Compact Modeling of Complex Smart Grid Dynamics: Application to Change Point Detection of Voltage Collapse

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The smart grid has many facets

- Large movement of power across geographically large areas
- Economic dispatch
- Line overloading
- Stochastic fluctuations induced by renewables
- Storage elements
- Integration with electric vehicles
- Phasor Measurement Unit (PMU) technology
- Privacy concern over smart meters
- Security ("black energy")
- etc.

Lots of mathematics & new concepts

- Is that all???

Plan of Action

- The catalyst: Evidence of fractal PMU signals
  - Review of Detrented Fluctuation Analysis
  - Texas & EPFL (Switzerland) normal PMU data
- Inadequacy of swing model
- *Why* are PMU signals fractal???
  - Fractional dynamics load modeling
  - Load aggregation
- Voltage stability
  - The loads are the “villains”
- Early warning of imminent blackout
  - *Increase* of Hurst exponent before blackout
  - *Statistical confirmation by Kendall tau*

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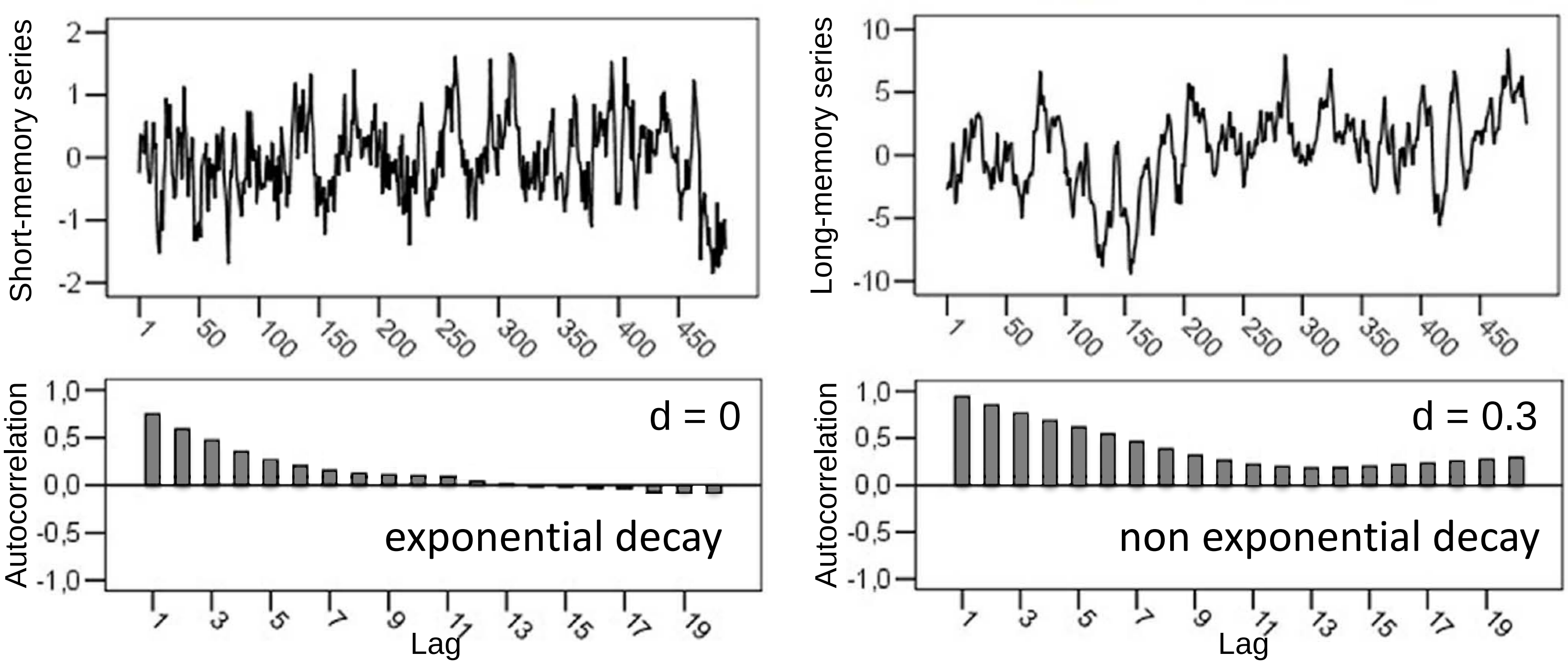
Long-Range Dependence or Memory (in PMU data)

- Long-range memory is one of the characteristics of fractal patterns. It relates to slow decay of the correlation as the lag between samples increase.



