iv) move towards θ with the chosen speed for T time units;[3] (v) after T time units pause according to assumption (e) and go to step (i).*

The following two Theorems calculate the expected hitting and meeting times for the Random Direction model. Our methodology is based on calculating the expected number of epochs until a static or mobile destination, respectively, is encountered.

Theorem 3.1. *The expected hitting time ET_{rd} for the Random Direction model is given by:*

$$ET_{rd} = \left(\frac{N}{2K\overline{L}}\right)\left(\frac{\overline{L}}{\overline{v}} + \overline{T}_{stop}\right). \tag{1}$$

Proof. Let a node A perform RD movement, starting from its stationary distribution. A's movement consists of a sequence of randomly and independently chosen epochs. Let further a second node B be static with uniformly chosen position, and let us calculate the probability that node A encounters node B during a given epoch i of length L_i. This epoch will "cover" an area of size $2KL_i$. If B lies anywhere within this area, then A "hits" B during this specific epoch. Furthermore, it is easy to see by the definition of the RD model, that the specific area of the network an epoch will cover is uniformly distributed around the whole network. Hence, the probability p_i of an epoch of length L_i hitting B is equal to $p_i = \frac{2KL_i}{N}$.

[2]This assumption ensures fast mixing of the corresponding process and simplifies analysis. Further, it is inline with the spirit of the model since RD has been introduced as a close alternative to RWP which has O(1) mixing time.

[3]If the boundary is reached, the node either reflects back or re-enters from the opposite side of the network (torus).

Let us now denote as N_{hit} the number of epochs until A hits B, and $P(N_{hit} > n)$ the probability that B has not been encountered after n epochs. Let further E_i, $i = 1 \ldots n$ denote the event that A doesn't hit B at the ith epoch given that the length of the epoch equals l_i, and $f_L(l_1, l_2, \ldots, l_n)$ denote the joint probability density function of the lengths of these first n epochs. Then:

$$P(N_{hit} > n) = \int \cdots \int P(E_1) \cdots P(E_n | E_1 \ldots E_n) f_L(l_1, \ldots, l_n) dl_1 \ldots dl_n.$$

Although consecutive epochs are not independent (the end of one epoch is the beginning of the next one), their lengths are i.i.d. and we can use the statistics of one epoch to describe all epochs. Further, while in general E_i is not independent of E_j, $j < i$ (to see this, consider very small epoch lengths, in which case RD resembles a random walk), we have assumed that the epoch lengths are large, specifically $O(\sqrt{N})$, such that the process mixes very fast (similar to RWP where the mixing time is exactly one epoch). Hence, E_i's are (approximately) independent (a similar argument has been made for RWP in [6]) and

$$P(N_{hit} > n) = \left(\int \left(1 - \frac{2Kl}{N}\right) f_L(l) dl\right)^n = \left(1 - \frac{2K\overline{L}}{N}\right)^n.$$

Consequently, the number of epochs needed till A hits B is geometrically distributed with average $\frac{N}{2K\overline{L}}$. Finally, the expected duration of each epoch is equal $\overline{T} + \overline{T}_{stop}$ (see assumptions (e),(f)), where $\overline{T} = \frac{\overline{L}}{\overline{v}}$ in the case of RD. □

Theorem 3.2. *The expected meeting time EM_{rd} for the Random Direction model is given by*

$$EM_{rd} = \frac{ET_{rd}}{p_m \hat{v}_{rd} + 2(1 - p_m)}, \tag{2}$$

where $\hat{v}_{rd}$ is the normalized relative speed for RD, and $p_m = \frac{\overline{T}}{\overline{T} + \overline{T}_{stop}}$ is the probability that a node is moving at any time.

Proof. Assume again that only one of the two nodes, say A, performs RD movement, while the second one, say B, is static. We first re-calculate the expected hitting time of Theorem 3.1 in a slightly different manner. Assume that node A performs RD movement in discrete steps of unit size, and let p_m denote the probability that A is moving at any of these steps. Then, with probability p_m any given step covers on average a new area of size $2K\overline{v}$, and with probability $1 - p_m$ it stands still. Thus, on average, each node step covers an area of $p_m 2K\overline{v}$, and the expected number of steps until the destination is found is equal to

$$ET'_{rd} = \frac{N}{p_m 2K\overline{v}}.$$

Note that this method of calculating the hitting time is equivalent to that of Thoerem 3.1, i.e. $ET'_{rd} = ET_{rd}$[4].

Now, to calculate the meeting time, we need to take into account that both A and B move *concurrently*. It is known that, for *generic* random walks on graphs, the meeting time between two walks is $\frac{1}{2}$ the respective hitting time of a single walk on the same graph [2]. This holds, because the

[4]One can see this by replacing p_m with its value $\frac{\overline{T}}{\overline{T} + \overline{T}_{stop}}$, which then gives the expected hitting time in the familiar form of $\frac{N}{2K\overline{L}}\left(\frac{\overline{L}}{\overline{v}} + \overline{T}_{stop}\right)$.

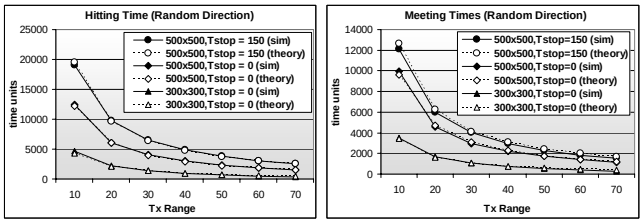

Figure 1: Comparison of theoretical and simulations results under the Random Direction model.

movements at consecutive steps are independent of each other. However, in the RD model a node keeps moving in the same direction for the duration of an epoch. The *relative* movement at consecutive steps is not independent. We thus need to calculate the expected relative speed $\|\vec{v_A} - \vec{v_B}\|$ between A and B. Due to the uniform choice of direction at every epoch, and the toroidal structure of the network, we can assume without loss of generality that the direction of $\vec{v_A}$ is fixed. In other words, $\vec{v_A} = (v_A, 0)$ and $\vec{v_B} = (v_B \cos\theta, v_B \sin\theta)$. If we assume, for simplicity, that $\vec{v_A} = \vec{v_B} = \overline{v}$, this gives us

$$\|\vec{v_A} - \vec{v_B}\| = \frac{\overline{v}}{2\pi} \int_0^{2\pi} \sqrt{(1+\cos\theta)^2 + \sin^2(\theta)} d\theta,$$

which is equal to $1.27\overline{v}$. (A little more calculus gives the general case for random speeds.) The normalized relative speed for RD mobility for the above case is $\hat{v}_{rd} = \frac{\|\vec{v_A} - \vec{v_B}\|}{\overline{v}} = 1.27$.

