CSCI 104
Recursive Graph & Tree Traversals Algorithms

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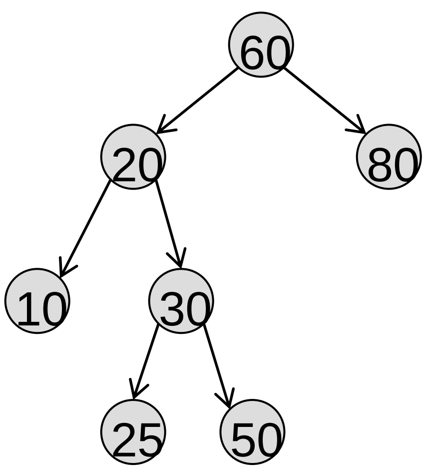
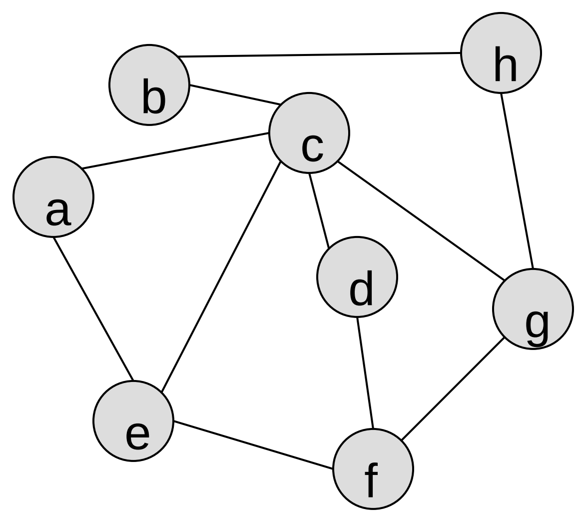
Traversal Algorithms

- Traversals should visit (and potentially apply some operation or processing to) each node once

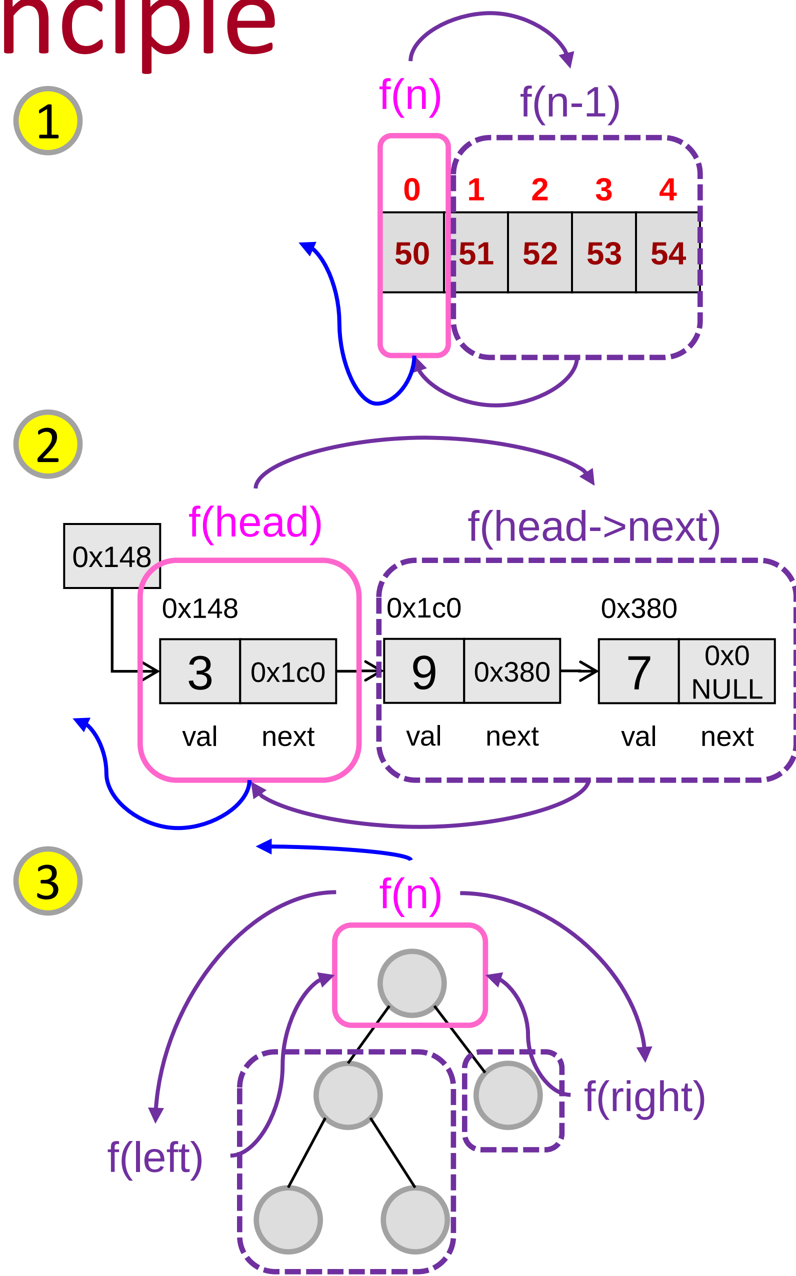




RECURSIVE TREE TRAVERSALS

Guiding Recursive Principle

- A useful principle when trying to develop recursive solutions is that the recursive code should handle only 1 element, which might be:
    1. An element in an array
    2. A node a linked list
    3. A node in a tree
    4. One choice in a sequence of choices
- Then use recursion to handle the remaining elements
- And finally combine the solution(s) to the recursive call(s) with the one element being handled



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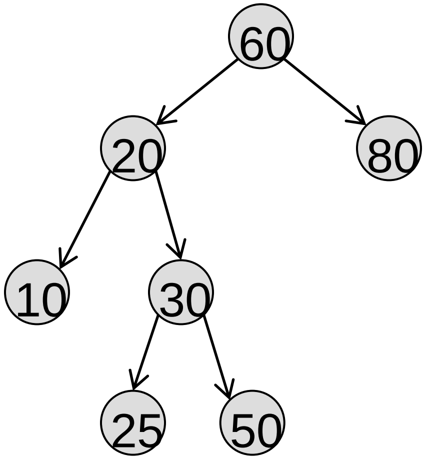
Recursive Tree Traversals

- A traversal iterates over all nodes of the tree
  - Usually using a depth-first, recursive approach
- Three general traversal orderings
  - Pre-order [Process root then visit subtrees]
  - In-order [Visit left subtree, process root, visit right subtree]
  - Post-order [Visit left subtree, visit right subtree, process root]

```
// Node definition
struct TNode
{
  int val;
  TNode *left, *right;
};
```




```
Preorder(TNode* t)
{  if t == NULL return
   process(t) // print val.
   Preorder(t->left)
   Preorder(t->right)
}
```

60 20 10 30 25 50 80

```
Inorder(TNode* t)
{  if t == NULL return
   Inorder(t->left)
   process(t) // print val.
   Inorder(t->right)
}
```

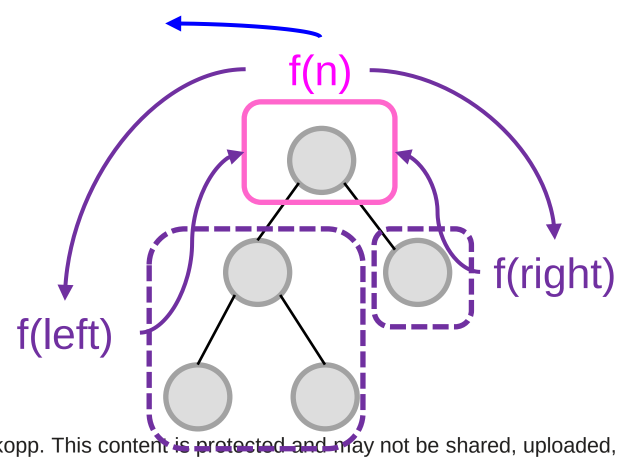
10 20 25 30 50 60 80

```
Postorder(TNode* t)
{  if t == NULL return
   Postorder(t->left)
   Postorder(t->right)
   process(t) // print val.
}
```

10 25 50 30 20 80 60

Example 1: Count Nodes

- Write a recursive function to **count how many nodes** are in the binary tree
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node
  - For in- or post-order traversals, determine how to use/combine results from recursion on children




```
// Node definition
struct Tnode {
  int val;
  TNode *left, *right;
};

int count(TNode* root)
{
  if( root == NULL ) _______________;
  else {



} }
```

Example 2: Prefix Sums

- Write a recursive function to **have each node store the sum of the values on the path from the root to each node.**
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node

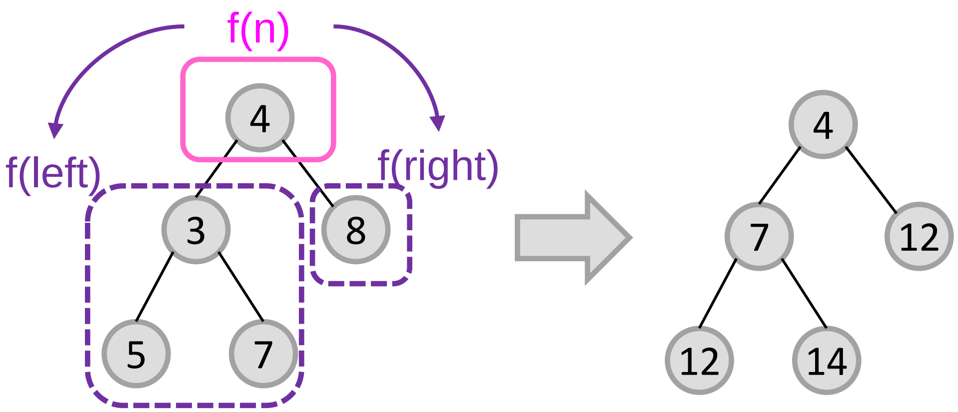
```
void prefixH(TNode* root, int psum)

void prefix(TNode* root)
{
  prefixH(root, 0);
}

void prefixH(TNode* root, int psum)
{
  if( root == NULL ) ________________;
  else {



  }
}
```

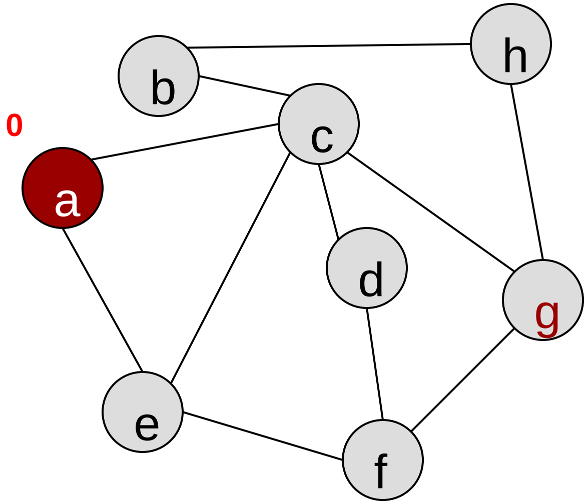


GENERAL GRAPH TRAVERSALS

BREADTH-FIRST SEARCH

Breadth-First Search

- Given a graph with vertices, V, and edges, E, and a starting vertex that we'll call u
- BFS starts at u ('a' in the diagram to the left) and fans-out along the edges to nearest neighbors, then to their neighbors and so on
- Goal: Find shortest paths (a.k.a. minimum number of hops or depth) from the start vertex to every other vertex



