CS103 Unit 8

Recursion

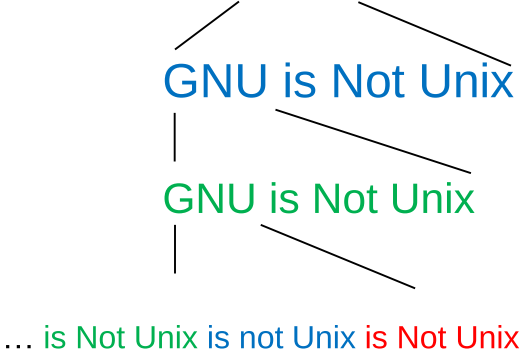
Mark Redekopp

Recursion

- Defining an object, mathematical function, or computer function in terms of *itself*

GNU

- Makers of gedit, g++ compiler, etc.
- GNU = GNU is Not Unix



GNU is Not Unix

GNU is Not Unix

… is Not Unix is not Unix is Not Unix

Recursion

- Problem in which the solution can be expressed in terms of itself (usually a smaller instance/input of the same problem) ***and a base/terminating case***
- Usually takes the place of a loop
- Input to the problem must be categorized as a:
  - Base case: Solution known beforehand or easily computable (no recursion needed)
  - Recursive case: Solution can be described using solutions to smaller problems of the same type
    - Keeping putting in terms of something smaller until we reach the base case
- Factorial: n! = n * (n-1) * (n-2) * … * 2 * 1
  - n! = n * (n-1)!
  - Base case: n = 1
  - Recursive case: n > 1 => n*(n-1)!

Recursive Functions

- Recall the system stack essentially provides separate areas of memory for each 'instance' of a function
- Thus each local variable and actual parameter of a function has its own value within that particular function instance's memory space

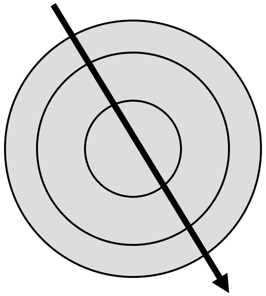
C Code:

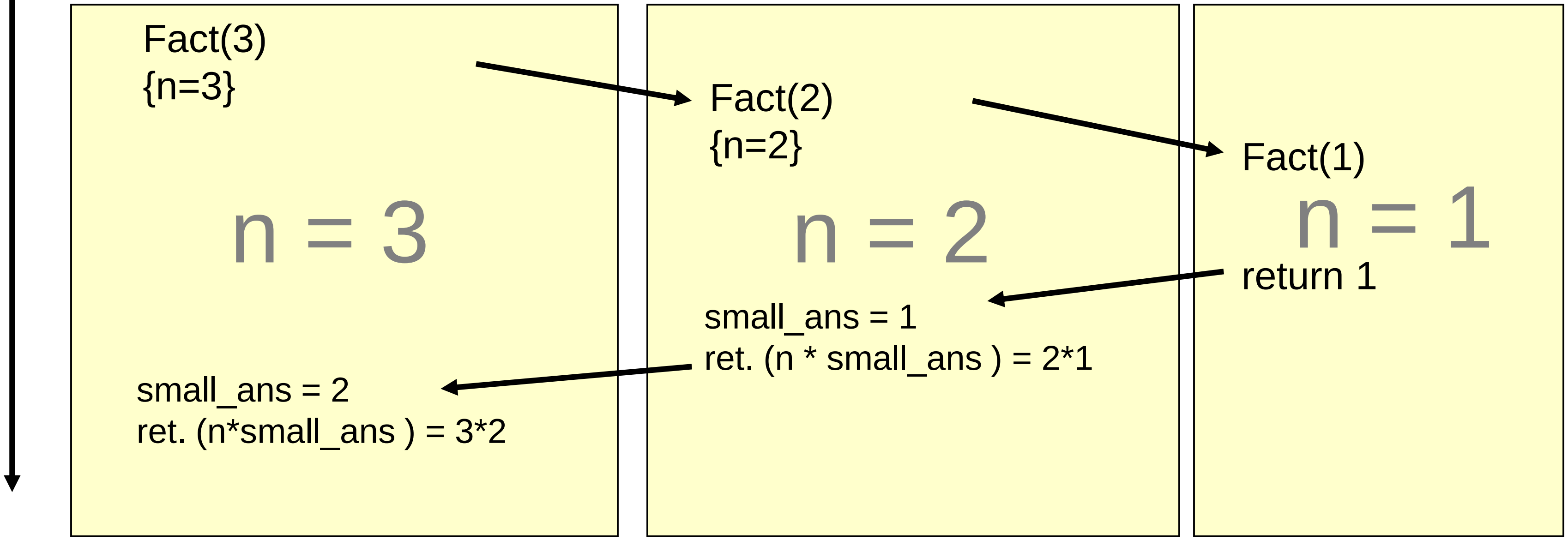
```
int fact(int n)
{
  // base case
  if(n == 1)
     return 1;

  // recursive case
  else {
     // calculate (n-1)!
     int small_ans = fact(n-1);

         // now ans = (n-1)!
         // so calculate n!
     return  n * small_ans;

     }
}
```