Long-Range Dependence or Memory

- There are several parameters that quantify the severity of the fractal behavior in a time series:
  - Number of incrementation or differentiation steps (d):

$$\text{ARFIMA}: \quad \left(1-\sum_{i=1}^{p}\phi_i L^i\right)(1-L)^d X_t = \left(1+\sum_{i=1}^{q}\theta_i L^i\right)\varepsilon_t, \quad \phi_1 = \text{AR}(1)$$

  - Power Spectral Density exponent (β):

$$S(f) \propto \frac{1}{f^{\beta}}$$

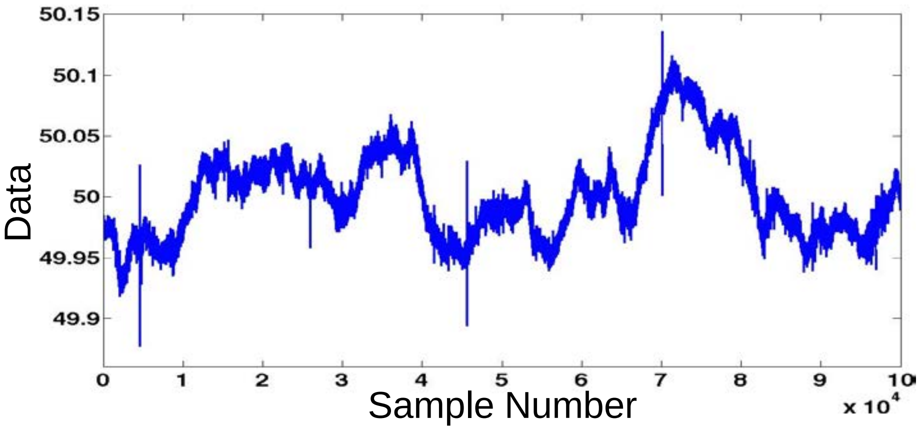
  - Hurst exponent (α):
    It relates to the autocorrelation of time series and the rate at which these decrease as the lag increases.

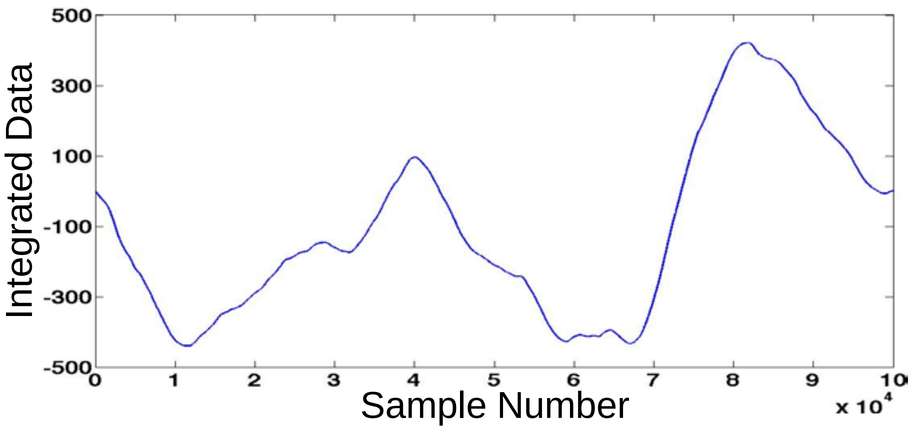
(2d+1)/2 = α = (β+1)/2

Detrended Fluctuation Analysis (DFA)

- Steps:
  1. Subtract average and integrate the data set:

$$y_{int}(k) = \sum_{i=1}^{k} (y(i) - y_{avg})$$

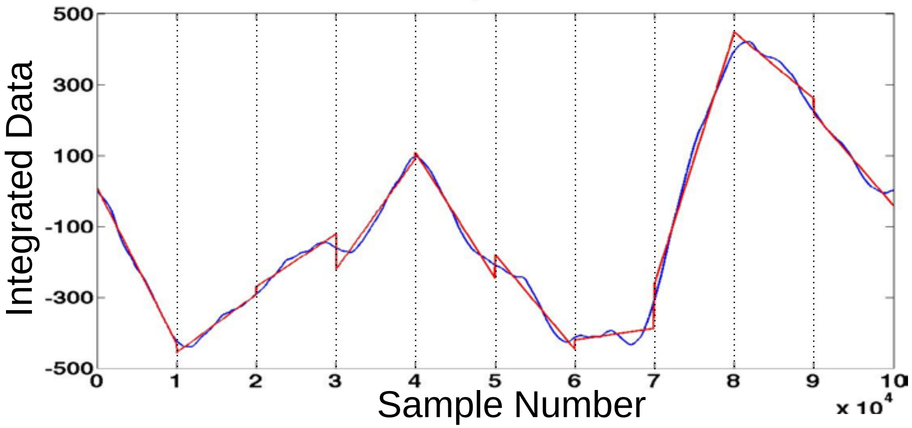


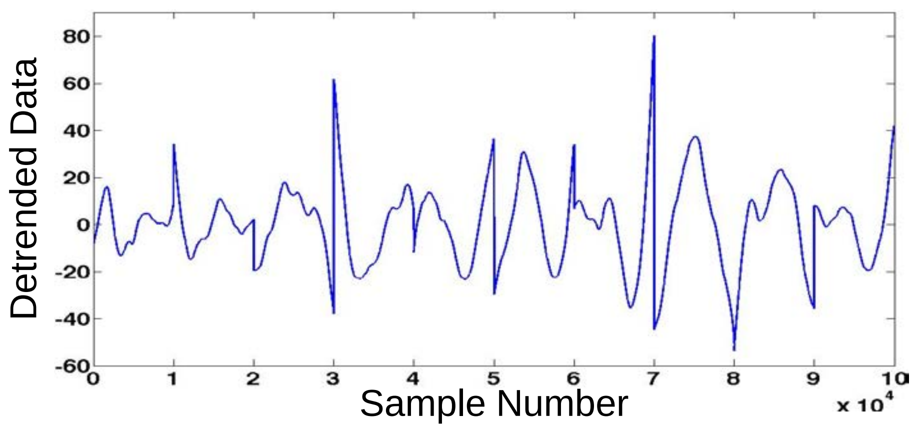


Detrended Fluctuation Analysis (DFA)

2. Divide the data into *n* equal-sized boxes and find the Linear Least Squares (LLS) line inside each box.
3. Subtract the LLS fitting from the integrated data to generate the detrended data:

$$y_{int}(k) - y_n(k) = y_d(k)$$





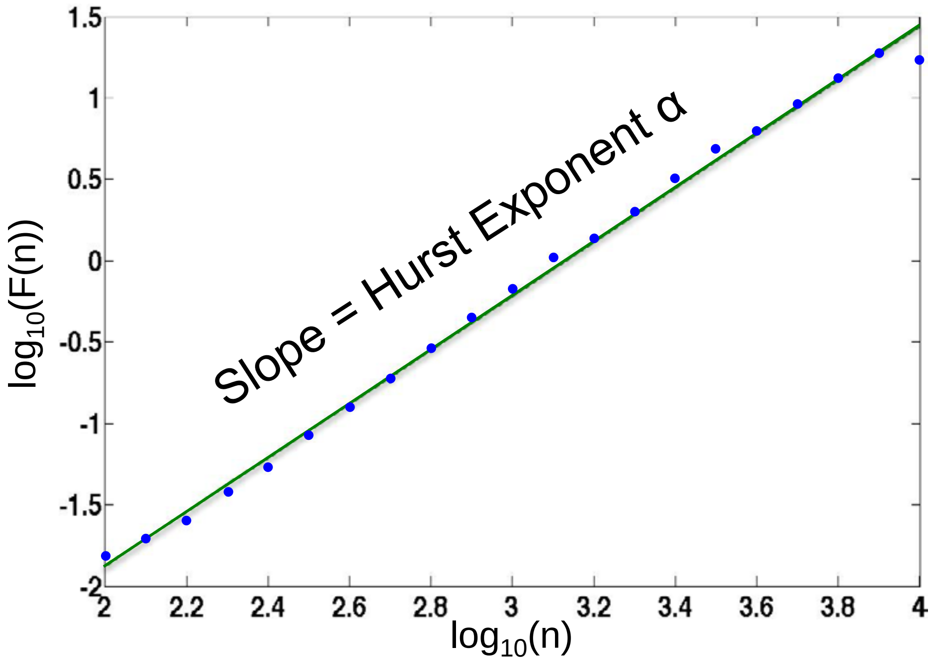
Detrended Fluctuation Analysis (DFA)