Depth 0: a

Breadth-First Search

- Given a graph with vertices, V, and edges, E, and a starting vertex, u
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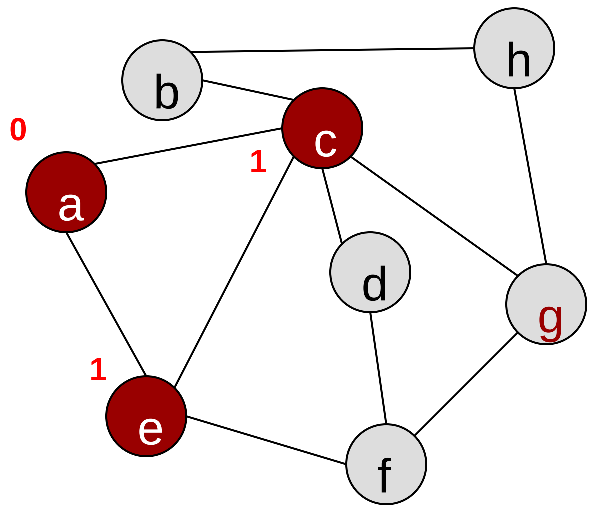

Depth 0: a
Depth 1: c,e

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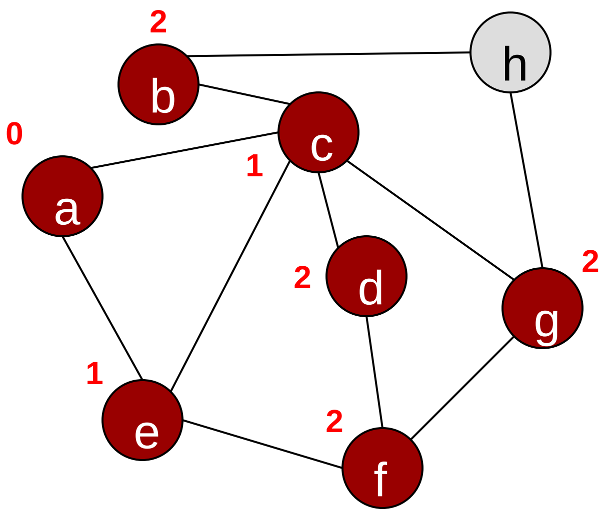

Depth 0: a
Depth 1: c,e
Depth 2: b,d,f,g

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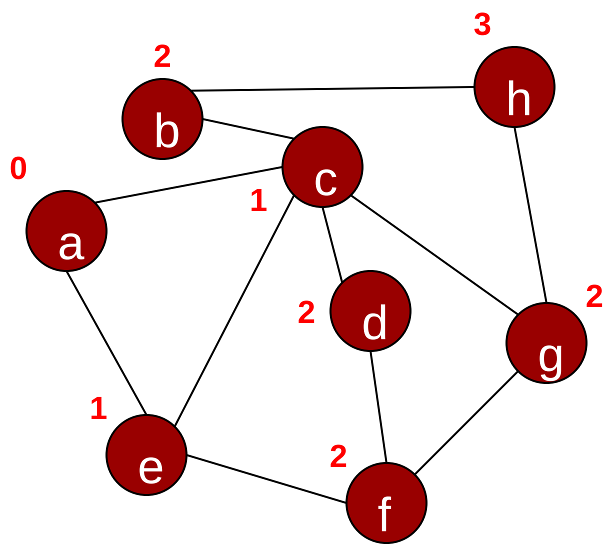


Depth 0: a
Depth 1: c,e
Depth 2: b,d,f,g
Depth 3: h

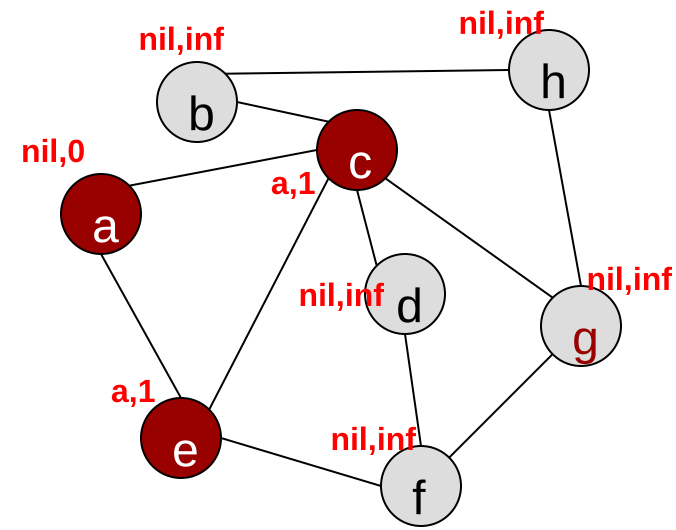
Developing the Algorithm

- Exploring all vertices in the order they are found implies we will explore vertices in First-In/First-Out order which implies use of a **Queue**
  - Important: BFS implies use of a **queue**
  - Put newly found vertices in the back and pull out a vertex from the front to explore next
- We don't want to put a vertex in the queue more than once…
  - "mark" a vertex the first time we encounter it (only allowing unmarked vertices to be put in the queue)
  - We can "mark" a vertex by **adding them to a set** OR by simply **setting some data member that indicates we've seen this vertex before**
- May also keep a "predecessor" structure or value per vertex that indicates which prior vertex found this vertex
  - Allows us to find a shortest-path back to the start vertex (i.e. retrace our steps)

Breadth-First Search

Algorithm:

```
BFS(G,u)
for each vertex v
     pred[v] = nil, d[v] = inf.
Q = new Queue
Q.enqueue(u),  d[u]=0
while Q is not empty
   v = Q.front(); Q.dequeue()
   foreach neighbor, w, of v:
       if pred[w] == nil // w not found
           Q.enqueue(w)
           pred[w] = v,  d[w] = d[v] + 1
```



v = a

Q: e c

Breadth-First Search

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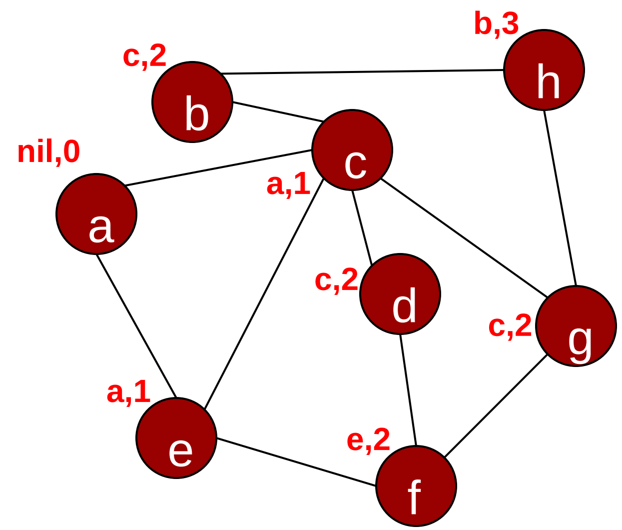
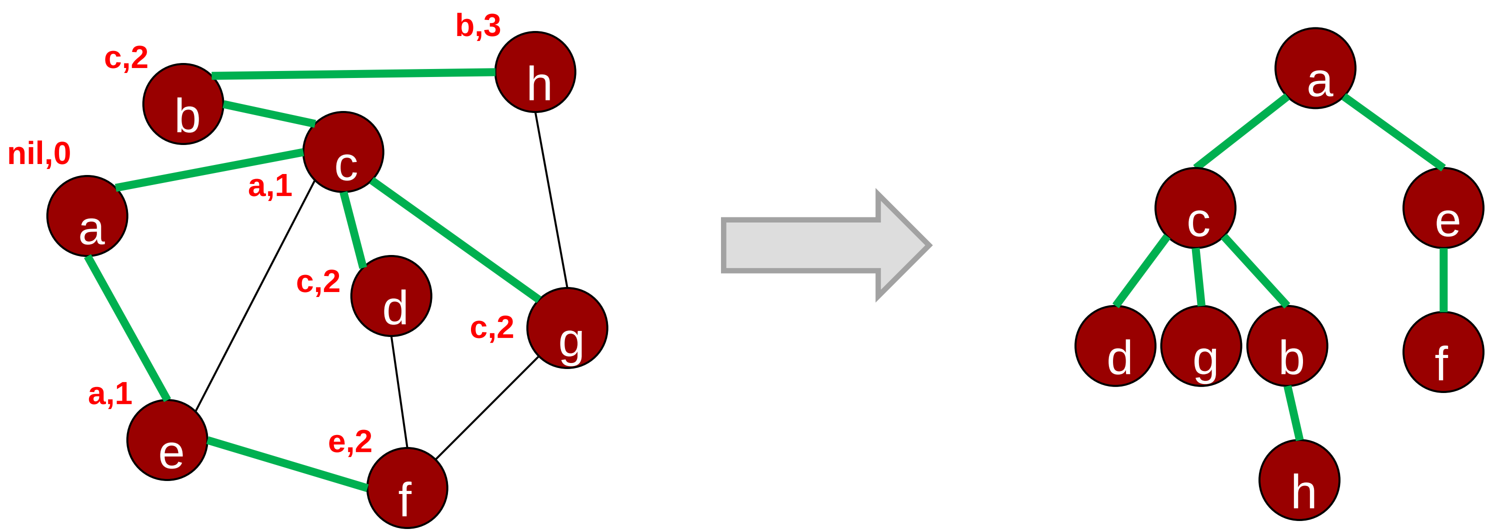


v = h

Q:

Breadth-First Search Trees

- BFS visits each node once and will induce a tree subgraph (as will DFS) from the original graph
  - BFS is tree of shortest paths from the source to all other vertices (in connected component)