Recursive Call Timeline

```
int fact(int n)
{
  if(n == 1)
     return 1;
  else {
     int small_ans = fact(n-1);
     return n * small_ans ;
} }
```

Time





Fact(3)
{n=3}

n = 3

small_ans = 2
ret. (n*small_ans) = 3*2

Fact(2)
{n=2}

n = 2

small_ans = 1
ret. (n * small_ans) = 2*1

Fact(1)

n = 1

return 1

- Value/version of n is implicitly “saved” and “restored” as we move from one instance of the ‘fact’ function to the next

Head vs. Tail Recursion

- Head Recursion: Recursive call is made before the real work is performed in the function body
- Tail Recursion: Some work is performed and then the recursive call is made

Tail Recursion

```
void doit(int n)
{
  if(n == 1) cout << "Stop";
  else {
   cout << "Go" << endl;
   doit(n-1);
  }
}
```

Head Recursion

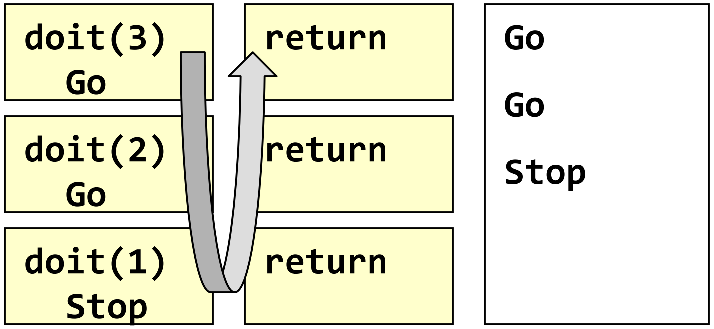
```
void doit(int n)
{
  if(n == 1) cout << "Stop";
  else {
   doit(n-1);
   cout << "Go" << endl;
  }
}
```

Head vs. Tail Recursion

- Head Recursion: Recursive call is made before the real work is performed in the function body
- Tail Recursion: Some work is performed and then the recursive call is made

Tail Recursion

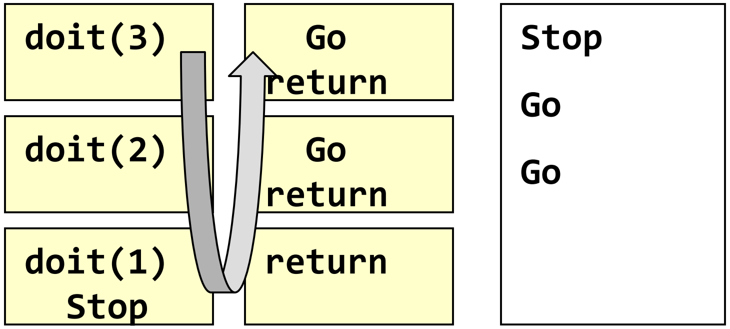
```
Void doit(int n)
{
  if(n == 1) cout << "Stop";
  else {
   cout << "Go" << endl;
   doit(n-1);
  }
}
```



```
doit(3)        return        Go
  Go
                             Go
doit(2)        return
  Go                         Stop

doit(1)        return
  Stop
```