$\hat{v}_{rd}\overline{v}$ is the relative speed between the nodes when *both* nodes are moving, which occurs with probability p_m^2. However, with probability $2p_m(1-p_m)$ only one of the node moves with relative speed $\overline{v}$, and with probability $(1-p_m)^2$ none of the nodes is moving. Consequently, the expected number of steps until the two walks meet equals

$$\begin{aligned} EM_{rd} &= \frac{N}{2K(p_m^2 \hat{v}_{rd}\overline{v} + 2p_m(1-p_m)\overline{v})} \\ &= \frac{ET'_{rd}}{p_m \hat{v}_{rd} + 2(1-p_m)}. \end{aligned}$$

□

Figure 1 compares analytical and simulation results for the expected hitting and meeting times, under the Random Direction model, for increasing transmission (Tx) range, and various network sizes and pause times. (Note that in this and all other plots throughout we normalize average speed to $\overline{v} = 1$.)

Remark: One might argue that counting epochs to calculate hitting times may not capture intra-epoch behavior. However, as can be inferred from the proof of Theorem 3.1, if $K \ll \sqrt{N}$, then the error introduced this way decreases as $\frac{1}{\sqrt{N}}$. This is also confirmed by the accuracy of the plots of Figure 1.

3.2 Random Waypoint Mobility Model

In the original Random Waypoint mobility model (RWP) [9] a node chooses a point in the network uniformly and moves toward it with some speed. When it reaches that point it pauses for a random amount of time, chooses another point in the network uniformly, and so on and so forth:

Definition 3.3 (Random Waypoint). *A node moving according to the RWP model performs the following steps: (i) choose a point ("waypoint") X in the network uniformly, (ii) move toward X with random speed chosen according to assumption (d), and (iii) when at X, pause a random amount of time according to assumption (e), and go to step (i).*

The stochastic properties of RWP have been extensively studied, as we mentioned earlier. In the following Lemma, we summarize some of these properties, derived in [6], which we use in our analysis.

Lemma 3.1. *Let a node move according to RWP, and let $f(x,y)$ denote the probability density function that a node is found at position (x,y) in the network. Then:*

- *The expected epoch length $\overline{L}$ can be derived in closed form (see [6]); for example, in a $\sqrt{N} \times \sqrt{N}$ square, $\overline{L} = 0.5214\sqrt{N}$;*
- *The stationary pdf $f(x,y)$ on a $\sqrt{N} \times \sqrt{N}$ square area is non-uniform, and can be approximated by*

$$f(x,y) \approx \frac{36}{N^3}\left(x^2 - \frac{N}{4}\right)\left(y^2 - \frac{N}{4}\right). \qquad (3)$$

  A better, yet more convoluted approximation can be found in [7];
- *The expected direction chosen at the beginning of an epoch is not uniform, but has a strong bias towards the center; Specifically, if θ is the angle measured from the direction pointing to the center of the network ($\theta = 0$), then the pdf $f_\Theta(\theta)$ is given by*

$$\begin{aligned} f_\Theta(\theta) = \frac{1}{4\pi|\sin^3\theta|}[|\sin\theta|(-2\cos^4\theta - 2\cos^3\theta|\cos\theta| \\ + \cos^2\theta + \cos\theta|\cos\theta| + 1) + \arcsin(|\sin\theta|)\cos\theta]. \end{aligned}$$

An interesting property of the RWP model is that nodes tend to move and slowly concentrate towards the center. This introduces complications in the analysis, since the common assumption of uniform node distribution does not hold. For this reason, hitting and meeting times under RWP mobility have not been widely studied. To the best of our knowledge, the only related effort in that direction is that of [32]. There, the authors come up with upper and lower bounds for the hitting time, and then derive an asymptotic expression based on these bounds. Here, we derive an accurate closed form expression.

Theorem 3.3. *Let $f(x,y)$ denote the stationary probability density function of the position (x,y) of a node that performs RWP movement. Then, the expected hitting time ET_{rwp} under RWP mobility is given by:*

$$ET_{rwp} = \left(\frac{\iint_U \frac{1}{f(x,y)} dxdy}{2KN\overline{L}}\right)\left(\overline{T} + \overline{T}_{stop}\right). \qquad (4)$$

Proof. See Appendix. □

As can be seen by the above Theorem, calculating the hitting time under RWP movement introduces an integral involving the stationary pdf for RWP, whose exact value is not known [6]. Furthermore, the solution of the integral raises convergence issues, due to $\frac{1}{f(x,y)} \to \infty$ at the edges of the network, and may not be calculable in closed form. The following Corollary of Theorem 3.3 uses the approximate expression of Eq.(3) for $f(x,y)$ and takes care of the edge phenomena to provide an approximate yet very accurate closed form expression for the hitting time.

COROLLARY 3.1. *The expected hitting time under the Random Waypoint model can be approximated by the following expression:*

$$ET_{rwp} \approx \left(\frac{g(N,K)}{2KN\overline{L}}\right)\left(\overline{T}+\overline{T}_{stop}\right), \qquad (5)$$

where $g(N,K) = \frac{\left(N^{\frac{3}{2}}+2(K-\sqrt{N})N\tanh^{-1}(1-\frac{2K}{\sqrt{N}})\right)^2}{9(K-\sqrt{N})^2}$.

PROOF. See Appendix. □

Despite the inherent difficulty in calculating the hitting time for this model, things become simpler when the destination performs RWP movement, as well. The position of B is not unifrom anymore, but is rather distributed according to $f(x,y)$. This, as we shall see, makes the meeting time expression independent of $f(x,y)$. In the following Theorem we derive the expected meeting time between two mobile nodes under RWP movement.

