EE 457 Unit 4a

Fixed Point Systems and Arithmetic

Unsigned
2's Complement
Sign and Zero Extension
Hexadecimal Representation

SIGNED AND UNSIGNED SYSTEMS

Signed Systems

- Several systems have been used
  - 2's complement system
  - 1's complement system
  - Sign and magnitude

Unsigned and Signed Variables

- Unsigned variables use unsigned binary (normal power-of-2 place values) to represent numbers

1	0	0	1	0	0	1	1
128	64	32	16	8	4	2	1

- Signed variables use the _____ system (_____ weight) to represent numbers

1	0	0	1	0	0	1	1

2's Complement System

- MSB has negative weight
- MSB determines sign of the number
  - 1 = negative
  - 0 = positive
- To take the negative of a number (e.g. -7 => +7 or +2 => -2), requires _____ _____
  - This is accomplished by _____

1001	x = -7
0110	Bit flip (1's comp.)
+ 1	Add 1
0111	-x = -(-7) = +7

Zero and Sign Extension

- Extension is the process of increasing the number of bits used to represent a number without changing its value

Unsigned = Zero Extension (Always add leading 0's):

111011 = 00111011

Increase a 6-bit number to 8-bit number by zero extending

2's complement = Sign Extension (Replicate sign bit):

pos. 011010 = 00011010

neg. 110011 = 11110011

Sign bit is just repeated as many times as necessary

Zero and Sign Truncation

- Truncation is the process of decreasing the number of bits used to represent a number without changing its value

Unsigned = Zero Truncation (Remove leading 0's):

~~00~~111011 = 111011

Decrease an 8-bit number to 6-bit number by truncating 0's. Can't remove a '1' because value is changed

2's complement = Sign Truncation (Remove copies of sign bit):

pos. ~~000~~11010 = 011010

neg. ~~111~~10011 = 10011

Any copies of the MSB can be removed without changing the numbers value. Be careful not to change the sign by cutting off ALL the sign bits.

Arithmetic & Sign

- You learned the addition (carry-method) and subtraction (borrow-method) algorithms in grade school
- Consider A + B…do you definitely use the addition algorithm?
  - What if A=(2), B=(-5)?
- Human add/sub algorithm depends on _____!!

Unsigned and Signed Arithmetic

- Addition/subtraction process is the same for both unsigned and signed numbers
  - Add columns right to left
  - Drop any final carry out
- This is the KEY reason we use 2's complement system to represent signed numbers
- Examples:

```
   1 1     If unsigned  If signed
  1001
+ 0011
  1100
```


Unsigned and Signed Subtraction

- Subtraction process is the same for both unsigned and signed numbers
  - Convert A – B to A + Comp. of B
  - Drop any final carry out
- Examples:

```
        If unsigned  If signed
  1100    (12)       (–4)
– 0010     (2)        (2)
```

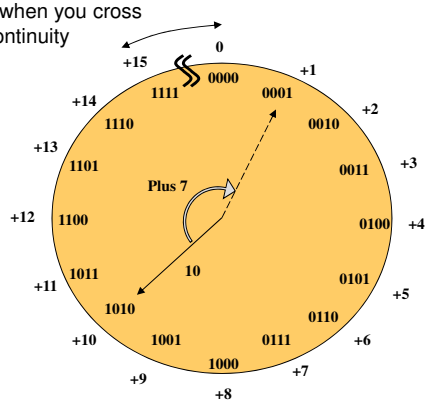
If unsigned If signed

Overflow

- Overflow occurs when the result of an arithmetic operation is too _____ _____ with the given number of bits
  - Unsigned overflow (___) occurs when adding or subtracting unsigned numbers
  - Signed (2's complement overflow) overflow (___) occurs when adding or subtracting 2's complement numbers

Unsigned Overflow

Overflow occurs when you cross this discontinuity



10 + 7 = 17

4 - 6 = 14

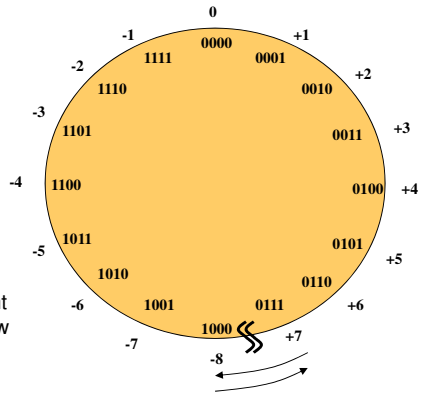
With 4-bit *unsigned* numbers we can only represent 0 – 15. Thus, we say overflow has occurred.

2's Complement Overflow

5 + 7 = +12

-6 + -4 = -10

With 4-bit *2's complement* numbers we can only represent -8 to +7. Thus, we say overflow has occurred.