Head Recursion

```
Void doit(int n)
{
  if(n == 1) cout << "Stop";
  else {
   doit(n-1);
   cout << "Go" << endl;
  }
}
```



```
doit(3)          Go          Stop
               return
                             Go
doit(2)          Go
               return        Go

doit(1)        return
  Stop
```

Recursive Functions

- Recall the system stack essentially provides separate areas of memory for each ‘instance’ of a function
- Thus each local variable and actual parameter of a function has its own value within that particular function instance’s memory space

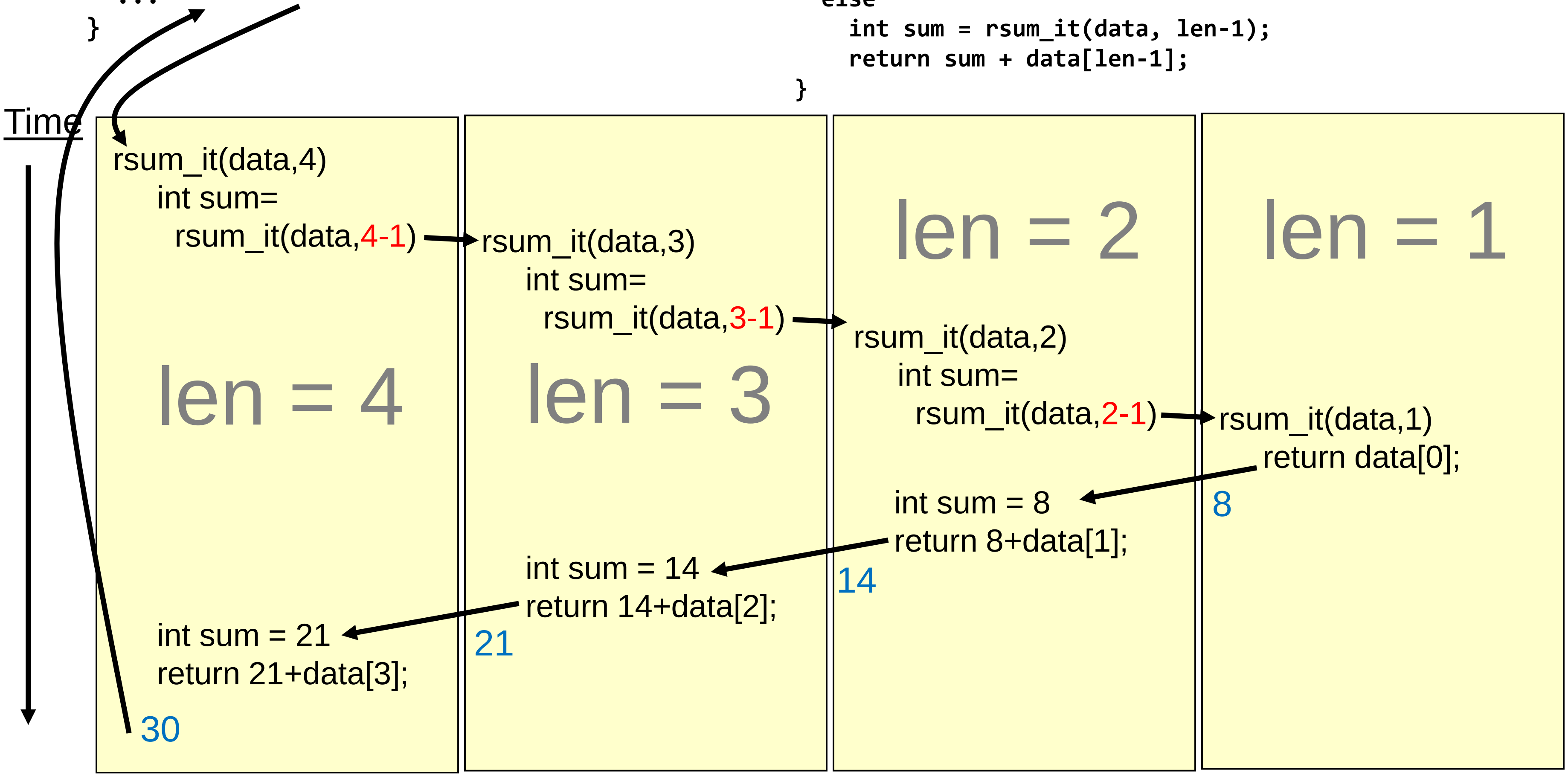
C Code:

```
int main()
{
  int data[4] = {8, 6, 7, 9};
  int sum1 = isum_it(data, 4);
  int sum2 = rsum_it(data, 4);
}

int isum_it(int data[], int len)
{
  sum = data[0];
  for(int i=1; i < len; i++){
    sum += data[i];
  }
}

int rsum_it(int data[], int len)
{
  if(len == 1)
    return data[0];
  else
    int sum = rsum_it(data, len-1);
    return sum + data[len-1];
}
```