Original graph, G

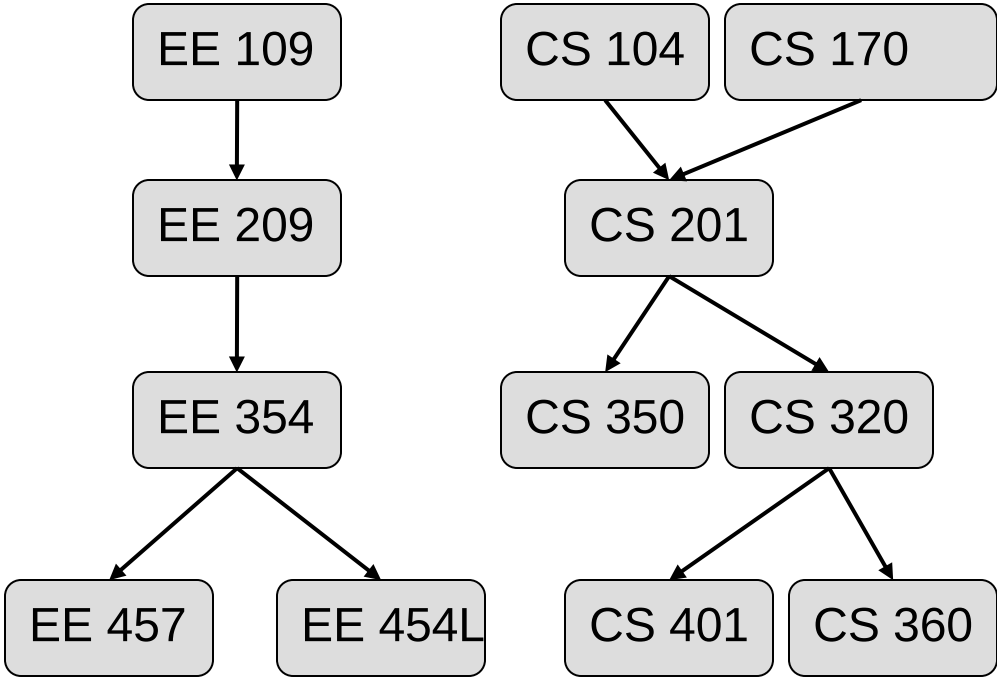
BFS Induced Tree

Topological Search

DEPTH FIRST SEARCH MOTIVATING EXAMPLE

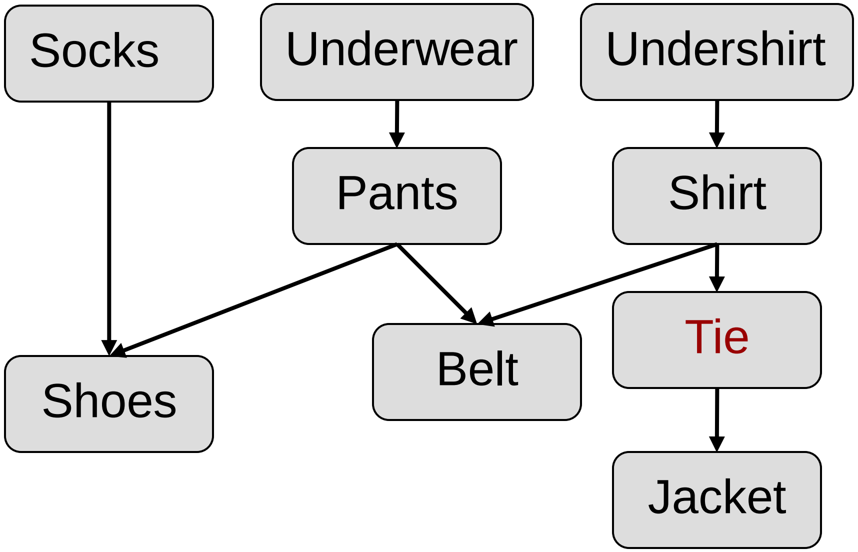
DFS Application: Topological Sort

- Breadth-first search doesn't solve all our problems.
- Given a graph of dependencies (tasks, prerequisities, etc.) **topological** sort creates a consistent ordering of tasks (vertices) where no dependencies are violated
- Many possible valid topological orderings exist
  - EE 109, EE 209, EE 354, EE 454, EE 457, CS104, PHYS 152, CS 201,…
  - CS 104, EE 109, CS 170, EE 209,…



Topological Sort

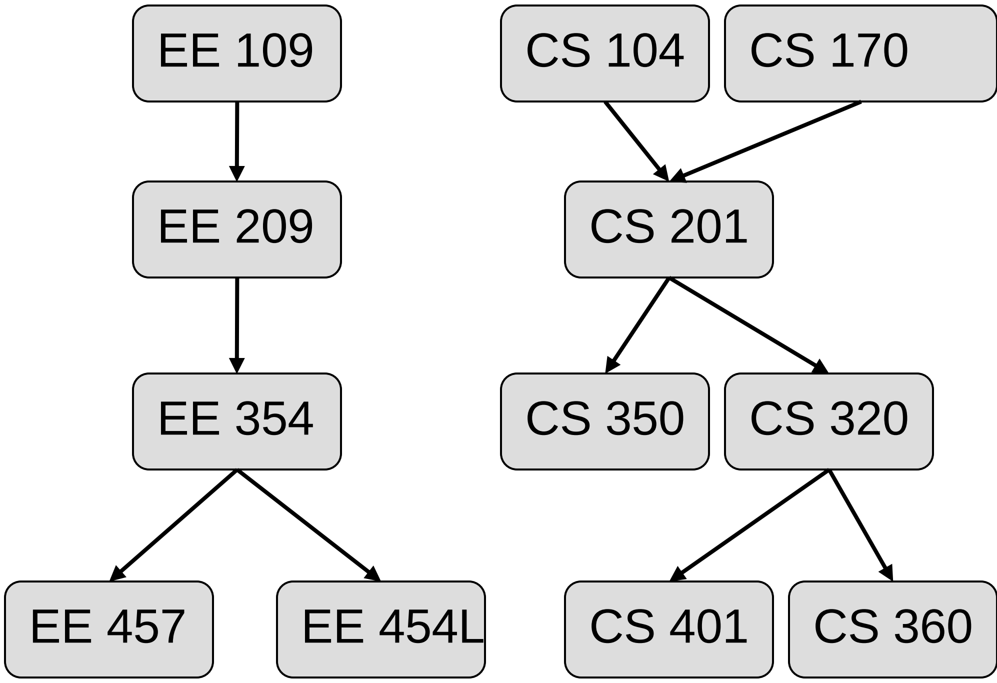
- Another example
  - Getting dressed
- More Examples:
  - Project management scheduling
  - Build order in a Makefile or other compile project
  - Cooking using a recipe
  - Instruction execution on an out-of-order pipelined CPU
  - Production of output values in a simulation of a combinational gate network



http://www.personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/GraphAlgor/topoSort.htm

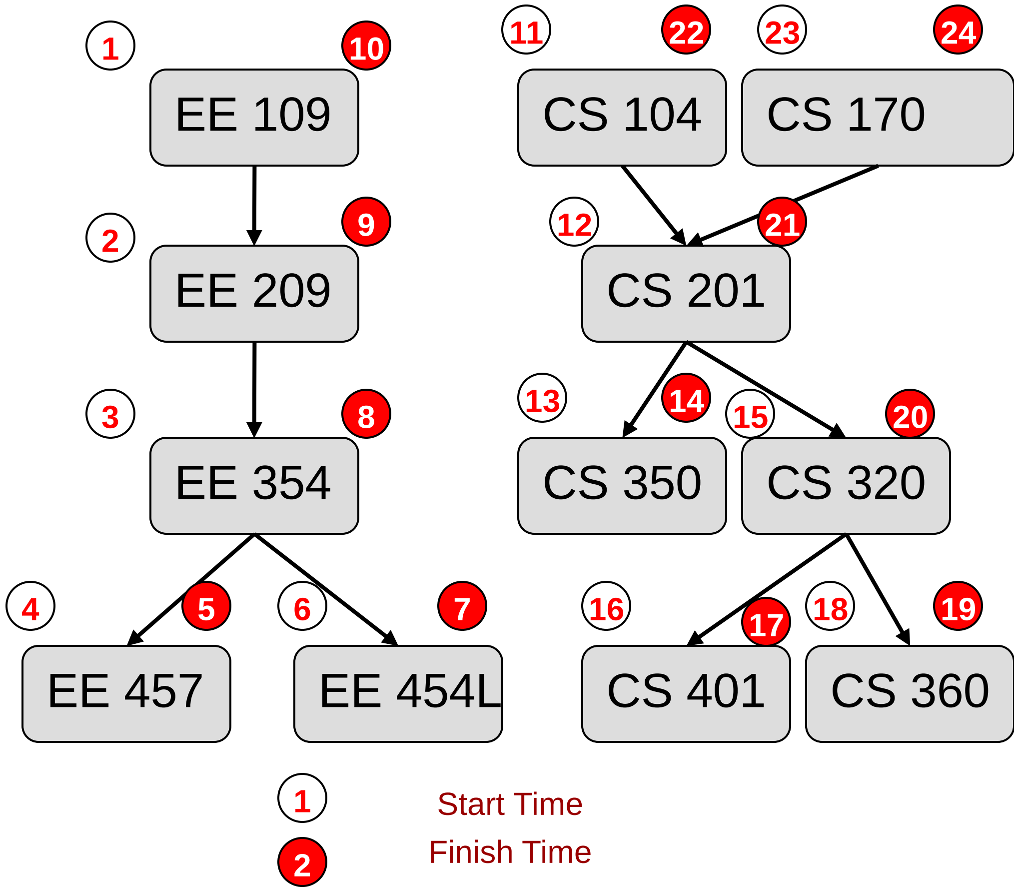
Topological Sort

- Does breadth-first search work?
  - No. What if we started at CS 170…
  - We'd go to CS 201L before CS 104
- All parent nodes need to be completed before any child node
- BFS only guarantees *some* parent has completed before child
- Turns out a Depth-First Search will be part of our solution



Depth First Search

- Explores ALL children before completing a parent
  - Note: BFS completes a parent before ANY children
- For DFS let us assign:
  - A start time when the node is first found
  - A finish time when a node is completed
- If we look at our nodes in reverse order of finish time (i.e. last one to finish back to first one to finish) we arrive at a…
  - Topological ordering!!!



Reverse Finish Time Order

CS 170, CS 104, CS 201, CS 320, CS 360, CS 477, CS 350, EE 109, EE 209L, EE 354, EE 454L, EE 457

DFS Algorithm

- DFS visits and completes all children before completing (and going on to a sibling)
- Process:
  - Visit a node
  - Mark as visited (started)
  - For each visited neighbor, visit it and perform DFS on all of their children
  - Only then, mark as finished
- Let's trace recursive DFS!!
- If cycles in the graph, ensure we don't get caught visiting neighbors endlessly
  - Use some status (textbooks use "colors" but really just some integer)
  - White = unvisited,
  - Gray = visited but not finished
  - Black = finished

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
     DFS-Visit (G, u, finish_list)
return finish_list
```

```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
       DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```

Source: "Introduction to Algorithms", Cormer, Leiserson, Rivest

Depth First-Search

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```
DFS-Visit (G, u, l)
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    u.color = BLACK
    l.append(u)
```

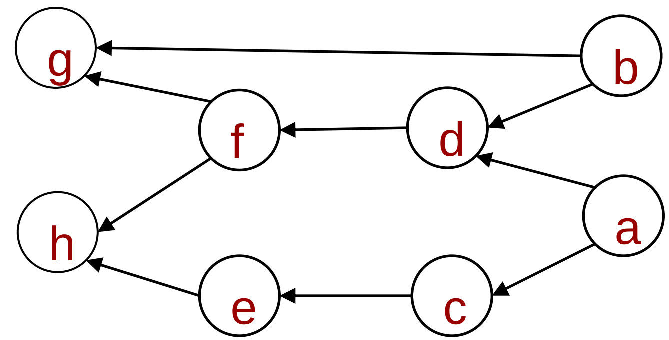

Depth First-Search

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```
DFS-Visit (G, u,l)
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  u.color = BLACK
  l.append(u)
```

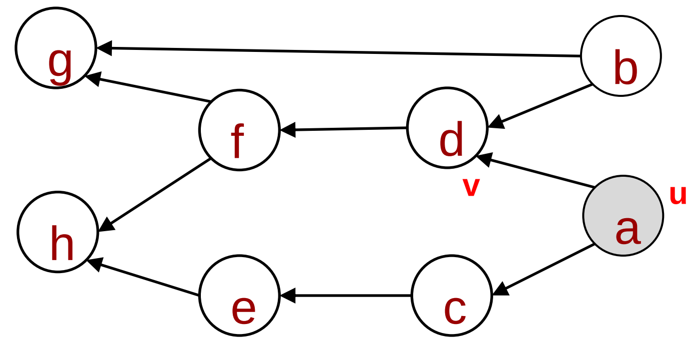

DFS-Visit(G,a,l):

