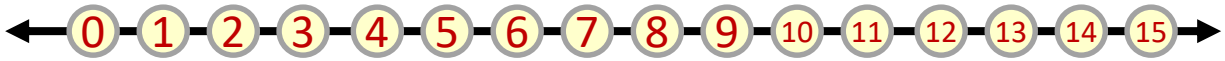
Unit 5

Programming Mathematics

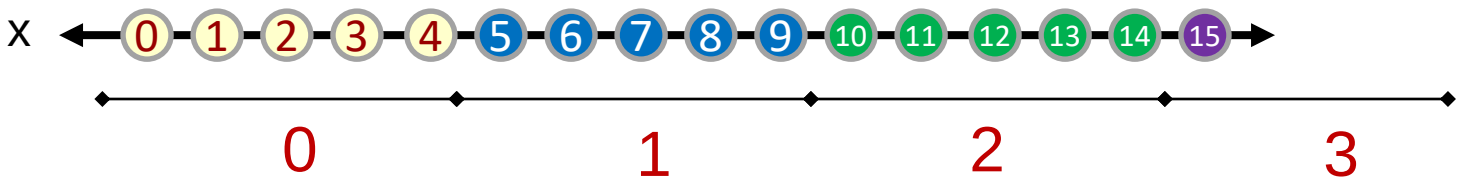
Division and Modulo Operations

- Division and modulo operations can be used to map a range of numbers to a smaller range



Integer Division

- Integer division maps a range of values to a common integer
- Consecutive values fall into the same class
- Example:
  - x / 5



Modulo Operations

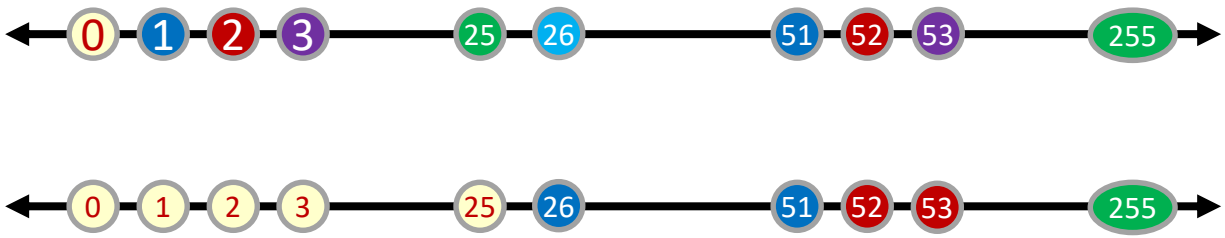
- Modulo has many uses and applications
- *x mod m* will yield numbers in the range [0 to m-1]
- Consecutive values fall into different classes
- Example:
  - x % 5

Exercise 1

- Use division or modulo to convert an input number, x, in the range 0-255 to proportional range of 0-9

Visualization of Exercise 1

- Convert an input number, x, in the range 0-255 to proportional range of 0-9
  - Using modulus: x % ____
  - Using division: x / ____



Exercise 2

- Simulate 2 random coin flips
  - cpp/var-expr/coins
- Use `rand()` to generate a random number.
  - `rand()` is defined in `<cstdlib>`
  - Returns a random integer between 0 and *about* 2^{31}
    - Really $+2^{31}-1$
  - Your job to convert it to some other random range (i.e. 0 or 1 for a coin flip)


```
#include <iostream>
#include <cstdlib>
using namespace std;

int main()
{
  // Generate a random number
  int r1 = rand();
  // And another
  int r2 = rand();

  cout << r1 << r2 << endl;

  return 0;
}
```

Exercise 3

- Make change (given 0-100 cents) convert to quarters, dimes, pennies
- cpp/var-expr/change

Exercise 4

- cpp/var-expr/in_n_days
  - Given the current day of the week (1-7) add n days and indicate what day of the week (1-7) it will be then
- Write out table of examples
  - Input => Desired Output
- Test by determining intput/output range
- Plug in several values, especially edge cases

```
int main()
{
  int cday, n;
  cin >> cday >> n;
  int day_plus_n = ____________________________;

  return 0;
}
```

n (assuming c_day=1)	Day_plus_n (desired)
1	2
2	3
3	4
4	5
5	6
6	7
7	1
8	2

n (assuming c_day=4)	Day_plus_n (desired)
1	5
2	6
3	7
4	1
5	2
6	3
7	4
8	5

Weighted Averages

- A common operation in statistics, machine learning, probability, and graphics is the weighted average of a set of data
- Example: Course Grade
  - Labs: 25% Your % = 100
  - MT: 35% Your % = 80
  - Final: 40% Your % = 90
- Grade = .25*100 + .35*80 + .4*90 = 89
- General formula for weights $\{w_0, w_1, \ldots, w_n\}$ and values $\{x_0, x_1, \ldots, x_n\}$

