## Unit 4e

## Sorting

## Task 12a - From Unit 3d

- Find the maximum value in an array and move it to the end of the array
- Questions:
- Do we scan through the array to find the maximum without moving it and swap it at the end ..or..
- Do we move it as we can through the array

Find the maximum value and move it to the end of the array.

| Index: | [0] | [1] | [2] | [3] | [4] | [5] | [6] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| out | 8 | 3 | 2 | 7 | 12 | 9 | 10 |

## Task 12a

Find the maximum value and move it to the end of the array.


## Task 12a

- What programming issues (mechanics) should you think about?
- Do we just need to track the maximum VALUE or the INDEX of the maximum value?
- Given that you can move the maximum number to the end of the array, how could this be used to SORT the entire array?

```
int main() {
    // setup array with data
    int n, val, data[100];
    cin >> n;
    for(int i=0; i < n; i++)
        { cin >> data[i]; }
    // now perform the given task
```

    // Print out results
    for(int i=0; i < n; i++)\{
        cout << data[i] << " ";
    \}
    cout << endl;
    return 0;
    \}

## Task 12b

Find the maximum value and move it to the end of the array.


## Task 12b

- What programming issues (mechanics) should you think about?
- Do we just need to track the maximum VALUE or the INDEX of the maximum value?
- Given that you can move the maximum number to the end of the array, how could this be used to SORT the entire array?

```
int main() {
    // setup array with data
    int n, val, data[100];
    cin >> n;
    for(int i=0; i < n; i++)
        { cin >> data[i]; }
    // now perform the given task
```

    // Print out results
    for(int i=0; i < n; i++)\{
        cout << data[i] << " ";
    \}
    cout << endl;
    return 0;
    \}

## Sorting

- Sorting requires us to move data around within an array

\section*{|  | List | $\mathbf{7}$ | $\mathbf{3}$ | $\mathbf{8}$ | $\mathbf{6}$ | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | Original}


|  | List | $\mathbf{1}$ | 3 | 5 | 6 | 7 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |

- Allows users to see and organize data more efficiently
- Behind the scenes it allows more effective searching of data
- There are MANY sorting algorithms out there, we will focus on two simple ones


## Bubble Sort

- Main Idea: Keep comparing neighbors, moving larger item up and smaller item down until largest item is at the top. Repeat on list of size n-1
- Have one loop to count each pass, (a.k.a. i) to identify which index we need to stop at
- Have an inner loop start at the lowest index and count up to the stopping location comparing neighboring elements and advancing the larger of the neighbors

List | 7 | 3 | 8 | 6 | 5 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Original

List | 3 | 7 | 6 | 5 | 1 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |

After Pass 1

| List |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 6 | 5 | 1 | 7 | 8 |

After Pass 2

| List |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 3 | 5 | 1 | 6 | 7 |

After Pass 3

| List | 3 | 1 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |

After Pass 4

| List | 1 | 3 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |

After Pass 5

## Bubble Sort Algorithm

```
void bsort(int mylist[], int size)
{
    int i, j ;
    for(i=... ){
        for(j=... ){
            if(mylist[j] > mylist[j+1]) {
            // swap mylist[j] & mylist[j+1]
    } } }
}
```

Pass 1




| 3 | 7 | 6 | 5 | 8 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |


| 3 | 7 | 6 | 5 | 1 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| swap |  |  |  |  |  |

Pass 2


| 3 | 6 | 5 | 1 | 7 | 8 | swap |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Pass n-2

| 3 | 1 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| j | i |  |  |  |  |
| 1 | 3 | 5 | 6 | 7 | 8 |
| swap |  |  |  |  |  |

Bubble Sort

Value

## Selection Sort

- Selection sort does away with the many swaps and just records where the min or max value is and performs one swap at the end
- The list/array can again be thought of in two parts
- Sorted
- Unsorted
- The problem starts with the whole array unsorted and slowly the sorted portion grows
- We could find the max and put it at the end of the list or we could find the min and put it at the start of the list
- Just for variation let's choose the min approach


## Selection Sort Algorithm

```
void ssort(int mylist[], int size)
{
    for(i=...){
        int min = i;
        for(j=... ){
            if(mylist[j] < mylist[min]) {
            min = j
        } }
            // swap mylist[i] & mylist[min]
}
```

Pass $1 \quad \min =0$

| 7 | 3 | 8 | 6 | 5 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\min =1$ |  |  |  |  |
|  | $j$ |  |  |  |  |


| 7 | 3 | 8 | 6 | 6 | 5 | 1 | min=1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| i |  | j |  |  |  |  |  |


| 7 | 3 | 8 | 6 | 5 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| i | j |  |  |  |  |
| j |  |  |  |  |  |


| 7 | 3 | 8 | 6 | 5 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| i | $\mathrm{min}=1$ |  |  |  |  |


| 7 | 3 | 8 | 6 | 5 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| i |  |  |  |  |  |
| $\mathrm{min}=5$ |  |  |  |  |  |


| 1 | 3 | 8 | 6 | 5 | 7 | swap |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Pass n-2 min=4