THEOREM 3.4. *The expected meeting time EM_{rwp} under Random Waypoint mobility is given by:*

$$EM_{rwp} = \frac{1}{p_m\hat{v}_{rwp}+2(1-p_m)}\frac{N}{2K\overline{L}}\left(\overline{T}+\overline{T}_{stop}\right), \qquad (6)$$

where $\hat{v}_{rwp} \approx 1.75$ is the normalized relative speed for RWP, and $p_m = \frac{\overline{T}}{\overline{T}+\overline{T}_{stop}}$ is the probability that a node is moving at any time.

PROOF. See Appendix. □

In Figure 2 we evaluate our analytical results regarding the expected hitting and meeting time for the Random Waypoint model against simulation results. As can be seen there, both expressions have a very good match with simulation results. The minor discrepancy in hitting time for very small transmission ranges is due to an approximation we make in the proof to account for edge phenomena (see proof of Corollary 3.1 in the Appendix).

4. COMMUNITY-BASED MOBILITY MODEL

In order to have a mobility model that better resembles real node movement, yet is still analytically tractable, we propose the "Community-based Mobility Model". This model consist of two states/phases, namely "local" state and "roaming" state, between which it alternates:

DEFINITION 4.1 (COMMUNITY-BASED MODEL). *All nodes move inside the network as follows:*

- *each node i has a local community C_i of size $\|C_i\| = c^2N, c \in (0,1]$; a node's movement consists of a sequence of local and roaming epochs.*

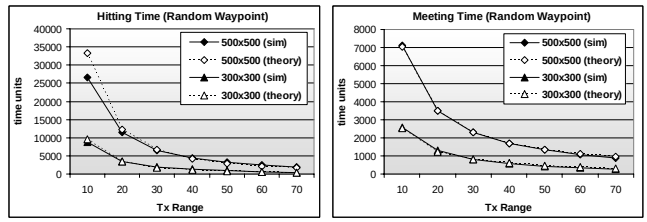

Figure 2: Comparison of theoretical and simulations results under the Random Waypoint model.

- *a **local epoch** is a Random Direction movement restricted inside area C_i with average epoch length $\overline{L}_c$ equal to the expected distance between two points uniformly chosen in C_i.*
- *a **roaming epoch** is a Random Direction movement inside the entire network with expected length $\overline{L}$.*
- (local state L) *if the previous epoch of node i was a local one, the next epoch is a local one with probability $p_l^{(i)}$, or a roaming epoch with probability $1-p_l^{(i)}$.*
- (roaming state R) *if the previous epoch of node i was a roaming one, the next epoch is a roaming one with probability $p_r^{(i)}$, or a local one with probability $1-p_r^{(i)}$.*

The Community-based mobility model can be represented by the simple two-state Markov Chain depicted in Figure 3. (Note that each node could also perform Random Waypoint movement during each of these states, instead of Random Direction.) As can be seen from its description, this model captures real life mobility characteristics observed in various traces [4, 18–20, 25] considerably better than the previous models. First, the locality of movement is captured by the existence of a community inside which each node spends a good amount of its time. Second, each node may also have different $p_r^{(i)}$ and $p_l^{(i)}$ parameters modelling a large range of different mobility characteristics per node. Finally, different nodes may have communities of very different sizes. These together allow for a large range of node heterogeneity to be captured. Although this is largely a qualitative argument, in the future we plan to perform a quantitative validation of this model against real mobility traces. (Note that in [24] a mobility model where each node has its own community to which it may move preferentially is also proposed. Although this model can capture some of the locality, it is not rich enough in terms of heterogeneity.)

Let us denote as $\pi_l^{(i)}$ and $\pi_r^{(i)}$ the probability that a given epoch of node i is a local or a roaming one, respectively. Then, from elementary Markov chain theory we get that

$$\pi_l^{(i)} = \frac{1-p_r^{(i)}}{2-p_l^{(i)}-p_r^{(i)}} \text{ and } \pi_r^{(i)} = \frac{1-p_l^{(i)}}{2-p_l^{(i)}-p_r^{(i)}}.$$

Table 2 summarizes some additional notation related to the community model.

4.1 Hitting Times under Homogeneous Mobility

We will first calculate the expected hitting and meeting times for the case where each node i has its own community

Table 2: Additional Notation for Section 4

C_i	community of node i: $\|C_i\| = c^2 N, c \in (0,1]$
$p_l^{(i)}$	probability that next epoch is local, given that previous epoch was local
$p_r^{(i)}$	probability that next epoch is roaming, given that previous epoch was roaming
$\pi_l^{(i)}$	probability that a given epoch is a local one
$\pi_r^{(i)}$	probability that a given epoch is a roaming one
$\overline{L}_c$	expected length of local epoch
$\overline{T}^l_{stop}$	expected pause time for a local epoch
$\overline{T}_l$	expected local epoch duration $(\overline{L}_c/\overline{v} + \overline{T}^l_{stop})$
$\overline{T}_r$	expected roaming epoch duration $(\overline{L}/\overline{v} + \overline{T}_{stop})$

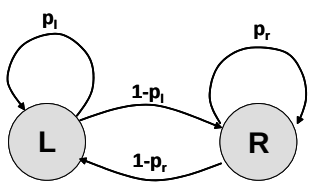

Figure 3: Community-based Mobility Model

C_i, but all nodes have the same mobility characteristics, that is, $p_l^{(i)} = p_l$ and $p_r^{(i)} = p_r$, $\forall i$. We deal with heterogeneous mobility in the next section.

Let's assume that a node A with community C_A moves according to the Community-based model, until it encounters a node B that is static with uniformly chosen position. If B's position is outside C_A, then A can only encounter B during a roaming epoch. Otherwise, if B lies inside C_A, A is expected to encounter B much faster, since it tends to move preferentially inside C_A. The following two Lemmas calculate the expected hitting time for each of these two subcases.