Depth First-Search

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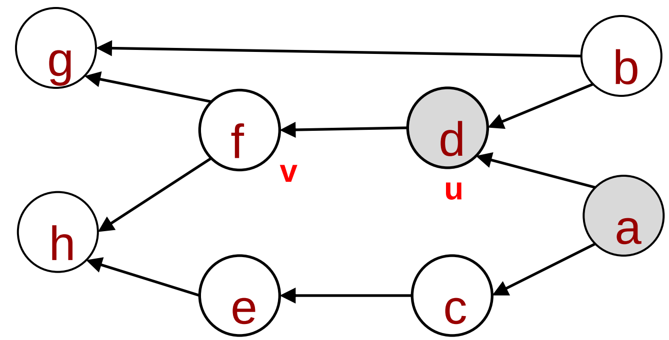

DFS-Visit(G,d,l):
DFS-Visit(G,a,l):

Depth First-Search

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DFS-Visit (G, u, l)
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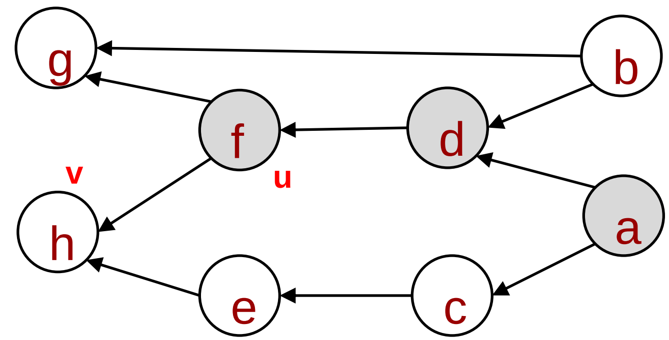
```



DFS-Visit(G,f,l):

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Depth First-Search

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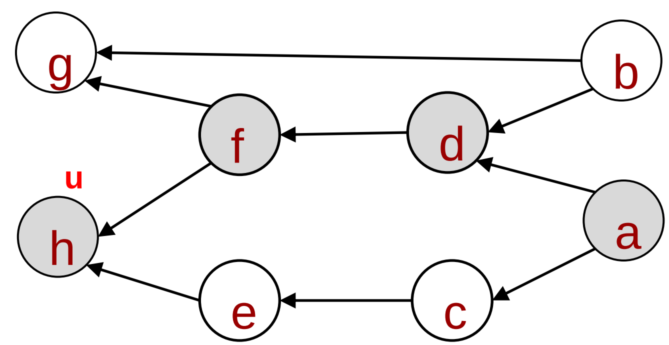

DFS-Visit(G,h,l):
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Depth First-Search

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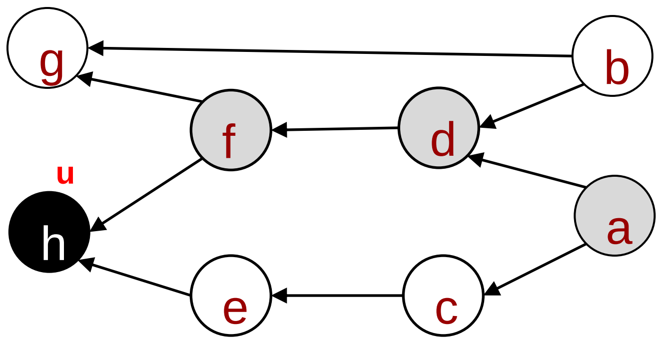

Finish_list:

h

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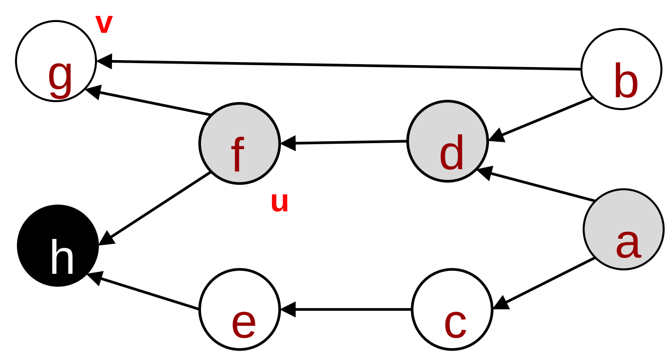

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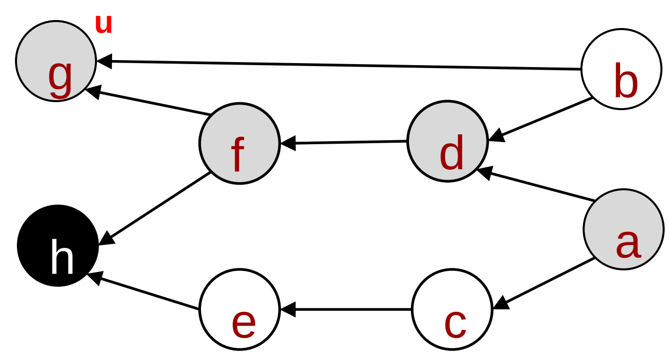

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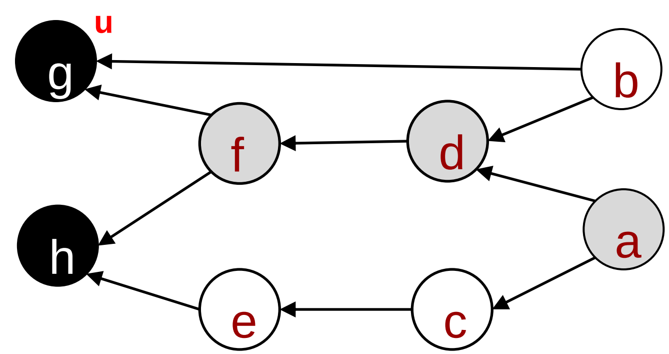

Finish_list:

h,
g

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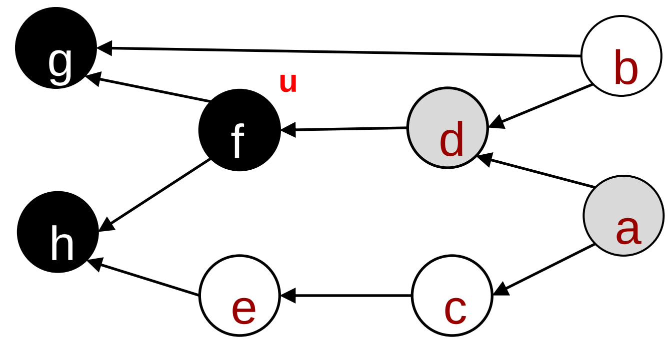

Finish_list:

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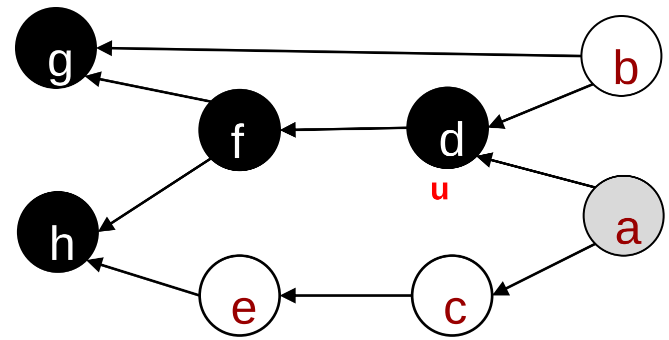

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g,
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  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```
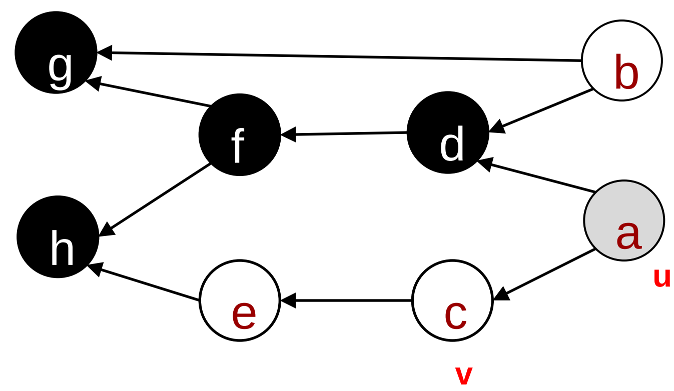



Finish_list:

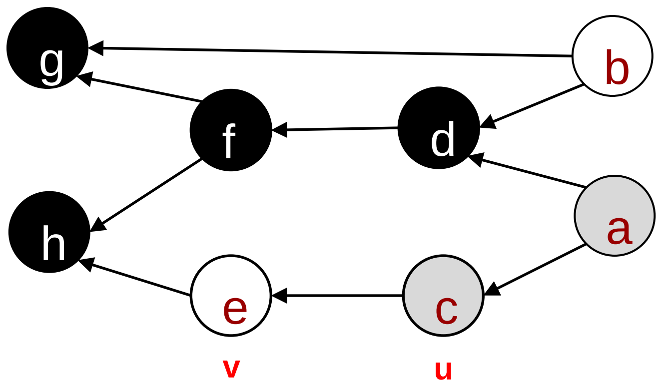
h,
g,
f,
d

DFS-Visit(G,a,l):

Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
return finish_list
```

```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```



Finish_list:

h,
g,
f,
d

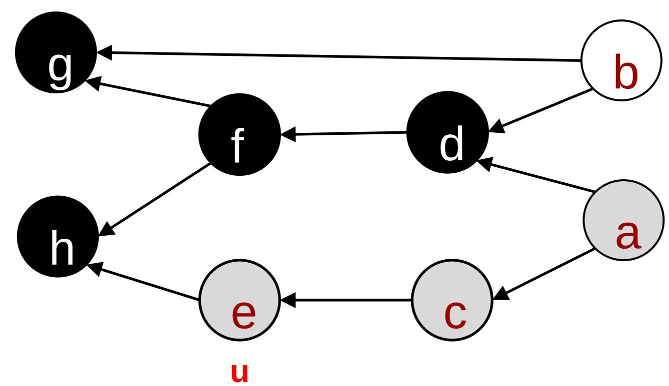
DFS-Visit(G,c,l):

DFS-Visit(G,a,l):

Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
return finish_list
```

```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
         DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```



Finish_list:

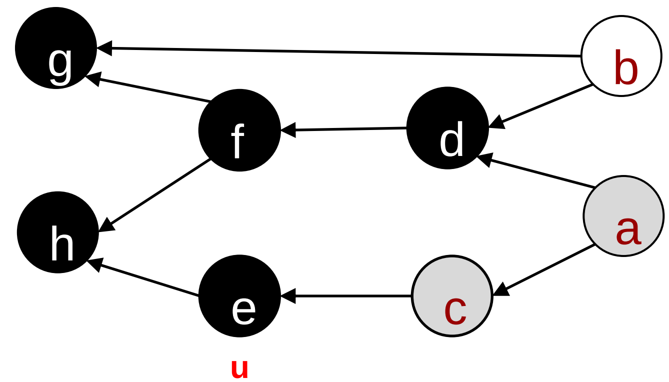
h,
g,
f,
d

DFS-Visit(G,e,l):
DFS-Visit(G,c,l):
DFS-Visit(G,a,l):

Depth First-Search