Recursive Call Timeline

```
int main(){
  int data[4] = {8, 6, 7, 9};
  int sum2 = rsum_it(data, 4);
  ...
}
```

```
int rsum_it(int data[], int len)
{
  if(len == 1)
    return data[0];
  else
    int sum = rsum_it(data, len-1);
    return sum + data[len-1];
}
```



Time

rsum_it(data,4)
int sum=
rsum_it(data,4-1)

len = 4

int sum = 21
return 21+data[3];
30

rsum_it(data,3)
int sum=
rsum_it(data,3-1)

len = 3

int sum = 14
return 14+data[2];
21

len = 2

rsum_it(data,2)
int sum=
rsum_it(data,2-1)

int sum = 8
return 8+data[1];
14

len = 1

rsum_it(data,1)
return data[0];
8

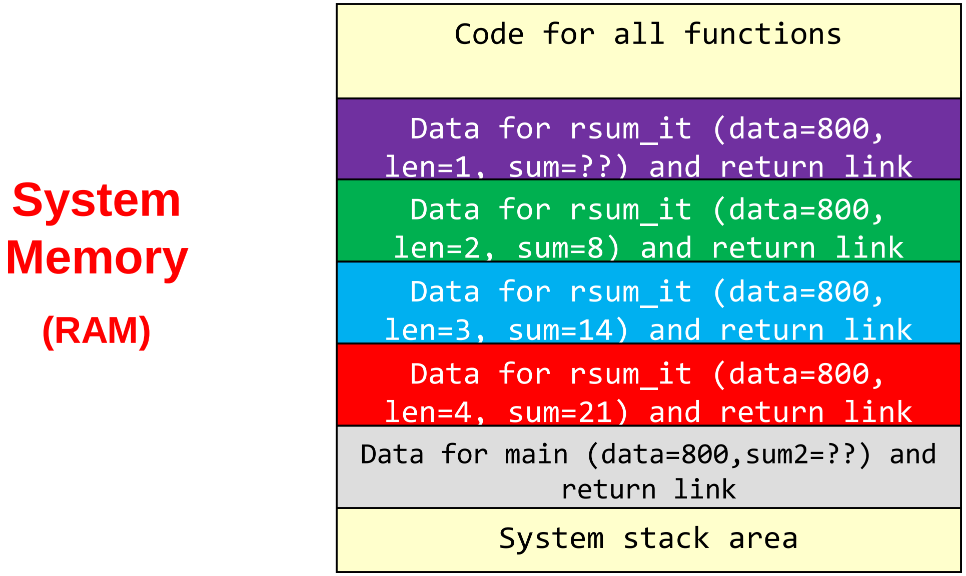
Each instance of rsum_it has its own len argument and sum variable