LEMMA 4.1. *Let a node A move according to the Community based Random Direction model. Let further B denote a second, static node, whose position is uniformly distributed outside A's community C_A. Then, the expected hitting time $ET^{(out)}_{comm}$ until A encounters B, is given by:*

$$ET^{(out)}_{comm} = ET_{rd} + \frac{1-p_r}{1-p_l}\frac{N}{2K\overline{L}}\overline{T}_l. \tag{7}$$

PROOF. Let N_l and N_r denote the number of times A visits the local state (L) and roaming state (R), respectively, before it finds B. Furthermore, let $N_{hit} = N_l + N_r$ denote the total number of epochs of any kind. Then, according to the law of large numbers, when $N_{hit} \to \infty$, $N_l \to \pi_l N_{hit}$ and $N_r \to \pi_r N_{hit}$.

Since B does not lie inside A's community, B can only be encountered while A is in the roaming state[5]. The expected number of roaming epochs needed until such a destination is met was found, in Theorem 3.1, to be equal to $\frac{2K\overline{L}}{N}$. This implies that A visits state R $E[N_r] = \frac{2K\overline{L}}{N}$ number of times before it meets B. The sum of the duration of these epochs is equal to ET_{rd}. Additionally, according to the previous argument based on the law of large numbers, A also visits state L *on average*

$$E[N_l] = \frac{\pi_l}{\pi_r}E[N_r] = \frac{1-p_r}{1-p_l}E[N_r]$$

times, before it meets B (given that A starts from its stationary distribution). The average time spent at state L, each time it is visited, is equal to $\frac{\overline{L}_c}{\overline{v}} + \overline{T}^l_{stop}$. Putting everything together gives us Eq.(7). □

LEMMA 4.2. *Let a node A move according to the Community based Random Direction model. Let further B denote a second, static node, whose position is uniformly distributed inside A's community C_A. Then, the expected hitting time $ET^{(in)}_{comm}$ until A encounters B, is given by:*

$$ET^{(in)}_{comm} \approx \frac{1}{1-\left[(1-p^l_{hit})^{\pi_l}(1-p^r_{hit})^{\pi_r}\right]}(\pi_l\overline{T}_l + \pi_r\overline{T}_r), \tag{8}$$

where $p^r_{hit} = \frac{2K\overline{L}}{N}$ and $p^l_{hit} = \frac{p^r_{hit}}{c}$.

PROOF. Let us count the number of steps in the Markov chain corresponding to the community model until B is found. Let further N_l and N_r denote again the number of local and roaming epochs elapsed, respectively, before B is encountered, and let $N_{hit} = N_l + N_r$ denote the total number of epochs. Finally, let $P(N_l, N_r)$ denote the probability that *at least* N_l local and N_r roaming epochs elapse before B is found. Then, $P(N_l, N_r) = (1-p^l_{hit})^{N_l}(1-p^r_{hit})^{N_r}$. According to the law of large numbers, when $N_{hit} \to \infty$, $N_l \to \pi_l N_{hit}$, $N_r \to \pi_r N_{hit}$, and $P(N_l, N_r) \to P(\pi_l N_{hit}, \pi_r N_{hit}) = P(N_{hit})$. Consequently,

$$\lim_{N_{hit}\to\infty} P(N_l, N_r) = P(N_{hit} > n) = (1-p^l_{hit})^{\pi_l n}(1-p^r_{hit})^{\pi_r n}.$$

This implies that the probability distribution of the total number of epochs N_{hit} (local or roaming) has a geometric tail with parameter

$$p_{hit} = 1 - \left[(1-p^l_{hit})^{\pi_l}(1-p^r_{hit})^{\pi_r}\right].$$

Consequently, when the average number of epochs necessary to find B is not too small, we can approximate the pdf of the total epochs with a geometric distribution with the above parameter p_{hit}. For this to occur we require that the transmission range is much smaller than the network dimensions, which is the case indeed in most situations of interest (i.e. when mobility is required to deliver a message). In this case, the expected number of epochs until B is encountered EN_{hit} is equal to $\frac{1}{p_{hit}}$. Finally, each of these epochs is a local one with probability π_l or a roaming one with probability π_r, and with duration $\overline{T}_l$ and $\overline{T}_r$, respectively. □

We can now go ahead and calculate the hitting time for the case where the destination's position is uniformly chosen over the entire network area.

THEOREM 4.1. *The expected hitting time ET_{comm} under the Community-based Mobility Model is given by:*

$$ET_{comm} = (1-c^2)ET^{(out)}_{comm} + c^2 ET^{(in)}_{comm}. \tag{9}$$

[5]we assume that the transmission range K of nodes is much smaller than the total network area N, and thus the probability that B is near the edge of C_A and thus can be encountered even while A is inside its community goes to 0 as $N \to \infty$.

PROOF. With probability $\frac{\|U\|-c^2\|U\|}{\|U\|} = 1 - c^2$ B's position is outside A's community C_A. In that case, B can only be encountered during a roaming phase, and the expected time until this occurs is given is $ET_{comm}^{(out)}$ (Lemma 4.1). Similarly, with probability c^2 B lies inside C_A, in which case the expected hitting time is given by Lemma 4.2. □

Finally, Theorem 4.2 gives the expected meeting time when both nodes are moving.

THEOREM 4.2. *The expected meeting time EM_{comm} under the Community-based Random Direction model is given by:*

$$EM_{comm} = \frac{ET_{comm}}{p_m^c \hat{v}_{rd} + 2(1 - p_m^c)}, \tag{10}$$

where $p_m^c = \frac{(1-p_r)\overline{L}_c/\overline{v}+(1-p_l)\overline{L}/\overline{v}}{(1-p_r)\overline{T}_l+(1-p_l)\overline{T}_r}$ is the probability that a node is moving at any time.

PROOF. See Appendix. □

So far we have assumed that the community of each node could be quite large, covering a considerable chunk of the network area. This might be the case, for example, when the community is a department building in a small college campus network, which implies that two nodes have a good chance of sharing the same community. However, in many real-life situations, each node tends to move most of the time in a very small area that's different for each node (e.g. at home), and that could be entirely covered by the node's antenna, while the network might be much larger (e.g. a city-wide wireless Metropolitan Area Network). In such situations, the probability that two nodes share the same community can be neglected. The following Corollary of Theorem 4.2 calculates the meeting time for the special case of small communities.