4. Find the Root Mean Square (RMS) fluctuation of the detrended data:

$$F(n) = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(y_d(k)\right)^2}$$

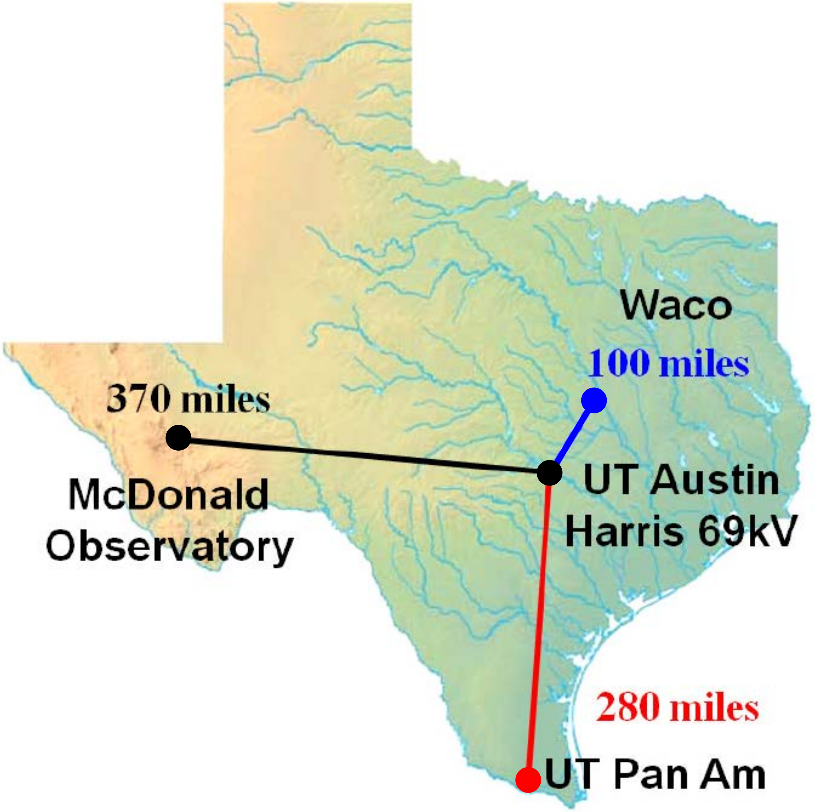
4. The second and third steps are repeated at different box sizes:

$$\alpha = \lim_{n\to\infty} \frac{\log_{10}(F(n))}{\log_{10}(n)}$$

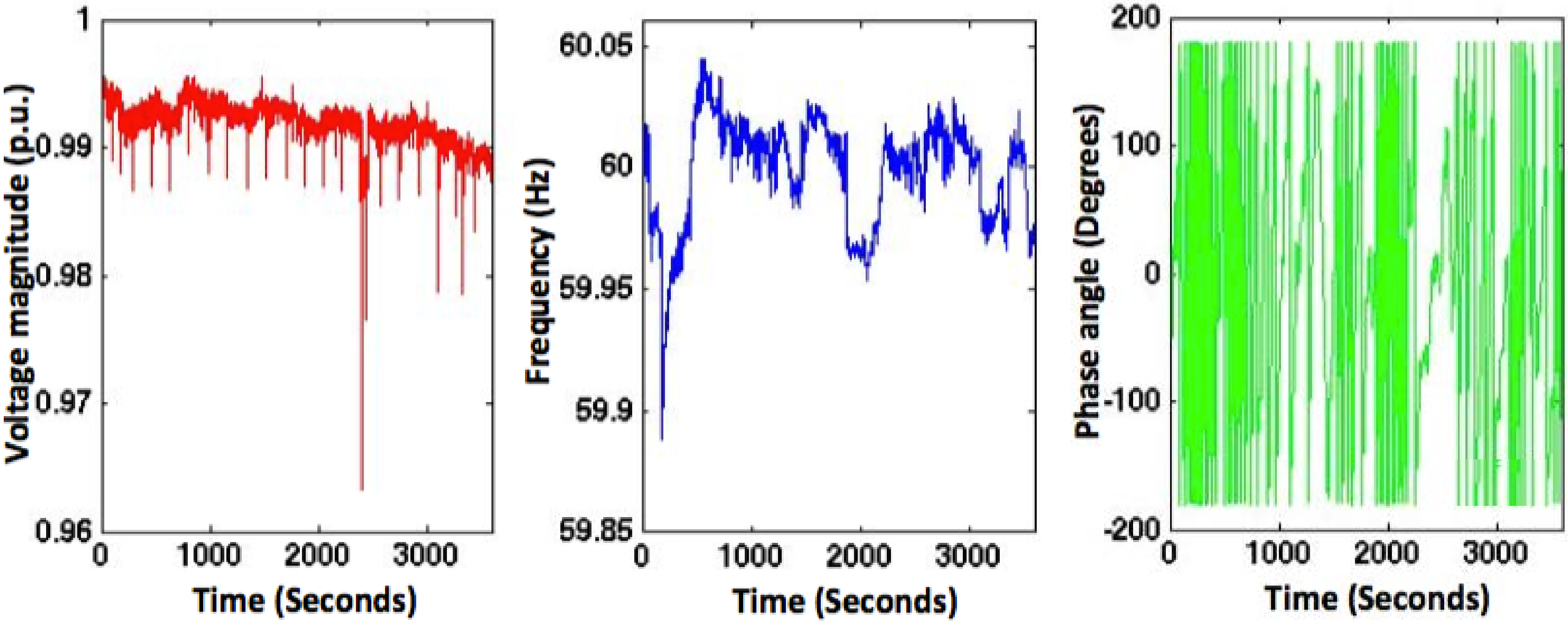


Texas Synchrophasor Network

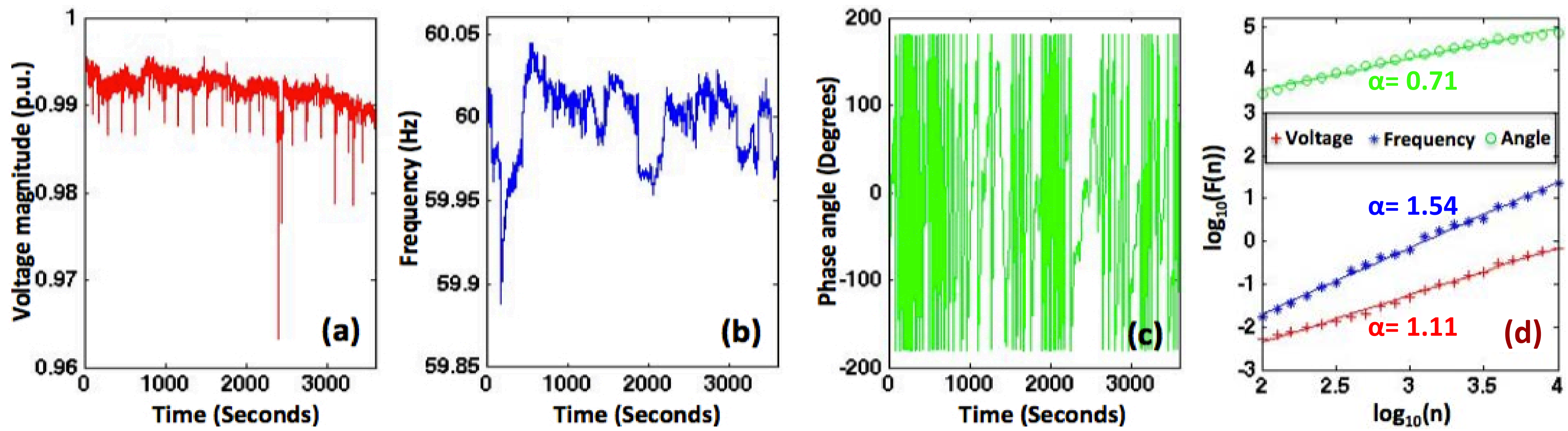
- Several PMUs are installed at 120V and 69KV over several locations:
  - Baylor University (Waco),
  - Harris Substation, and
  - McDonald Observatory.
- The data we analyzed here are
  - voltage magnitude,
  - frequency, and
  - phase angle.
- The sampling rate of the data is 30 samples/second.



PMU Time Series (Texas)