```
DFS-All (G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
       DFS-Visit (G, u, finish_list)
  return finish_list
```

```
DFS-Visit (G, u, l)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
          DFS-Visit (G, v)
    u.color = BLACK
    l.append(u)
```



Finish_list:

h,
g,
f,
d.
e

DFS-Visit(G,e,l):
DFS-Visit(G,c,l):
DFS-Visit(G,a,l):

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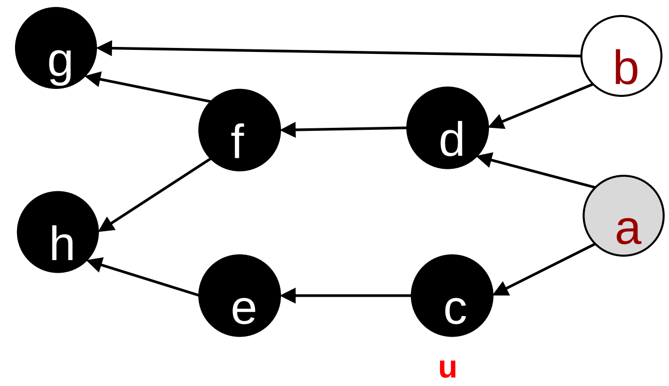
Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
     DFS-Visit (G, u, finish_list)
return finish_list
```




```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```

Finish_list:

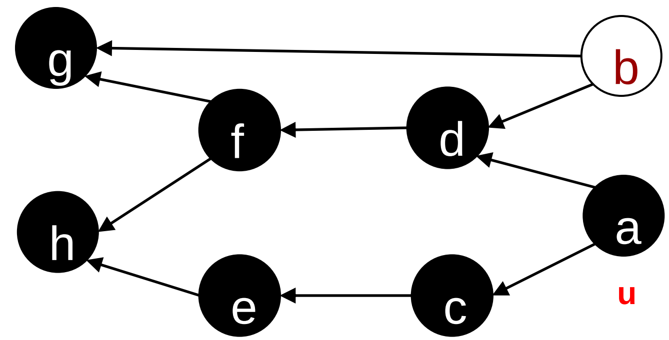
h, g, f, d. e, c

DFS-Visit(G,c,l):
DFS-Visit(G,a,l):

Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
     DFS-Visit (G, u, finish_list)
return finish_list
```

```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```



Finish_list:

h, g, f, d. e, c, a

DFS-Visit(G,a,l):

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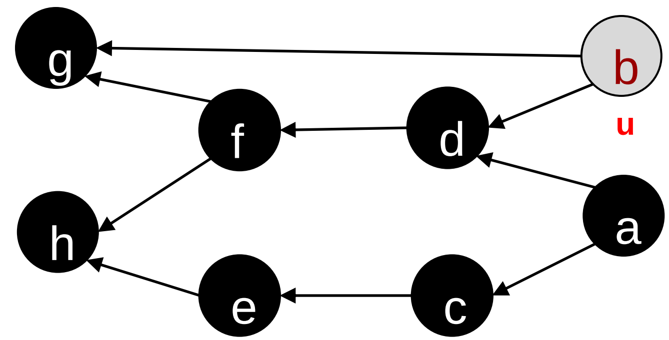
Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
     DFS-Visit (G, u, finish_list)
return finish_list
```

May iterate through many complete vertices before finding b to launch a new search from




```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```

Finish_list:

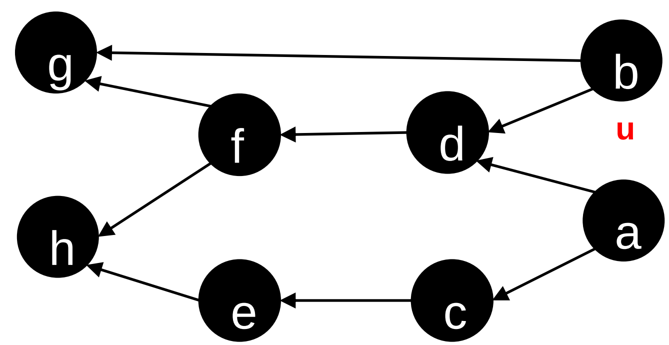
h,
g,
f,
d.
e,
c,
a

DFS-Visit(G,b,l):

Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
     DFS-Visit (G, u, finish_list)
return finish_list
```

```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```



Finish_list:

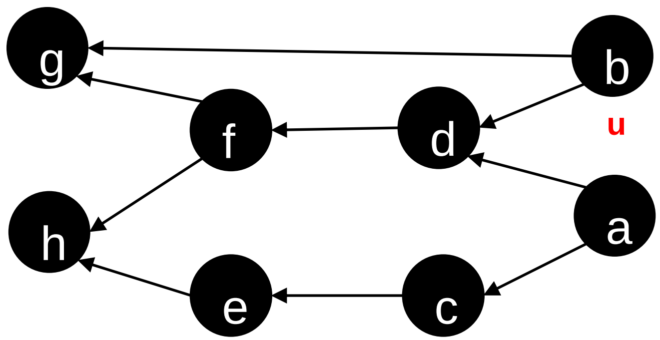
h, g, f, d. e, c, a, b

DFS-Visit(G,b,l):

Depth First-Search

```
DFS-All (G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
return finish_list
```

```
DFS-Visit (G, u, l)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  l.append(u)
```



Finish_list:

h, g, f, d. e, c, a, b

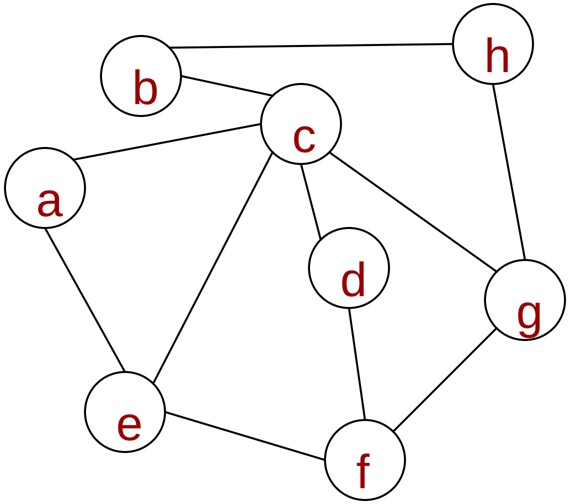
With Cycles in the graph

ANOTHER EXAMPLE (IF TIME)

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

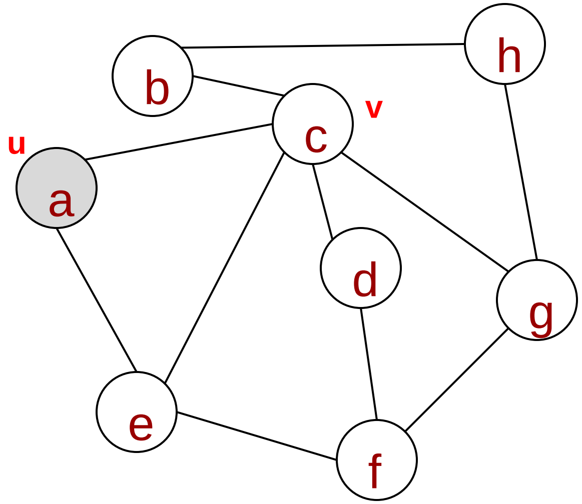
```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```



Depth First-Search

```
Toposort(G)
1  for each vertex u
2    u.color = WHITE
3  finish_list = empty_list
4  for each vertex u do
5     if u.color == WHITE then
6        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
1    u.color = GRAY
2    for each vertex v in Adj(u) do
3      if v.color = WHITE then
4         DFS-Visit (G, v)
5    u.color = BLACK
6    finish_list.append(u)
```

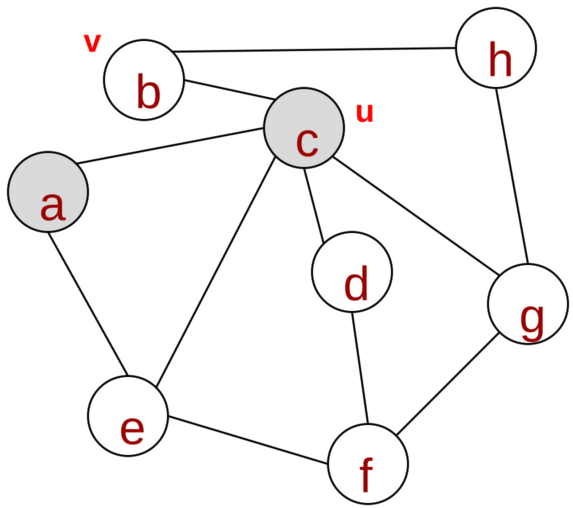


DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
          DFS-Visit (G, v)
    u.color = BLACK
    finish_list.append(u)
```



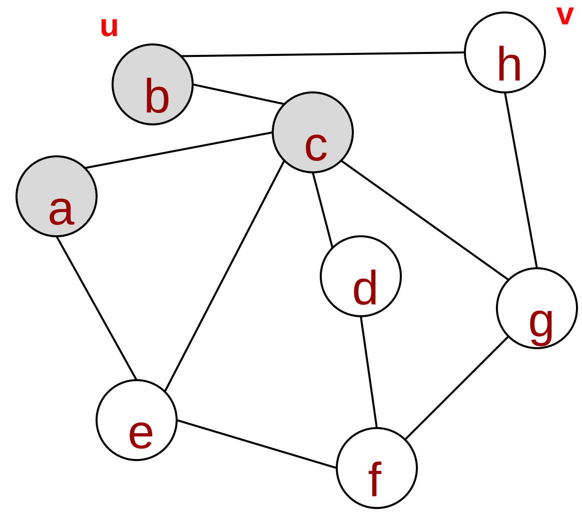
DFS-Visit(G,c):

DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
u.color = GRAY
for each vertex v in Adj(u) do
  if v.color = WHITE then
     DFS-Visit (G, v)
u.color = BLACK
finish_list.append(u)
```