COROLLARY 4.1 (SMALL COMMUNITY). *When the community size of nodes is much smaller than the network area ($\|C_i\| \ll N$), the expected meeting time $EM_{comm}^{(small)}$ under the Community-based Random Direction model is given by:*

$$EM_{comm}^{(small)} = \frac{ET_{rd} + \frac{1-p_r}{1-p_l}\frac{N}{2K\overline{L}}\overline{T}^l_{stop}}{p_m^c \hat{v}_{rd} + 2(1 - p_m^c)}, \tag{11}$$

where $p_m^c = \frac{(1-p_l)\overline{L}/\overline{v}}{(1-p_r)\overline{T}^l_{stop}+(1-p_l)\overline{T}_r}$.

PROOF. Eq.(11) follows easily from Eq.(10),(9), and (7) by replacing $c \approx 0$ and $\overline{T}_l \approx \overline{T}^l_{stop}$. □

Figure 4 compares analytical and simulation results for the expected hitting time under the Community-based Random Direction model, for small and large communities (for the large community case all pause times are zero and $P_l = 0.9, P_r = 0.5$). As can be seen there theory matches simulations quite closely. We have also observed similar accuracy for the respective meeting time results.

4.2 Hitting Time for Heterogeneous Nodes

In the following theorem we calculate the expected meeting time in a heterogeneous community model, where each node (i) has its own community, and (ii) has its own mobility parameters. This is a model where nodes do not move in an identical manner.

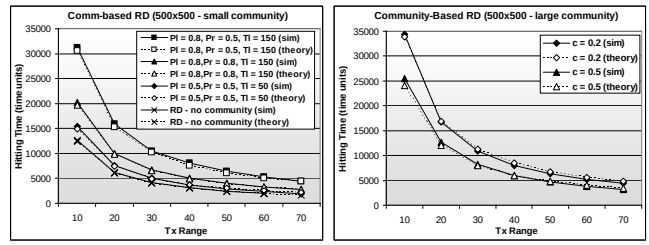

Figure 4: Hitting Times under the Community-based RD model for small (left) and large (right) communities.

THEOREM 4.3. *Let two nodes A and B move according to the Community-based Random Direction, each with its own community C_i and model parameters $(p_l^{(i)}, p_r^{(i)})$, $i \in \{A, B\}$. Let further $ET_{comm}(i \to j)$ denote the expected hitting time under the Community-based model for the case where only node i is mobile and j is static. Then the expected meeting time $EM_{comm}^{(h)}$ between A and B is given by*

$$EM_{comm}^{(h)} = \frac{ET_{comm}(A \to B)}{p_m^{(B)}\hat{v}_{rd} + (1 - p_m^{(B)}) + \frac{p_m^{(B)}}{p_m^{(A)}}(1 - p_m^{(A)})} \tag{12}$$

$$= \frac{ET_{comm}(B \to A)}{p_m^{(A)}\hat{v}_{rd} + (1 - p_m^{(A)}) + \frac{p_m^{(A)}}{p_m^{(B)}}(1 - p_m^{(B)})}, \tag{13}$$

$p_m^i = \frac{(1-p_r^{(i)})\overline{L}_c/\overline{v}+(1-p_l^{(i)})\overline{L}/\overline{v}}{(1-p_r^{(i)})\overline{T}_l+(1-p_l^{(i)})\overline{T}_r}$.

PROOF. See Appendix. □

It is important to note that Theorem 4.3 can be generalized to capture heterogeneity in any of the previous models discussed. For example, let two nodes A and B move according to any waypoint-based mobility model MM, but with different mobility characteristics (e.g. RWP movement with different pause time or different average speed), and let $E_AT_B^{(mm)}$ and denote the respective hitting times from A to B, under the given mobility model. Then the expected meeting time $EM^{(mm)}$ between the two non-identically moving nodes is given by

$$EM^{(mm)} = \frac{E_AT_B^{(MM)}}{p_m^{(B)}\hat{v}_{mm} + (1 - p_m^{(B)}) + \frac{p_m^{(B)}}{p_m^{(A)}}(1 - p_m^{(A)})},$$

where p_m^A and p_m^B denote the probabilities that nodes A and B, respectively, are moving at any given time instant. We have validated the above analytical expressions against simulations, as well, and have observed similar good accuracy.

Remark: Throughout this section, we have dealt only with 1st order statistics. It would also be interesting to calculate higher order statistics and the pdf for the different models. Such statistics would be useful, for example, to provide performance guarantees. Due to lack of space, we intend to look further into this issue in future work. However, based on the analysis of this section, we reckon that the exponential/geometric distribution might be a good approximation, at least for the tail of the respective distribution, for most if not all of the hitting times calculated. Thus, higher order

statistics shouldn't be difficult to derive. The results of the next section provide fairly good evidence to that direction.

5. DELAY OF MOBILITY-ASSISTED ROUTING

In this section, we will look into how our results regarding the encounter times can be integrated with a general theoretical framework that can be used to analyze the performance of mobility-assisted routing. Our goal is to demonstrate that our meeting time results can be easily plugged into generic equations regarding the delay of different routing schemes, and derive performance results for a specific mobility model without resorting to simulations or curve fitting.

5.1 Upper and Lower Bounds on Delay

The meeting time for a given mobility model corresponds to the expected delay of one of the simplest mobility-assisted routing schemes, namely Direct Transmission. In Direct Transmission, the source of a message holds on to it until it comes within range of the destination itself. This scheme has the largest expected delay among all possible encounter-based schemes (that are non-adversarial). Consequently, the meeting times derived in the previous section for a given mobility model constitute an upper bound on the delay of any scheme under this model.

Additionally, the algorithm that minimizes the expected delivery delay is an "oracle-based" algorithm. It is aware of all future movement of nodes and computes the *optimal set of forwarding decisions* (i.e. time and next hop), which delivers a message to its destination in the minimum amount of time. Note also that, under the assumption of no contention (i.e. infinite buffer capacity and bandwidth), *epidemic routing* [38] will find the same paths as the oracle-based scheme, and thus achieve this minimum expected delay. The properties of this algorithm have been widely studied [29, 31, 33, 35, 42]. The following Lemma bounds the expected delay of any mobility-assisted routing scheme, under a given mobility model (proofs can be found in any of the above papers).