## Selection Sort

## Value

Courtesy of wikipedia.org
List Index

## OPERATIONS ON A SORTED ARRAY

## Insertion to a Sorted Array

- Another option rather than sorting an unordered array us to always insert new data into the correct location of the array
- See example below
- To insert, we must
- Iterate until we find the appropriate location to place the new value
- Make room for the new value by shifting the remaining items back a spot


|  | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | insert(6) | 3 | 7 | 8 |
|  |  |  |  |  |
|  | $-\cdots$ |  |  |  |



## Removing from a Sorted Array

- Erasing / removing item at any location other than the very last item requires us to copy all items behind the removed item to the previous slot

| To delete/remove the item at location 2 requires us to move everyone else up | $\begin{array}{lllllll}0 & 1 & 2 & 3 & 4 & 5\end{array}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 51 | م2 | 53 | 54 | 60 |
|  | ¢ 人 |  |  |  |  |  |
|  | 30 | 51 | 53 | 54 | 60 |  |

School of Engineering

## COMPLEXITY \& RUNTIME

## Time Complexity

- Coming up with AN algorithm to solve a problem is often not TOO hard
- Coming up with a GOOD algorithm to solve a problem can be a bit harder
- We need a way to judge how "GOOD" an algorithm is
- For us "GOOD" will mean how long the algorithm takes to solve the problem
- We will count steps of work and come up with an answer in terms of $n$, where $n$ is the size of the input/problem


## Bubble Sorting

- Recall the bubble sort
- How much work do our nested loops require us to do
- Think of each step/iteration as 1 unit of time/work

$$
\begin{array}{|l|l|l|l|l|l|}
\hline \text { List } 7 & 3 & 8 & 6 & 5 & 1 \\
\hline
\end{array}
$$

Original List is length $\mathbf{N}$ ( $\mathrm{N}=6$ for this example)

Original

List | 3 | 7 | 6 | 5 | 1 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Pass 1 $\qquad$ steps)

| List | 3 | 6 | 5 | 1 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Pass 2 ( $\qquad$ steps)

| List | 3 | 5 | 1 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Pass 3 $\qquad$ steps)

| List | 3 | 1 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |

Pass 4 $\qquad$ steps)

List | 1 | 3 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Pass 5 $\qquad$ steps)

| List | 1 | 3 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$\qquad$ steps)

## Complexity of Sort Algorithms

- Bubble Sort \& Selection Sort
- 2 Nested Loops
- Execute outer loop $n$ times
- For each outer loop iteration, inner loop runs $i$ times.
- Time complexity is proportional to $n^{2}$
- Other sort algorithms can run in time proportional to:
$n * \log _{2}(n)$



## Importance of Time Complexity

- It makes the difference between effective and impossible
- Many important problems currently can only be solved with exponential run-time algorithms (e.g. $O\left(2^{n}\right)$ time)
- Usually algorithms are only practical if they run in polynomial time (e.g. O(n) or O(n²) etc.)

| $\mathbf{N}$ | $\mathbf{O}(\mathbf{1})$ | $\mathbf{O}\left(\log _{2} n\right)$ | $\mathbf{O}(\mathbf{n})$ | $\mathbf{O}\left(\mathbf{n}^{*} \log _{2} \mathbf{n}\right)$ | $\mathbf{O}\left(\mathbf{n}^{2}\right)$ | $\mathbf{O}\left(\mathbf{2}^{\mathbf{n}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 1 | 2 | 2 | 4 | 4 |
| 20 | 1 | 4.3 | 20 | 86.4 | 400 | $1,048,576$ |
| 200 | 1 | 7.6 | 200 | $1,528.8$ | 40,000 | $1.60694 E+60$ |
| 2000 | 1 | 11.0 | 2000 | $21,931.6$ | $4,000,000$ | \#NUM! |

## SOLUTIONS

## Task 12a - Sol

- What programming issues (mechanics) should you think about?
- Do we just need to track the maximum VALUE or the INDEX of the maximum value?
- Given that you can move the maximum number to the end of the array, how could this be used to SORT the entire array?
- Repeat the process for the first n -1 elements, then repeat for the first n - 2 elements, etc.

```
int main() {
    // setup array with data
    int n, val, data[100];
    cin >> n;
    for(int i=0; i < n; i++)
        { cin >> data[i]; }
    // now perform the given task
    int cmax = 0;
    for(int i=1; i < n; i++) {
        if(data[i] > data[cmax]){
                cmax = i;
        }
    }
    // swap the max and end element
    int temp = data[n-1];
    data[n-1] = data[cmax];
    data[cmax] = temp;
    // Print out results
    for(int i=0; i < n; i++){
        cout << data[i] << " ";
    }
    cout << endl;
    return 0;
}
```


## Task 12b - Sol

- What programming issues (mechanics) should you think about?
- Do we just need to track the maximum VALUE or the INDEX of the maximum value?
- Given that you can move the maximum number to the end of the array, how could this be used to SORT the entire array?
- Repeat the process for the first n -1 elements, then repeat for the first n - 2 elements, etc.

```
int main() {
    // setup array with data
    int n, val, data[100];
    cin >> n;
    for(int i=0; i < n; i++)
        { cin >> data[i]; }
    // now perform the given task
    for(int i=0; i < n-1; i++) {
        if(data[i] > data[i+1]){
                int temp = data[i];
                data[i] = data[i+1];
                data[i+1] = temp;
        }
    }
    // Print out results
    for(int i=0; i < n; i++){
        cout << data[i] << " ";
    }
    cout << endl;
    return 0;
}
```