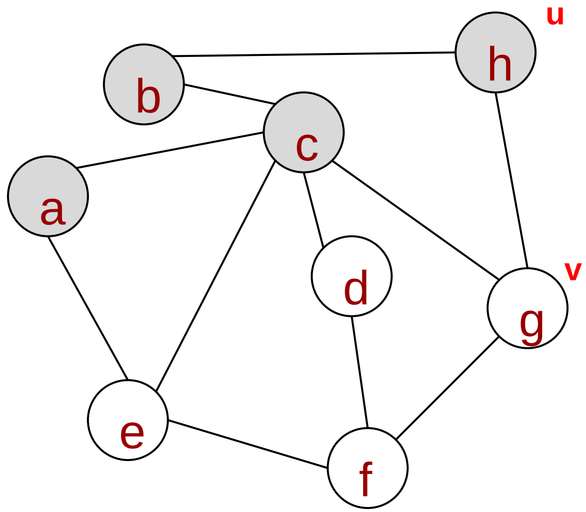


DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
         DFS-Visit (G, v)
    u.color = BLACK
    finish_list.append(u)
```



DFS-Visit(G,h):

DFS-Visit(G,b):

DFS-Visit(G,c):

DFS-Visit(G,a):

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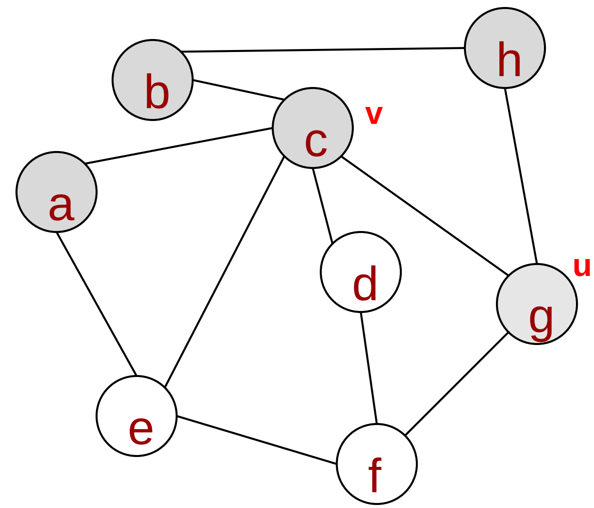
Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
   u.color = GRAY
   for each vertex v in Adj(u) do
     if v.color = WHITE then
        DFS-Visit (G, v)
   u.color = BLACK
   finish_list.append(u)
```



DFS-Visit(G,g):

DFS-Visit(G,h):

DFS-Visit(G,b):

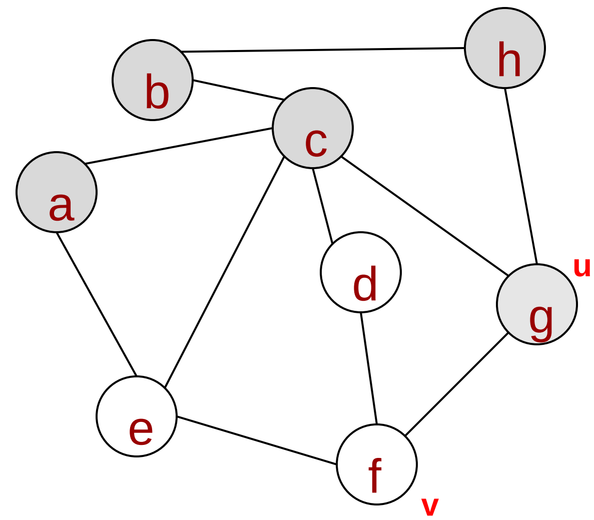
DFS-Visit(G,c):

DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
         DFS-Visit (G, v)
    u.color = BLACK
    finish_list.append(u)
```

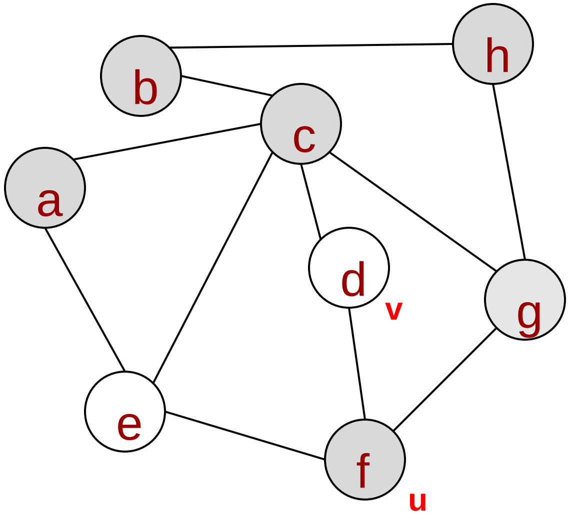


DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
         DFS-Visit (G, v)
    u.color = BLACK
    finish_list.append(u)
```

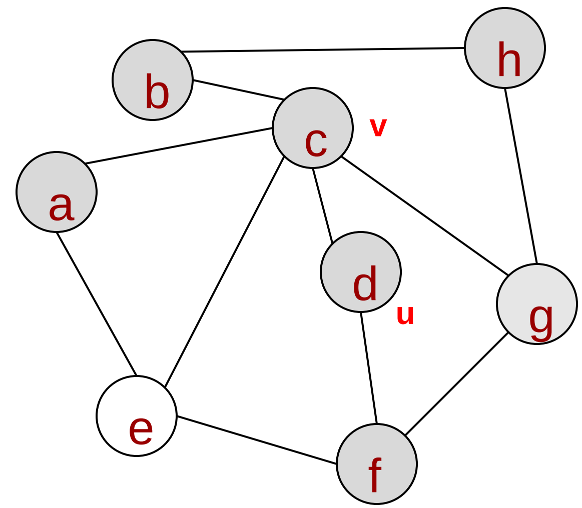


DFS-Visit(G,f):
DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
       DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```



DFS-Visit(G,d):
DFS-Visit(G,f):
DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

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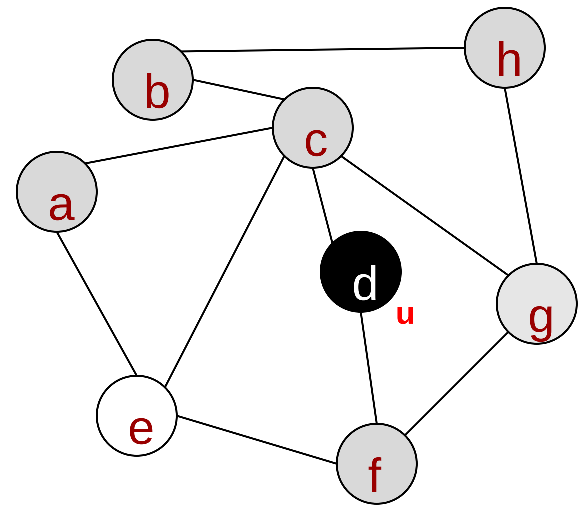
Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
       DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```



DFSQ:

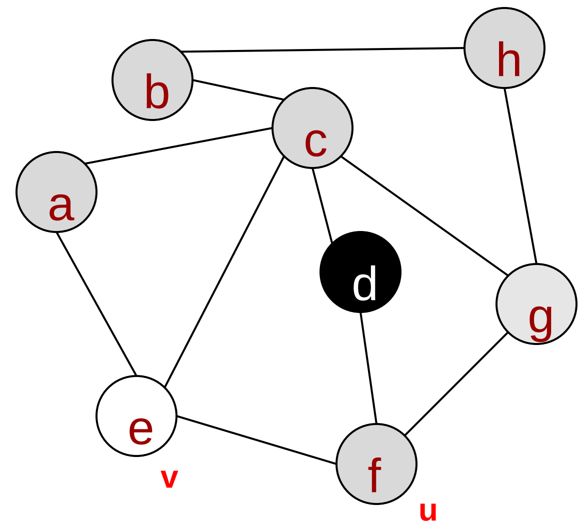
d

DFS-Visit(G,d):
DFS-Visit(G,f):
DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
         DFS-Visit (G, v)
    u.color = BLACK
    finish_list.append(u)
```



DFSQ:

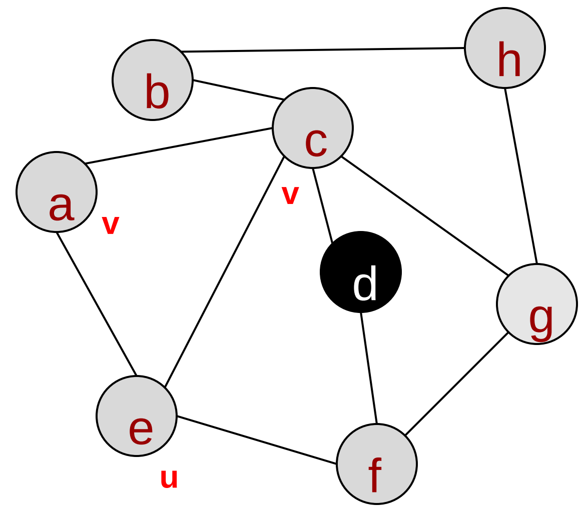
d

DFS-Visit(G,f):
DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
u.color = GRAY
for each vertex v in Adj(u) do
  if v.color = WHITE then
     DFS-Visit (G, v)
u.color = BLACK
finish_list.append(u)
```



DFSQ:

d

DFS-Visit(G,e):

DFS-Visit(G,f):

DFS-Visit(G,g):

DFS-Visit(G,h):

DFS-Visit(G,b):

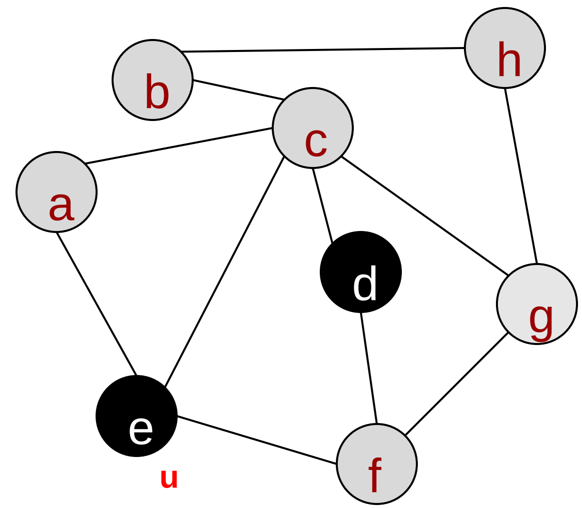
DFS-Visit(G,c):

DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
  for each vertex u
    u.color = WHITE
  finish_list = empty_list
  for each vertex u do
     if u.color == WHITE then
        DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
    u.color = GRAY
    for each vertex v in Adj(u) do
      if v.color = WHITE then
          DFS-Visit (G, v)
    u.color = BLACK
    finish_list.append(u)
```



DFSQ:

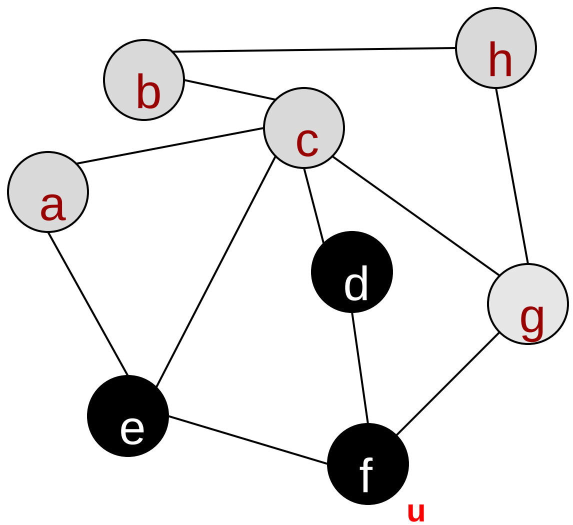
d
e

DFS-Visit(G,e):
DFS-Visit(G,f):
DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
       DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```



DFSQ:

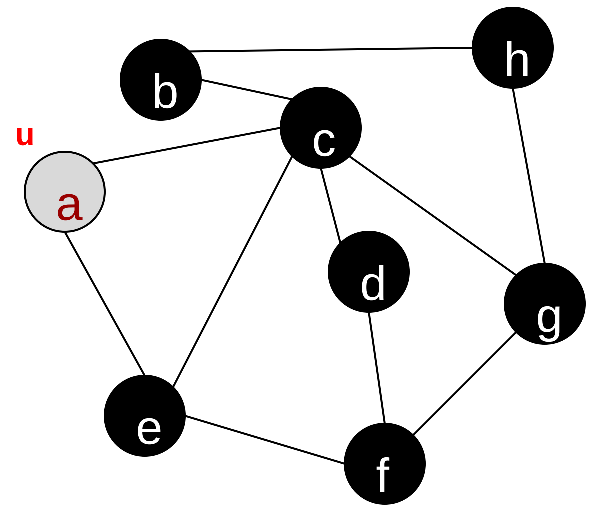
d
e
f

DFS-Visit(G,f):
DFS-Visit(G,g):
DFS-Visit(G,h):
DFS-Visit(G,b):
DFS-Visit(G,c):
DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```



DFSQ:

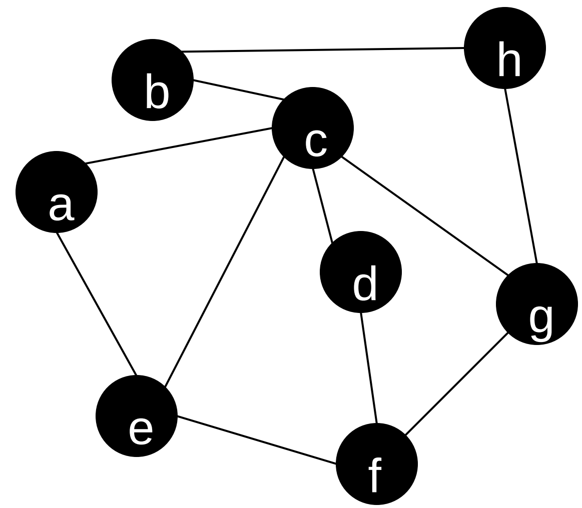
d
e
f
g
h
b
c

DFS-Visit(G,a):

Depth First-Search