LEMMA 5.1. *Let M nodes move according to a given mobility model with exponentially distributed meeting times. Then, the expected message delivery time of any routing algorithm ED is*

$$\frac{H_{M-1}}{(M-1)} EM^{mm} \leq ED \leq EM^{mm} \tag{14}$$

where H_n is the nth Harmonic Number, i.e, $H_n = \sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$.

Lemma 5.1 implies that we can replace the values we calculated for the meeting times under different mobility models in Eq.(14) and derive closed-form expressions for the best-case and worst-case delays of any mobility-assisted scheme, under the given mobility model[6]. To the best of our knowledge, this has only been performed for the case of Random Walk mobility [35]. For other mobility models (e.g. Random Waypoint, etc.) no such closed form expressions exist. Instead, simulations are run for a specific set of network parameters to estimate EM^{mm} directly or by fitting the theoretical curves to the simulation ones.

[6]In the case of Community-based mobility with large communities Eq.(14) has to be slightly modified; However, the resulting expression is more convoluted without providing much additional insight, so we choose to omit it.

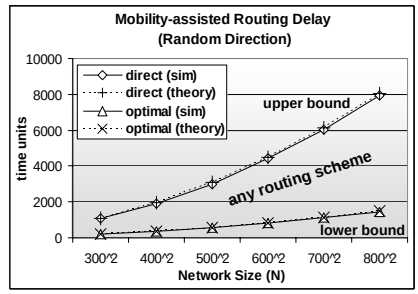

Figure 5: Upper and lower bounds on the delay of any mobility-assisted routing scheme under Random Direction mobility.

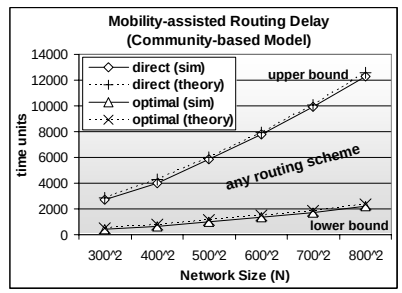

Figure 6: Upper and lower bounds on the delay of any mobility-assisted routing scheme under Community-Based mobility.

In Figure 5 we compare our analytical results, based on Lemma 5.1 and the expressions derived in Sections 3, 4, to simulation results, for the Random Direction model. Figure 6 does the same for the Community-based Random Direction model ($p_l = 0.8$, $p_{roam} = 0.5$, $\overline{T}_{stop} = 0$, $\overline{T}^l_{stop} = 150$) with small communities. As can be seen by both plots, our theoretical results for the optimal delay match very closely with simulations results. (We have observed similar good accuracy for the respective Random Waypoint results, so we do not include plots for it due to the limited space.) This implies not only that our meeting time expressions for different mobility models are accurate, but that *derivative delay expressions based on these meeting times, and pertaining to the delay of more complicated mobility-assisted routing schemes are also accurate.*

5.2 Delay of Other Routing Schemes

In this section, we're going to look into a couple of examples of mobility-assisted routing schemes, namely a single-copy (i.e. only one copy of each message is routed) and a multiple-copy one (i.e. more than one copies of the same message are routed in parallel). We first look into a multiple copy scheme, where the source of a message distributes only a fixed number of copies L, one to each of the first L distinct relays it encounters. Although a number of different names have been used for variants of this scheme, we'll refer to it here simply as the *L-copy scheme*. This scheme has been found to achieve delays comparable to the optimal scheme,

without the overhead of transmitting a copy to every node in the network. Different approximations and bounds for its delay have been calculated (e.g. [31, 33, 34]). Lemma 5.2 gives a recursive method, first proposed in [34] for Random Walk mobility, and adapted here for any mobility model, to calculate the delay of the L-copy scheme.

LEMMA 5.2. *Let ED_L^{mm} denote the expected delay of the L-copy scheme, under mobility model "MM". Let further $ED(i)$ denote the expected remaining delay for this scheme, after i message copies have been distributed. Then, ED_L^{mm} can be calculated by the following system of recursive equations, where $ED_L^{mm} \equiv ED(1)$:*

$$\begin{aligned} ED(i) &= \frac{EM_{mm}}{M-1} + \frac{M-i}{M-1} ED(i+1), i \in [1, L-1], \\ ED(L) &= \frac{EM_{mm}}{L}. \end{aligned} \tag{15}$$

Next, we analyze the expected delay for a scheme that uses only a single copy per message.

DEFINITION 5.1 (RANDOMIZED ROUTING). *In the Randomized routing algorithm, a node A hands over a message to another node B it encounters with probability $p > 0$.*

Randomized routing could be thought of as Direct Transmission, with the difference that a message may occasionally jump to a new relay, whenever such an appropriate relay is encounter. This and other similar single-copy schemes [35] that occasionally handover the message to a new relay based on some criteria can be analyzed using the methodology of Theorem 3.2. Due to lack of space we only present results for the delay of Randomized Routing under RD mobility and we omit the proof. Similar results can be derived for the rest of the mobility models, as well.

LEMMA 5.3. *Let M nodes perform Random Direction mobility, and let $\overline{v}_{rd}$ denote the* average *relative speed between two nodes. Let further a node A have a message to deliver to another node B. Then, the expected time ED_{rand} until the message is delivered to B using Randomized routing with probability p is given by:*

$$ED_{rand} = \frac{N}{2K\left[(1-pp_{tx})\overline{v}_{rd} + pp_{tx}\frac{2}{3}K\right]}, \tag{16}$$

where $p_{tx} = 1 - \left(1 - \frac{\pi K^2}{N}\right)^{M-1}$ is the probability that at least one node is within range of the current message relay at any time, and $\overline{v}_{rd} = p_m^2 \hat{v}_{rd}\overline{v} + 2p_m(1-p_m)\overline{v}$.

Figure 7 compares analytical and simulation results for the expected delay of Randomized routing with $p = 1$ (left plot) and the L-copy scheme (right plot), under Random Direction mobility with $T_{stop} = 0$.