Details of Long-range dependence in PMU data



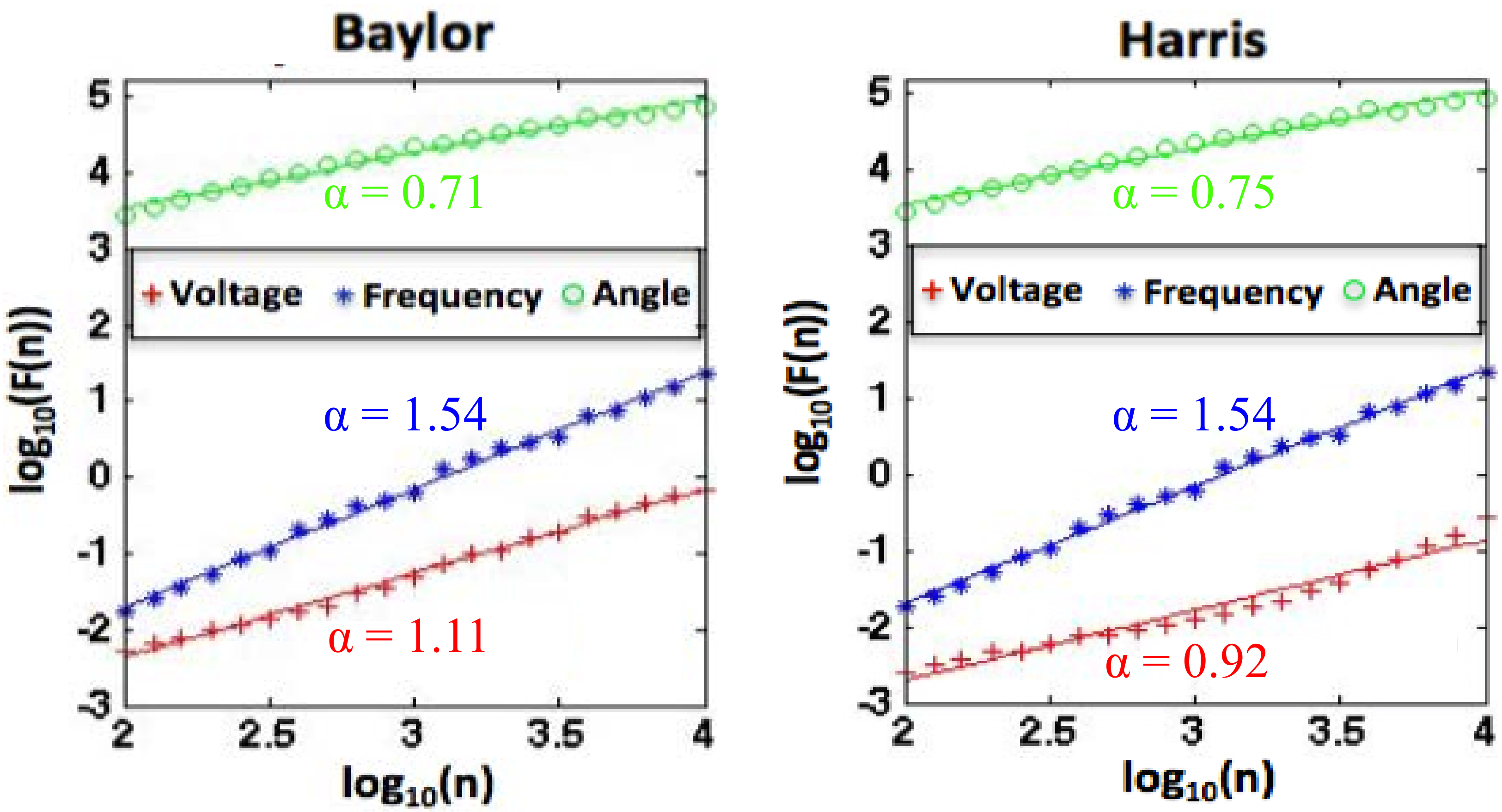
- Voltage Magnitude (V) → $\alpha \approx 1.00$ → Pink noise $(1/f)$
- Frequency (f) → $\alpha \approx 1.50$ → Brownian noise $(1/f^2)$
- Phase Angle (θ) → $\alpha \approx 0.70$ → Long-range/Power-law

$$\beta = 2\alpha - 1$$

Hurst Exponent (Texas)