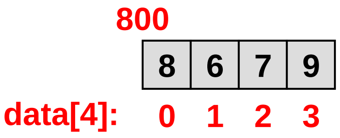
Every instance of a function has its own copy of local variables

System Stack & Recursion

- The system stack makes recursion possible by providing separate memory storage for the local variables of each running instance of the function

```
int main()
{
  int data[4] = {8, 6, 7, 9};
  int sum2 = rsum_it(data, 4);
}

int rsum_it(int data[], int len)
{
  if(len == 1)
    return data[0];
  else
    int sum =
       rsum_it(data, len-1);
    return sum + data[len-1];
}
```


System Memory
(RAM)
Code for all functions
Data for rsum_it (data=800, len=1, sum=??) and return link
Data for rsum_it (data=800, len=2, sum=8) and return link
Data for rsum_it (data=800, len=3, sum=14) and return link
Data for rsum_it (data=800, len=4, sum=21) and return link
Data for main (data=800,sum2=??) and return link
System stack area



Exercise

- Exercises
  - Count-down
  - Count-up

Recursion Double Check

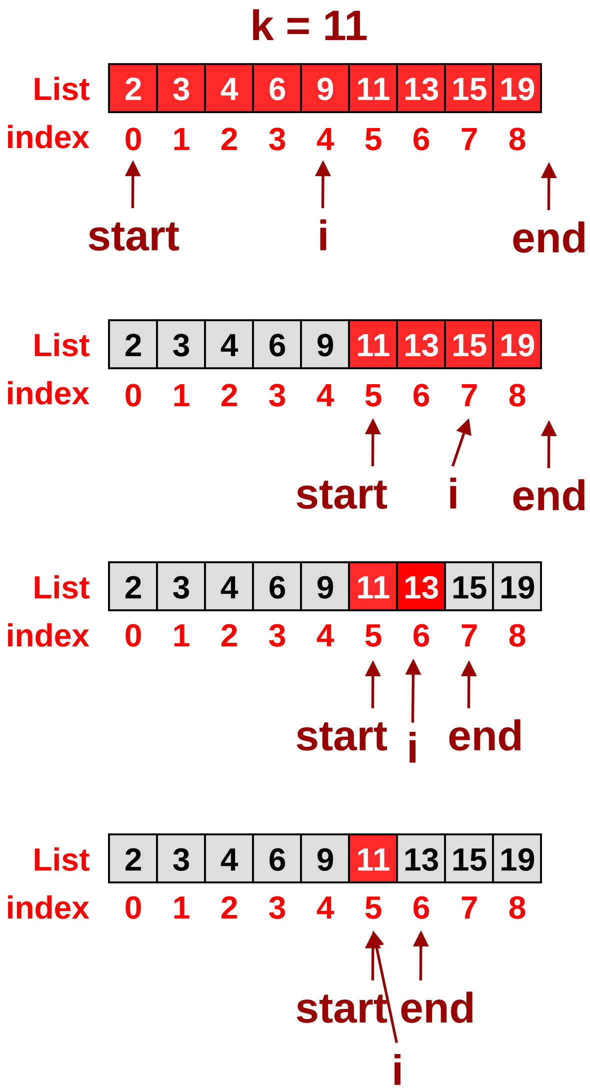
- When you write a recursive routine:
  - Check that you have appropriate base cases
    - Need to check for these first before recursive cases
  - Check that each recursive call makes progress toward the base case
    - Otherwise you'll get an infinite loop and stack overflow
  - Check that you use a 'return' statement at each level to return appropriate values back to each recursive call
    - You have to return back up through every level of recursion, so make sure you are returning something (the appropriate thing)