```
Toposort(G)
for each vertex u
  u.color = WHITE
finish_list = empty_list
for each vertex u do
   if u.color == WHITE then
      DFS-Visit (G, u, finish_list)
```

```
DFS-Visit (G, u)
  u.color = GRAY
  for each vertex v in Adj(u) do
    if v.color = WHITE then
        DFS-Visit (G, v)
  u.color = BLACK
  finish_list.append(u)
```



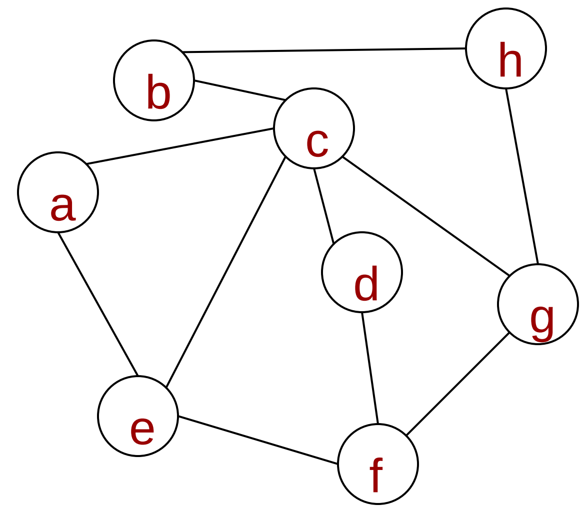
DFSQ:

d
e
f
g
h
b
c
a

ITERATIVE VERSION

Depth First-Search

```
DFS (G,s)
for each vertex u
  u.color = WHITE
st = new Stack
st.push_back(s)
while st not empty
   u = st.back()
   if u.color == WHITE then
     u.color = GRAY
     foreach vertex v in Adj(u) do
        if v.color == WHITE
           st.push_back(v)
   else if u.color != WHITE
     u.color = BLACK
     st.pop_back()
```



st: | **a** |

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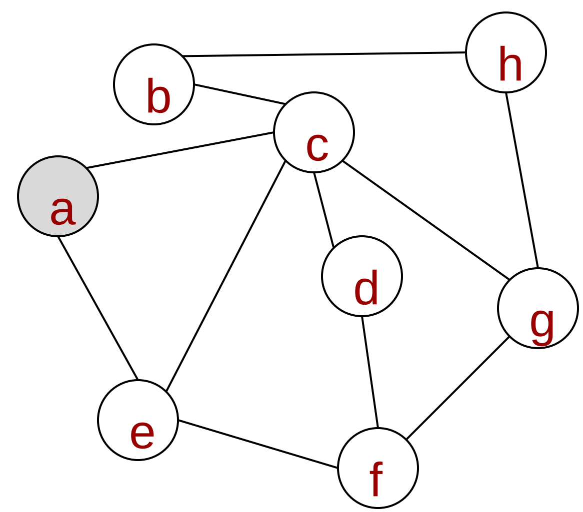
Depth First-Search

```
DFS (G,s)
  for each vertex u
    u.color = WHITE
  st = new Stack
  st.push_back(s)
  while st not empty
     u = st.back()
     if u.color == WHITE then
       u.color = GRAY
       foreach vertex v in Adj(u) do
          if v.color == WHITE
             st.push_back(v)
    else if u.color != WHITE
       u.color = BLACK
       st.pop_back()
```

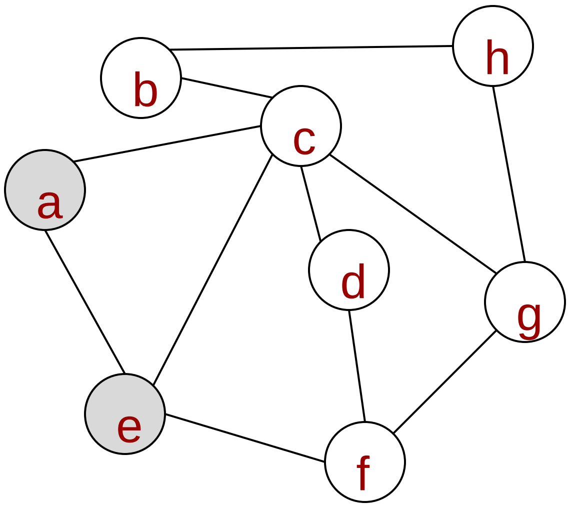


st:

a	c	e

Depth First-Search

```
DFS (G,s)
for each vertex u
  u.color = WHITE
st = new Stack
st.push_back(s)
while st not empty
   u = st.back()
   if u.color == WHITE then
     u.color = GRAY
     foreach vertex v in Adj(u) do
        if v.color == WHITE
           st.push_back(v)
  else if u.color != WHITE
     u.color = BLACK
     st.pop_back()
```



st:

a	c	e	c	f

Depth First-Search

```
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for each vertex u
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while st not empty
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   if u.color == WHITE then
     u.color = GRAY
     foreach vertex v in Adj(u) do
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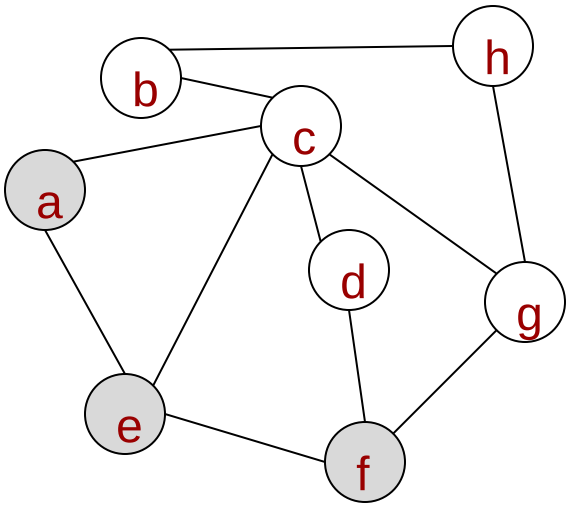

st:

a	c	e	c	f	d	g

Depth First-Search

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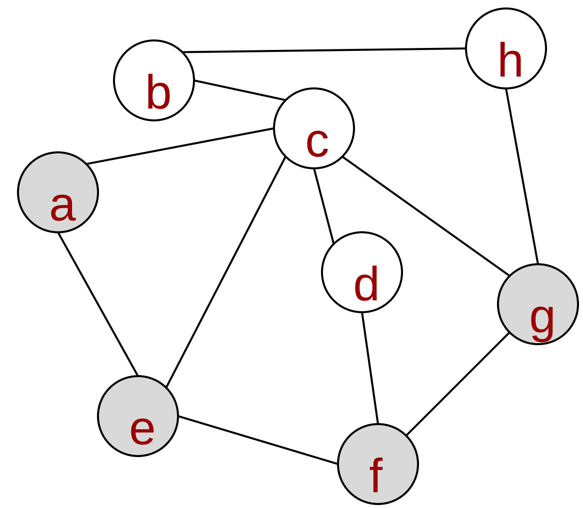

st:

a	c	e	c	f	d	g	c	h

Depth First-Search

```
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  for each vertex u
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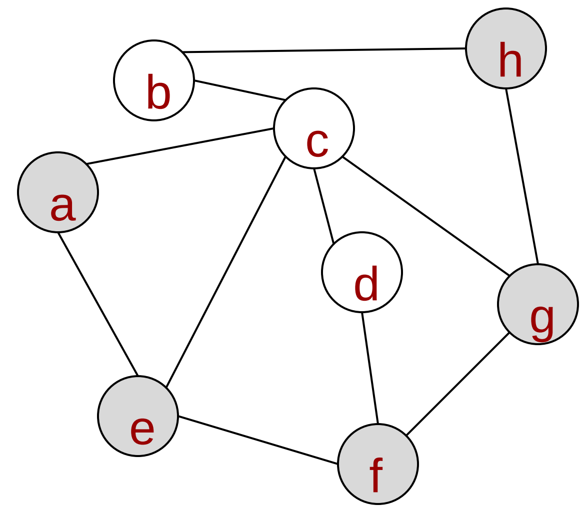

st:

a	c	e	c	f	d	g	c	h	b

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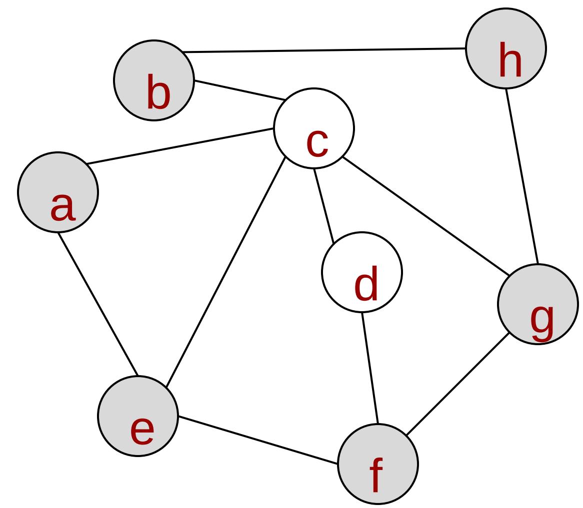

st:

a	c	e	c	f	d	g	c	h	b	c

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Depth First-Search

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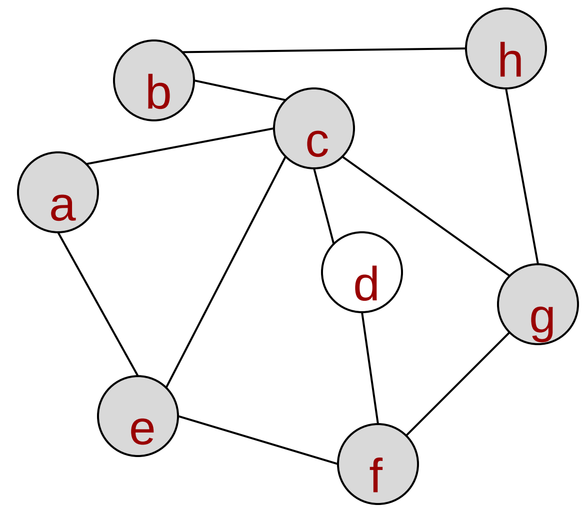

st:

a	c	e	c	f	d	g	c	h	b	c	d

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Depth First-Search

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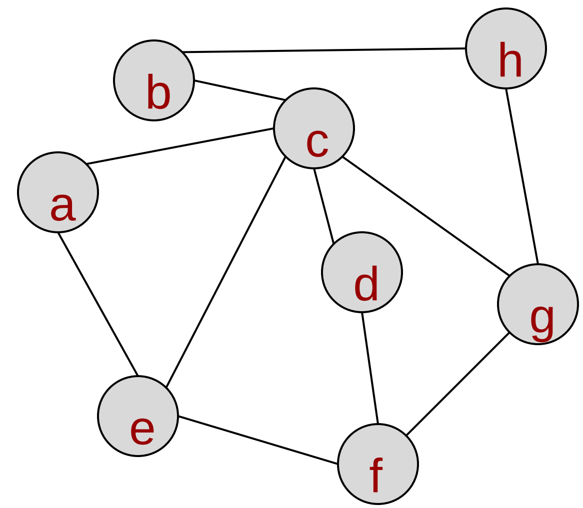

st:

a	c	e	c	f	d	g	c	h	b	c	d

Depth First-Search

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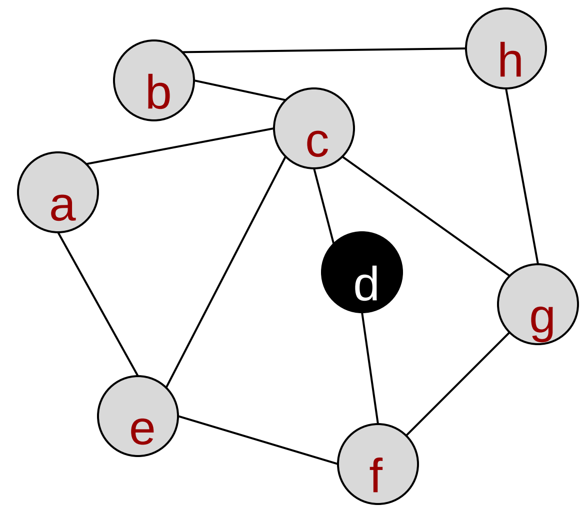

st:

a	c	e	c	f	d	g	c	h	b	c	d

Depth First-Search

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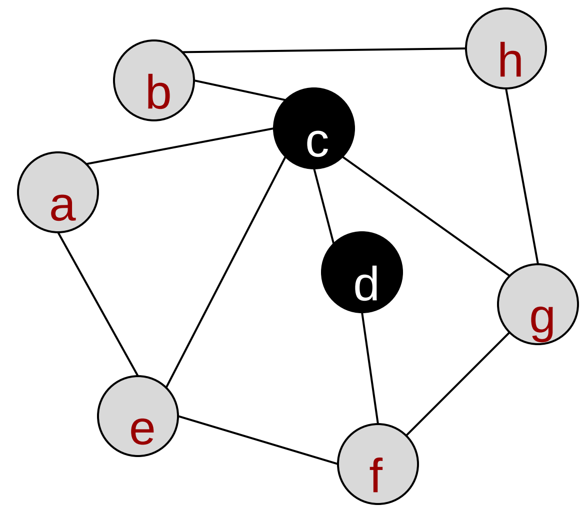
          if v.color == WHITE
             st.push_back(v)
    else if u.color != WHITE
       u.color = BLACK
       st.pop_back()
```



st:

a	c	e	c	f	d	g	c	h	b	c

Depth First-Search

```
DFS (G,s)
  for each vertex u
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     u = st.back()
     if u.color == WHITE then
       u.color = GRAY
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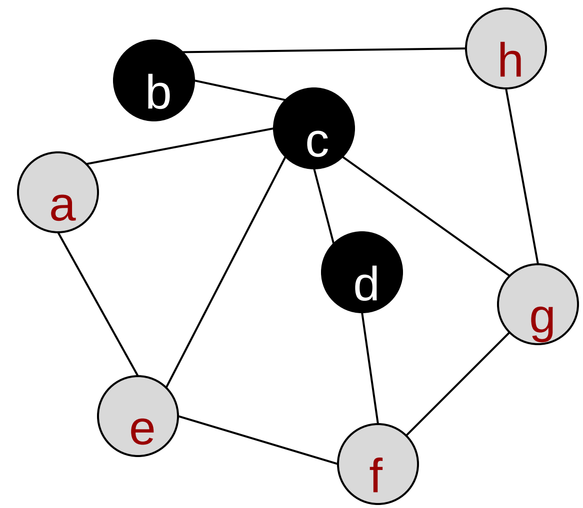
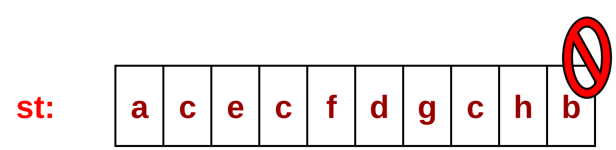
          if v.color == WHITE
             st.push_back(v)
  else if u.color != WHITE
       u.color = BLACK
       st.pop_back()
```





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Depth First-Search

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DFS (G,s)
for each vertex u
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st = new Stack
st.push_back(s)
while st not empty
   u = st.back()
   if u.color == WHITE then
     u.color = GRAY
     foreach vertex v in Adj(u) do
        if v.color == WHITE
           st.push_back(v)
  else if u.color != WHITE
     u.color = BLACK
     st.pop_back()
```

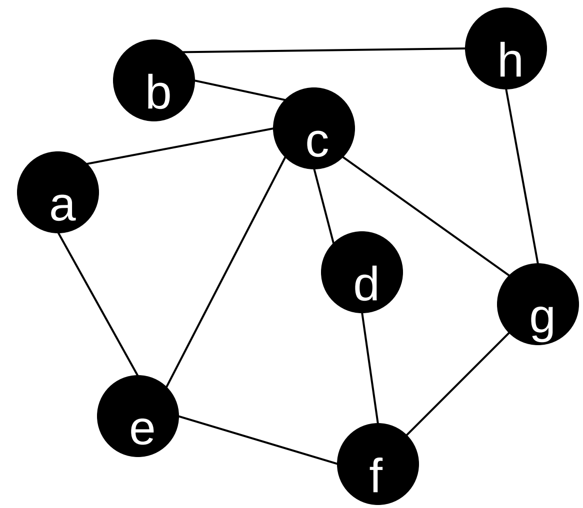

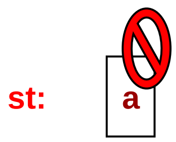

BFS vs. DFS Algorithm

- BFS and DFS are more similar than you think
  - Do we use a FIFO/Queue (BFS) or LIFO/Stack (DFS) to store vertices as we find them

```
BFS-Visit (G, start_node)
  for each vertex u
    u.color = WHITE
    u.pred = nil
  bfsq = new Queue
  bfsq.push_back(start_node)
  while bfsq not empty
    u = bfsq.pop_front()
    if u.color == WHITE
      u.color = GRAY
      foreach vertex v in Adj(u) do
        bfsq.push_back(v)
```

```
DFS-Visit (G, start_node)
  for each vertex u
    u.color = WHITE
    u.pred = nil
  st = new Stack
  st.push_back(start_node)
  while st not empty
    u = st.top(); st.pop()
    if u.color == WHITE
      u.color = GRAY
      foreach vertex v in Adj(u) do
        st.push_back(v)
```

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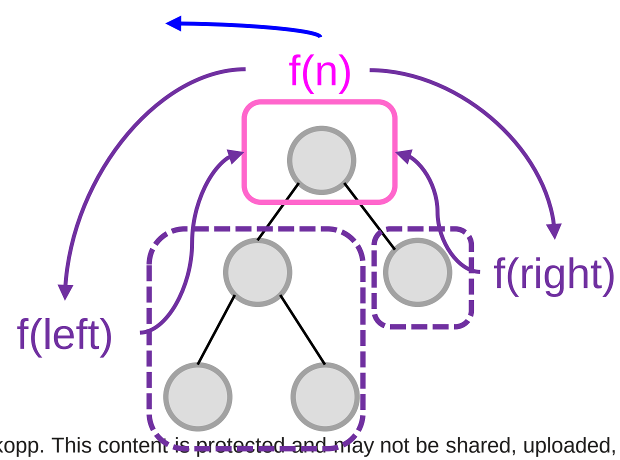
SOLUTIONS

Example 1: Count Nodes

- Write a recursive function to **count how many nodes** are in the binary tree
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node
  - For in- or post-order traversals, determine how to use/combine results from recursion on children

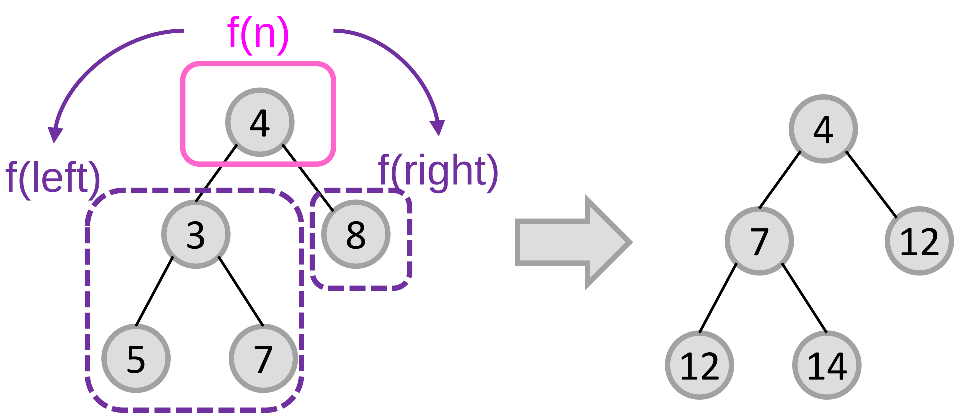



```
// Node definition
struct Tnode {
  int val;
  TNode *left, *right;
};

int count(TNode* root)
{
  if( root == NULL ) return 0;
  else {
    return 1 + count(root->left) +
               count(root->right);
  }
}
```

Example 2: Prefix Sums

- Write a recursive function to **have each node store the sum of the values on the path from the root to each node.**
  - Only process 1 node at a time
  - Determine pre-, in-, or post-order based on whose answers you need to compute the result for your node




```
void prefixH(TNode* root, int psum)

void prefix(TNode* root)
{
  prefixH(root, 0);
}

void prefixH(TNode* root, int psum)
{
  if( root == NULL ) return;
  else {
    root->val += psum;
    prefixH(root->left,  root->val);
    prefixH(root->right, root->val);
  }
}
```