$$0.5 \le \alpha \le 1: \quad \text{long range with power law}$$

$$\alpha > 1: \quad \text{long range but no power law}$$



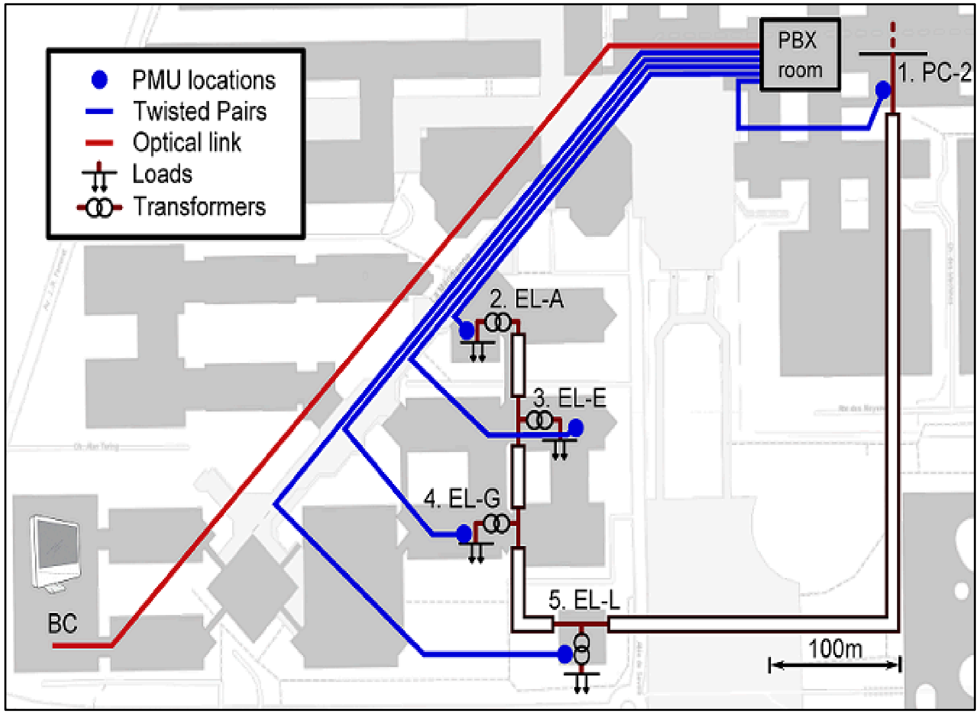
Hurst Exponent (Texas)

Data Set	Baylor			Harris			McDonald		
	V	f	θ	V	f	θ	V	f	θ
#1	1.11	1.54	0.71	0.92	1.54	0.75	1.32	1.54	0.74
#2	1.11	1.53	0.66	0.81	1.53	0.63	1.30	1.53	0.64
#3	1.05	1.45	0.67	0.91	1.45	0.76	1.37	1.45	0.73
#4	0.91	1.49	0.63	0.89	1.49	0.64	1.32	1.49	0.64

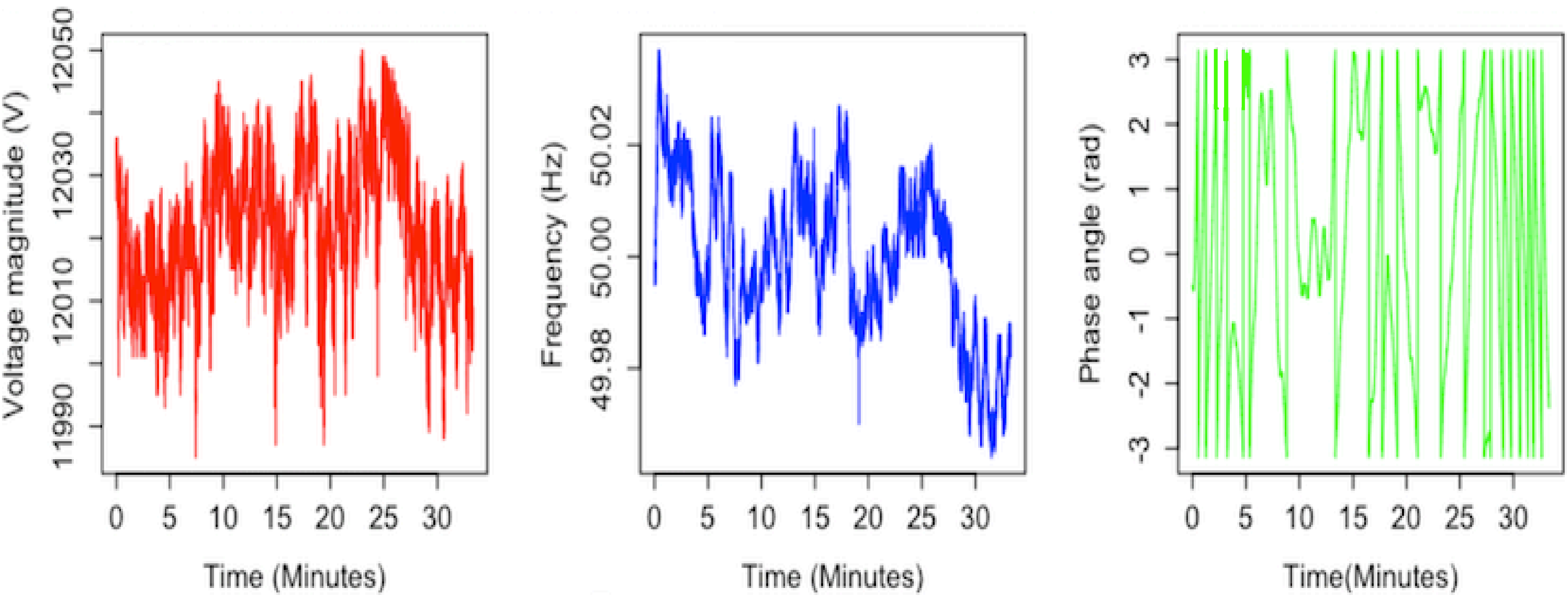
- Frequency and angle data are consistent across the 3 stations.
- Voltage definitely has higher Hurst exponent at McDonald… Why???
  - Proximity of wind farm?
  - Is the Hurst exponent of voltage a sign of *penetration of renewables* in the larger grid?