6. CONCLUSIONS

In this paper, we have analyzed the encounter statistics for some commonly used mobility models. We have derived accurate closed form solutions for all the respective hitting and meeting times between different nodes. Additionally, we have proposed a mobility model that is rich enough to capture real-world mobility characteristics more accurately than many existing models, and have calculated various hitting time results for it, as well. Finally, we have demonstrated how these results can be used in a more general framework to analyze the delay of different mobility-assisted routing schemes, that is, schemes that require the node to carry a message for (potentially long) periods of time. Such schemes have been recently recognized to be very helpful in improving the performance of regular wireless networks or to enable data delivery in networks that are disconnected for the majority of time. We believe that this work can help in better understanding the particular advantages and shortcomings of various approaches in different settings, and can facilitate the design of new, improved protocols.

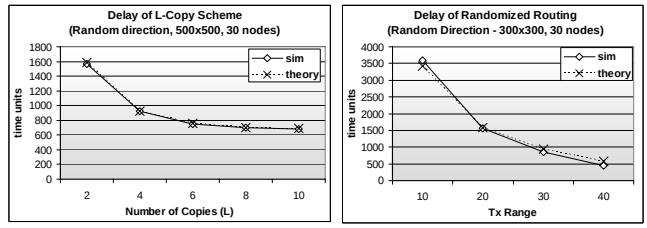

Figure 7: Simulation and analytical results for the expected delay of Randomized and L-copy routing schemes.

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APPENDIX

PROOF. (**Theorem 3.3**) Let a node A perform RWP movement, starting from its stationary distribution, and let a second node B be static with uniformly chosen position. Let us further look at a randomly chosen epoch with endpoints X_s and X_f, and let us denote as $X^{\perp}_{s-f}$ the point of line (X_s, X_f) that's closest to B. If we also denote the epoch with these endpoints as (X_s, X_f) then B is hit by epoch (X_s, X_f) if and only if $\|X_B - X^{\perp}_{s-f}\| \leq K$. The probability of this event occurring depends on $X_B = (x_B, y_B)$, because the area covered by epoch (X_s, X_f) is not uniformly distributed. We denote this probability as $P_{hit|B}$. $P_{hit|B}$ is higher when B is near the center and smaller when B is near the edges.

Let now S denote the set of all possible epochs, that is, $S = \{(X_s, X_f) : X_s, X_f \in U\}$. Let further $S_{hit|B}$ denote the set of all epochs that "hit" X, that is, $S_{hit|B} = \{(X_s, X_f) : \|X_B - X^{\perp}_{s-f}\| \leq K\}$. The size of a given set of epochs S_e is calculated by $\|S_e\| = \iint\limits_{S_e} 1 dX_s dX_f$. It is not difficult to see then, that $P_{hit|B}$ is given by:

$$P_{hit|B} = \frac{\|S_{hit|B}\|}{\|S\|} = \frac{\|S_{hit|B}\|}{N^2}.$$

Using a similar argument as in the proof of Theorem 3.1, we can use the statistics of a single epoch to calculate the expected number of epochs until A comes within range of B. Thus, if $N_{hit|B}$ is a random variable denoting the number of epochs until B is encountered, conditioned on B's position, then $N_{hit|B}$ is geometrically distributed with mean $EN_{hit|B} = \frac{N^2}{\|S_{hit|B}\|}$. Averaging over all possible locations $X_B = (x_B, y_B)$ (using linearity of expectations) we calculate that the unconditional expected number of epochs EN_{hit} is

$$\begin{aligned} EN_{hit} &= \iint\limits_U EN_{hit|B} P\{X_B = (x_B, y_B)\} dx_B dy_B \\ &= \iint\limits_U \frac{N^2}{\|S_{hit|B}\|} \frac{1}{N} dxdy = \iint\limits_U \frac{N}{\|S_{hit|B}\|} dxdy. \end{aligned}$$

Let us now define the set $S^*_{hit|B}$, which contains all *lines* in U with end-points drawn uniformly, whose X_B is a part. In other words $S^*_{hit|B} = \{(X_s, X_f) : \|X_B - X^{\perp}_{s-f}\| \leq \delta, \delta \to 0\}$. It is easy to see that, in discrete space, *for every line* $(X_s, X_f) \in S^*_{hit|B}$ *there are* $2K$ *other lines of same length and parallel to it in* $S^*_{hit|B}$ (ignoring boundary effects). Or, in continuous space, $\|S_{hit|B}\| \approx 2K\|S^*_{hit|B}\|$.

Now, a point $B = (x, y) \in U$ is covered by a percentage $\frac{\|S^*_{hit|B}\|}{N^2}$ of all lines. Further, each line covers, on average, another $\overline{L}$ points in addition to the given one, so $f(x, y) \approx \frac{\|S^*_{hit|B}\|}{N^2 \overline{L}}$. Consequently, $\|S_{hit|B}\| \approx \frac{2KN^2\overline{L}}{f(x,y)}$, and

the expected number of epochs EN_e until A encounters a static node B is

$$\frac{\iint\limits_U \frac{1}{f(x,y)} dxdy}{2KN\overline{L}}$$

Finally, the expected duration of each of these epochs is given in assumption (d). □

PROOF. (**Corollary 3.1**) As is shown in [6] Eq.(3) is quite a good approximation of the stationary pdf $f(x,y)$ away from the network edges. However, we cannot simply replace this value in the hitting time equation and calculate its value, because the integral $\iint\limits_U \frac{1}{f(x,y)} dxdy$ would not converge. The reason for this is that $f(x,y) \to 0$ at the network edges, implying that if the destination is very near the edge, it would take an infinite number of epochs to find it. However, this is not the case in reality, since nodes have a non-zero transmission range K. Hence, we can assume that the value of $\frac{1}{f(x,y)}$ everywhere inside a strip of width K from the network edge is constant and equal to its value K distance far from the edge. Referring to the proof of Theorem 3.3 this simply implies that the probability of a given epoch hitting a point is constant inside that strip (this assumption becomes increasingly valid as K increases).

Based on this observation, we can now brake the integral as follows:

$$\iint\limits_U \frac{1}{f(x,y)} dxdy = \frac{36}{N^3}(I_1^2 + 4I_1I_2 + 4I_2^2),$$

where $I_1 = \int_{-\sqrt{N}/2+K}^{\sqrt{N}/2-K} \left(x^2 - N/4\right)^{-1} dx$, and $I_2 = \int_{\sqrt{N}/2-K}^{\sqrt{N}/2} [(\sqrt{N}/2 - K)^2 - N/4]^{-1} dx$.