Overflow occurs when you cross this discontinuity

Testing for Overflow

- Most fundamental test
  - Check if answer is wrong (i.e. Positive + Positive yields a negative)
- Unsigned overflow (**C**) test
  - If _____
- Signed (2's complement) overflow (**V**) test
  - Only occurs if
    - _____ … or…
    - _____
  - Alternate test: See following slides

Alternate Signed Overflow Test

A & B	A3	B3	S3	C3	C4	V
Both Positive	0	0	0			
			1			
One Positive & One Negative	0	1	0			
			1			
	1	0	0			
			1			
Both Negative	1	1	0			
			1			

- Check if _____

Overflow in Addition

- Overflow occurs when the result of the addition cannot be represented with the given number of bits.
- Tests for overflow:
  - Unsigned: if Cout = 1
  - Signed: if p + p = n or n + n = p

	If unsigned	If signed
1101		
+ 0100		

	If unsigned	If signed
0110		
+ 0101		

Overflow in Subtraction

- Overflow occurs when the result of the subtraction cannot be represented with the given number of bits.
- Tests for overflow:
  - Unsigned: if Cout = 0
  - Signed: if addition is p + p = n or n + n = p

If unsigned If signed

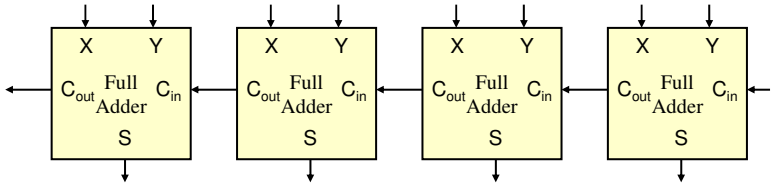
```
  0111
- 1000
```

If unsigned If signed

Addition – Full Adders

- Show the following addition operation

```
  0110 = X
+ 0111 = Y
```

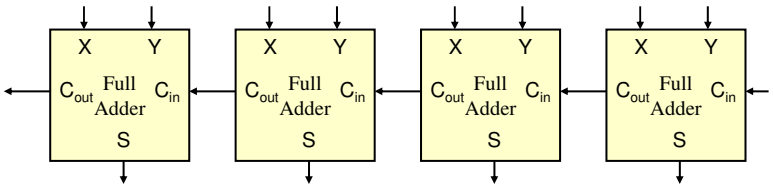


Performing Subtraction w/ Adders

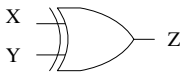
- To subtract
  - Flip bits of Y
  - Add 1

```
  0101 = X
- 0011 = Y
  1101
```

```
  0101
+ 1100
     1
  0010
```



XOR Gate Review



XOR

$Z = X \oplus Y$

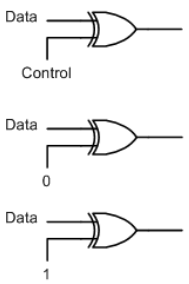
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

True if an odd # of inputs are true
2 input case: True if inputs are different

XOR Conditional Inverter

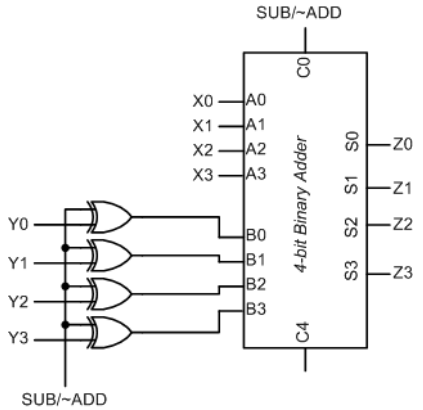
- If one input (say, X) to an XOR gate is 0, then Z=_____
- If one input (say, X) to an XOR gate is 1, then Z=_____
- Use one input as a control input which can conditionally pass or invert the other input

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0



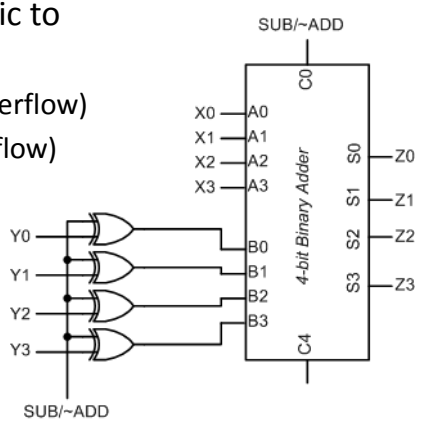
Adder/Subtractor

- Using XOR gates before one set of adder inputs we can
  - Selectively pass or invert Y
  - Add an extra '1' via the Carry-in
- If SUB/~ADD=0,
  - Z = X+Y
- If SUB/~ADD=1,
  - Z = X-Y



Adder/Subtractor

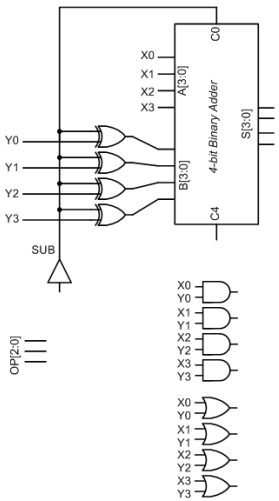
- Exercise: Add appropriate logic to produce
  - **C** (unsigned overflow)
  - **V** (signed overflow) flags

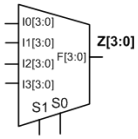


ALU Design

Complete the ALU design given the function table below

OP[2:0]	Z
000	X+Y
001	X-Y
011	SLT: Z=1, if X<Y Z=0, other
100	AND
110	OR
Others	Z = und.