$$weightedAvg = \frac{\sum_i (w_i \cdot x_i)}{\sum_i w_i}$$

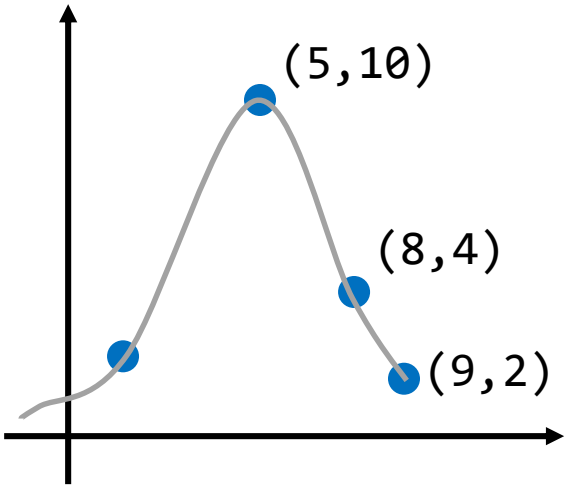
Exercise

- Compute your semester's GPA assuming you take 4 classes and use the values shown in the table for letter grades
  - cpp/var-expr/gpa
- Data: grade values for each course
- Weights: _____

Letter	Value
A	4.0
A-	3.7
B+	3.3
B	3.0
B-	2.7
C+	2.3
C	2.0
C-	1.7
D+	1.3
D	1.0
F	0.0

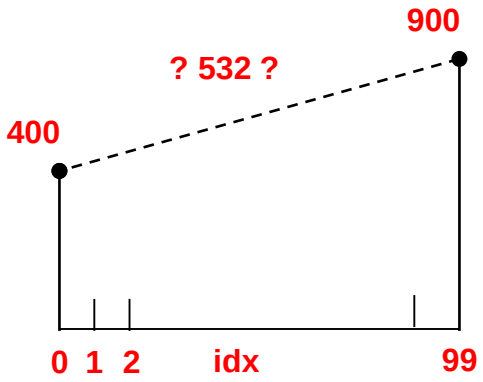
Application: Interpolation

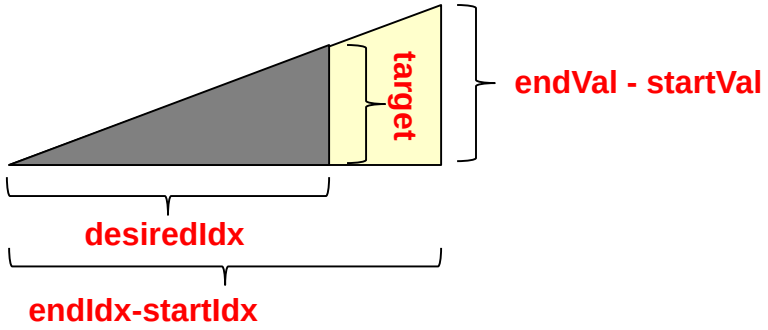
- Interpolation refers to creating or inferring new data points from a set of known datapoints
  - Finding the equation of a curve that goes through various known points
  - Determining what colors to display when you zoom in on a low-res photo
  - Predicting a function/pattern from known set of data



Linear Interpolation

- If I have a range of 100 numbers (y0-y99) where the first is 400 and the last is 900, at what index would I expect 532 (my target) to be?





$$\frac{(EndIdx - StartIdx+1)}{(EndVal - StartVal)} = \frac{desiredIdx-startIdx}{target-startVal}$$

$$(target-startVal) * \frac{(EndIdx - StartIdx+1)}{(EndVal - StartVal)} + startIdx = desiredIdx$$

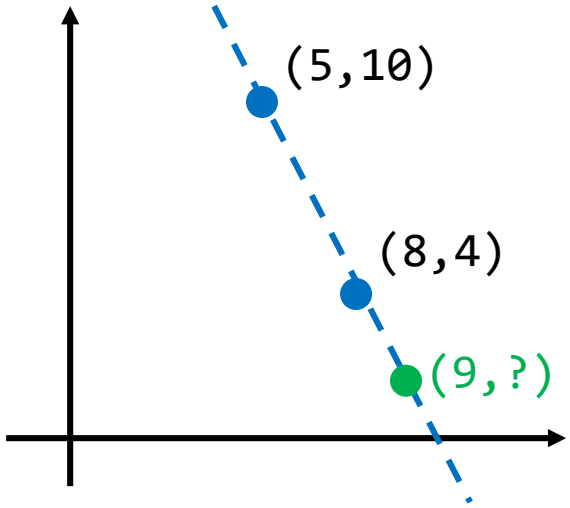
$$(532-400)*(100/500) + 0 = desiredIdx$$

$$132*0.2 = desiredIdx$$

$$26.4 = desiredIdx$$

Interpolation (1)

- Given two points (x1,y1) and (x2,y2) and an x0 value, determine y0 such that (x0,y0) lines on the line given by (x1,y1) and (x2,y2)
  - cpp/var-expr/interp1d



Interpolation (2)

- Suppose 0=black, 255=white
- If we have black at x=0.0 and white at x=1.0 what shade of gray is at location $x=x_0$

Interpolation (2)

- 2D-Interpolation to determine what color to display when you zoom in on set of pixel values
- Determine a heat/color map based on a set of known measurements
- Look up bilinear interpolation for more details