Using some calculus we can easily calculate the values of I_1 and I_2 as

$$I_1 = \frac{(4\tanh^{-1}(1 - \frac{2K}{\sqrt{N}}))^2}{\sqrt{N}}, \text{ and } I_2 = \frac{1}{K - \sqrt{N}}.$$

Replacing these values in the above integral equation and performing some calculations gives us $g(N,K)$. □

PROOF. (**Theorem 3.4**) When the destination, let B, is also moving according to the Random Waypoint model, the position X_B of the destination at any time is distributed according to $f(x,y)$, instead of uniformly (f(x,y) again denotes the stationary node distribution for RWP). Let us then first calculate the expected hitting time until a static node, distributed according to $f(x,y)$ is encountered by A. Let's also assume initially that the pause time at every epoch is 0 (i.e. $\overline{T}_{stop} = 0$.). Finally, let $N_{hit|B}$ be again the random variable denoting the number of epochs until B is encountered, conditioned on B's position, and N_{hit} be the respective unconditional variable. We can now change the steps of the proof of Theorem 3.3 accordingly to calculate the expected number of epochs EN_{hit} until B is found as follows: $EN_{hit} = \iint\limits_U N_{hit|B} P\{X_B(x,y)\} dxdy \Rightarrow EN_{hit} = \iint\limits_U \frac{N^2}{\|S_{hit|B}\|} f(x,y) dxdy \Rightarrow EN_{hit} = \iint\limits_U \frac{1}{2K\overline{L} f(x,y)} f(x,y) dxdy \Rightarrow EN_{hit} = \frac{N}{2K\overline{L}}$, since $S_{hit|B} = \frac{2K\overline{L}}{N^2} f(x,y)$. ($EN_{hit}$ is smaller here than when B's position was uniformly distributed.) Taking into account that the expected epoch duration is given by $\frac{\overline{L}}{\overline{u}}$ we get that the expected hitting time until a static node, distributed according to stationary RWP pdf, is encountered is $\frac{N}{2K\overline{v}}$.

Now, if A also pauses at the end of every epoch (i.e. $T_{stop} > 0$), we calculate the expected hitting time using the alternative method of Theorem 3.2, and find it equal to $ET'_{rwp} = \frac{N}{2Kp_m\overline{v}}$. The rest of the proof is the same as that of Theorem 3.2, only replacing the normalized relative speed $\hat{v}_{rwp}$ for $\hat{v}_{rd}$. However, note that, to calculate $\hat{v}_{rwp}$, we need to integrate over the angle distribution for the velocities of *both* A ($\vec{v_A}$) *and* B ($\vec{v_B}$) from Lemma 3.1. Unlike the Random Direction case, this needs to be done numerically. For the case of $v_A = v_B = \overline{v}$ it is $\hat{v}_{rwp} \approx 1.754$. □

PROOF. (**Theorem 4.2**) The proof is similar to that of Theorem 3.2. We only need to recalculate the respective probability of a node moving for the community model, p_m^c, at any time. Each local epoch last on average $\overline{T}_l = \frac{\overline{L}_c}{\overline{v}} + \overline{T}^l_{stop}$ time units, and each roaming epoch $\overline{T}_r = \frac{\overline{L}}{\overline{v}} + \overline{T}_{stop}$ time units. Furthermore, it is easy to see (as in all previous cases) that the community-based mobility process is also ergodic. Consequently, the probability p_m^c is equal to the percentage of time a node is in a moving state in an infinite evolution of the mobility process. Let then N_r and N_l denote the total number of roaming and local epochs having occurred in the above evolution, and $N_{hit} = N_r + N_l$ denote the total number of epochs till the destination is found. In this case

$$p_m^c = \lim_{N_{hit}\to\infty} \frac{N_l \frac{\overline{L}_c}{\overline{v}} + N_r \frac{\overline{L}}{\overline{v}}}{N_l\overline{T}_l + N_r\overline{T}_r}.$$

We know also that $N_r \to \pi_r N_{hit}$ and $N_l \to \pi_l N_{hit}$ as $N_{hit} \to \infty$ (law of large numbers). Hence,

$$p_m^c = \frac{\pi_l \frac{\overline{L}_c}{\overline{v}} + \pi_r \frac{\overline{L}}{\overline{v}}}{\pi_l\overline{T}_l + \pi_r\overline{T}_r} = \frac{(1-p_r)\frac{\overline{L}_c}{\overline{v}} + (1-p_l)\frac{\overline{L}}{\overline{v}}}{(1-p_r)\overline{T}_l + (1-p_l)\overline{T}_r}$$

□

PROOF. (**Theorem 4.3**) Let us denote as $EN^{(0)}$ the expected number of epochs till hitting time, when node A moves constantly (i.e. pause time in any phase is 0). Using a similar argument as in Lemma 3.4, when node A pauses also between epochs, then the resulting hitting time becomes $ET = \frac{EN^{(0)}}{p_m^{(A)}\overline{v}}$, since $p_m^{(A)}\overline{v}$ is the expected area covered within a single time unit.

When node B is also mobile we need to take into account the fact that either both nodes are mobile (probability $p_m^A p_m^B$) and move with $\hat{v}_{rd}\overline{v}$ relative speed (see Theorem 3.2), or only one of the two nodes is moving (probability $p_m^A(1-p_m^B)+p_m^B(1-p_m^A)$) with relative speed $\overline{v}$, or both nodes stand (probability $(1-p_m^A)(1-p_m^B)$ with relative speed 0. Consequently, the expected meeting time EM will be given by

$$\begin{aligned} EM &= \frac{EN^{(0)}}{p_m^A p_m^B \overline{v}\hat{v}_{rd} + [p_m^A(1-p_m^B) + p_m^B(1-p_m^A)]\overline{v}} \\ &= \frac{ET}{p_m^{(B)}\hat{v}_{rd} + (1-p_m^{(B)}) + \frac{p_m^{(B)}}{p_m^{(A)}}(1-p_m^{(A)})}. \end{aligned}$$

The procedure is entirely symmetrical, so we could use the hitting time of B to A, instead, replacing the appropriate quantities in the above equations. □