NON-REQUIRED MATERIAL

Hexadecimal Representation

- Since values in modern computers are many bits, we use hexadecimal as a shorthand notation (4 bits = 1 hex digit)
  - 11010010 = D2 hex
  - 0111011011001011 = 76CB hex
- To interpret the value of a hex number, you must know what underlying binary system is assumed (unsigned, 2's comp. etc.)

Translating Hexadecimal

- Hex place values (16^2, 16^1, 16^0) can ONLY be used if the number is positive.
- If hex represents unsigned binary simply apply hex place values
  - B2 hex = $11*16^1 + 2*16^0 = 178_{10}$
- If hex represents signed value (2's comp.)
  - First determine the sign to be pos. or neg.
    - Convert the MS-hex digit to binary to determine the MSB (e.g. for B2 hex, B=1011 so since the MSB=1, B2 is neg.)
    - In general, hex values starting 0-7 = pos. / 8-F = neg.
  - If pos., apply hex place values (as if it were unsigned)
  - If neg., take the **16's complement** and apply hex place values to find the neg. number's magnitude

Taking the 16's Complement

- Taking the 2's complement of a binary number yields its negative and is accomplished by finding the 1's complement (bit flip) and adding 1
- Taking the 16's complement of a hex number yields its negative and is accomplished by finding the 15's complement and adding 1
  - 15's complement is found by subtracting each digit of the hex number from F_{16}

Original value B2:	FF	
	- B2	Subtract each digit from F
	4D	15's comp. of B2
	+ 1	Add 1
16's comp. of B2:	4E	16's comp. of B2

Translating Hexadecimal

- Given 6C hex
  - If it is unsigned, apply hex place values
    - 6C hex = $6*16^1 + 12*16^0 = 108_{10}$
  - If it is signed…
    - Determine the sign by looking at MSD
      - 0-7 hex has a 0 in the MSB [i.e. positive]
      - 8-F hex has a 1 in the MSB [i.e. negative]
      - Thus, 6C (start with 6 which has a 0 in the MSB is positive)
    - Since it is positive, apply hex place values
      - 6C hex = $6*16^1 + 12*16^0 = 108_{10}$

Translating Hexadecimal

- Given FE hex
  - If it is unsigned, apply hex place values
    - FE hex = $15*16^1 + 14*16^0 = 254_{10}$
  - If it is signed…
    - Determine sign => Negative
    - Since it is negative, take 16's complement and then apply place values
      - 16's complement of FE = 01 + 1 = 02 and apply place values = 2
      - Add in sign => -2 = FE hex

Finding the Value of Hex Numbers

- B2 hex representing a signed (2's comp.) value
  - Step 1: Determine the sign: Neg.
  - Step 2: Take the 16's comp. to find magnitude

    FF - B2 + 1 = 4E hex
  - Step 3: Apply hex place values ($4E_{16} = +78_{10}$)
  - Step 4: Final value: B2 hex = -78_{10}
- 7C hex representing a signed (2's comp.) value
  - Step 1: Determine the sign: Pos.
  - Step 2: Apply hex place values ($7C_{16} = +124_{10}$)
- 82 hex representing an unsigned value
  - Step 1: Apply hex place values ($82_{16} = +130_{10}$)

Hex Addition and Overflow

- Same rules as in binary
  - Add left to right
  - Drop any carry (carry occurs when sum > F_{16})
- Same addition overflow rules
  - Unsigned: Check if final Cout = 1
  - Signed: Check signs of inputs and result

```
  1  1
  7AC5
+ C18A
  3C4F
```

If unsigned: Overflow, Cout = 1 — If signed: No Overflow, p + n

```
  0 1  1
  6C12
+ 549F
  C0B1
```

If unsigned: No Overflow, Cout = 0 — If signed: Overflow, p + p = n

Hex Subtraction and Overflow

- Same rules as in binary
  - Convert A – B to A + Comp. of B
  - Drop any final carry out
- Same subtraction overflow rules
  - Unsigned: Check if final Cout = 0
  - Signed: Check signs of addition inputs and result

```
  B1ED         1
- 76FE   =>    B1ED
               8901
             +    1
               3AEF
```

If unsigned
No Overflow
Cout = 1

If signed
Overflow
n + n = p

```
  0001         0
- 0002   =>    0001
               FFFD
             +    1
               FFFF
```

If unsigned
Overflow
Cout = 0

If signed
No Overflow
p + n

Credits

- These slides were derived from Gandhi Puvvada's EE 457 Class Notes