Loops & Recursion

- Is it better to use recursion or iteration?
  - ANY problem that can be solved using recursion can also be solved with iteration and other appropriate data structures
- Why use recursion?
  - Usually clean & elegant. Easier to read.
  - Sometimes generates much simpler code than iteration would
  - Sometimes iteration will be almost impossible
  - The power of recursion often comes when each function instance makes ***multiple*** recursive calls
- How do you choose?
  - Iteration is usually faster and uses less memory
  - However, if iteration produces a very complex solution, consider recursion

Recursive Binary Search

- Assume remaining items = [start, end)
  - start is inclusive index of start item in remaining list
  - End is exclusive index of start item in remaining list
- binSearch(target, List[], start, end)
  - Perform base check (empty list)
    - Return NOT FOUND (-1)
  - Pick mid item
  - Based on comparison of k with List[mid]
    - EQ => Found => return mid
    - LT => return answer to BinSearch[start,mid)
    - GT => return answer to BinSearch[mid+1,end)



k = 11

List	2	3	4	6	9	11	13	15	19
index	0	1	2	3	4	5	6	7	8

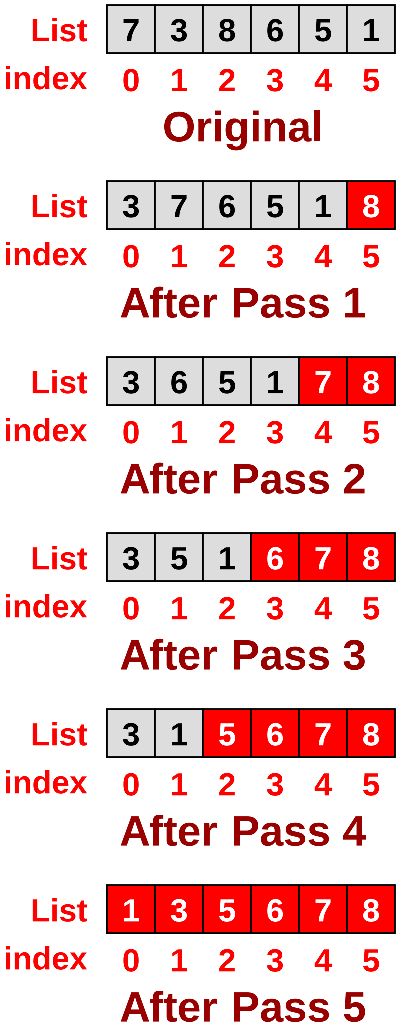
List	2	3	4	6	9	11	13	15	19
index	0	1	2	3	4	5	6	7	8

List	2	3	4	6	9	11	13	15	19
index	0	1	2	3	4	5	6	7	8

List	2	3	4	6	9	11	13	15	19
index	0	1	2	3	4	5	6	7	8

Sorting

- If we have an unordered list, sequential search becomes our only choice
- If we will perform a lot of searches it may be beneficial to sort the list, then use binary search
- Many sorting algorithms of differing complexity (i.e. faster or slower)
- Bubble Sort (simple though not terribly efficient)
  - On each pass through thru the list, pick up the maximum element and place it at the end of the list. Then repeat using a list of size n-1 (i.e. w/o the newly placed maximum value)



List	7	3	8	6	5	1
index	0	1	2	3	4	5

Original

List	3	7	6	5	1	8
index	0	1	2	3	4	5

After Pass 1

List	3	6	5	1	7	8
index	0	1	2	3	4	5

After Pass 2

List	3	5	1	6	7	8
index	0	1	2	3	4	5

After Pass 3

List	3	1	5	6	7	8
index	0	1	2	3	4	5

After Pass 4

List	1	3	5	6	7	8
index	0	1	2	3	4	5

After Pass 5

Exercise

- Exercises
  - Text-based fractal

Flood Fill

- Imagine you are given an image with outlines of shapes (boxes and circles) and you had to write a program to shade (make black) the inside of one of the shapes. How would you do it?
- Flood fill is a recursive approach
- Given a pixel
  - Base case: If it is black already, stop!
  - Recursive case: Call floodfill on each neighbor pixel
  - Hidden base case: If pixel out of bounds, stop!