PMU-Based Monitoring in EPFL
(Ecole Polytechnique Fédérale de Lausanne)

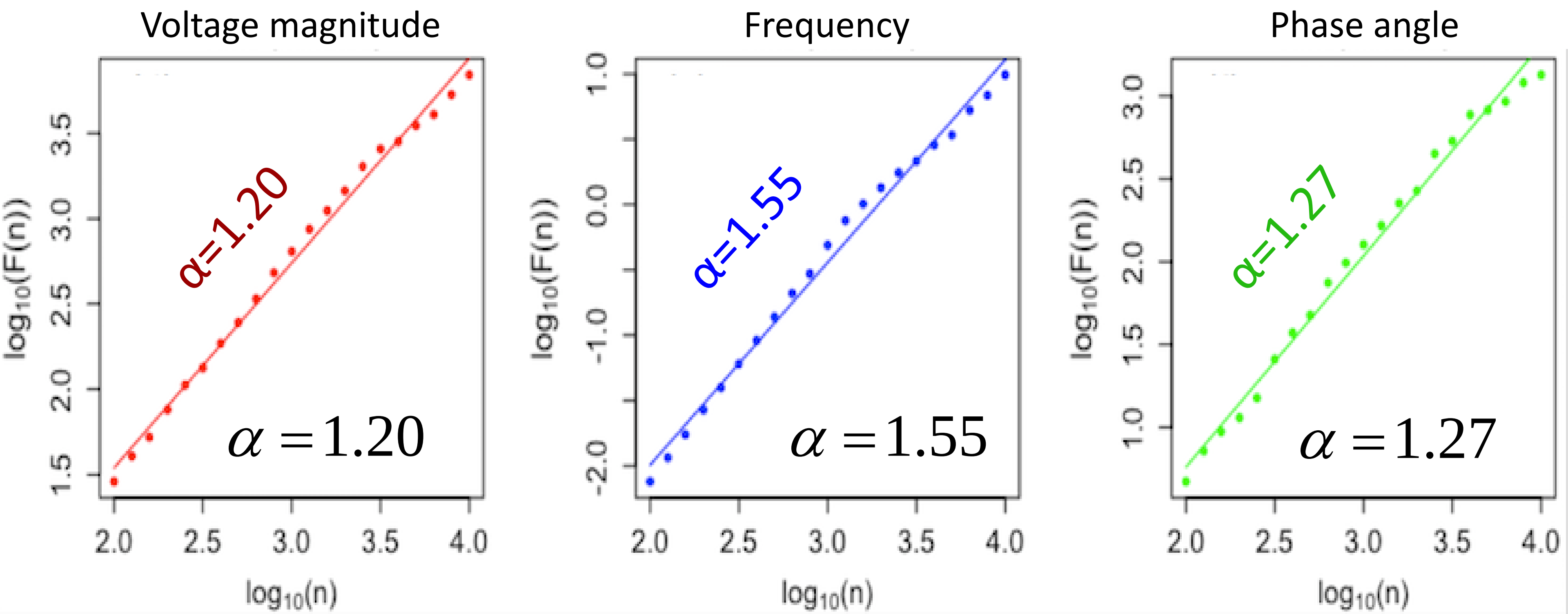
- PMUs installed in EPFL campus perform real time monitoring of the EPFL pilot smart grid.
- The PMUs were installed on medium voltage buses (12KV)
- The sampling rate is 50 samples/second



PMU Time Series (EPFL)

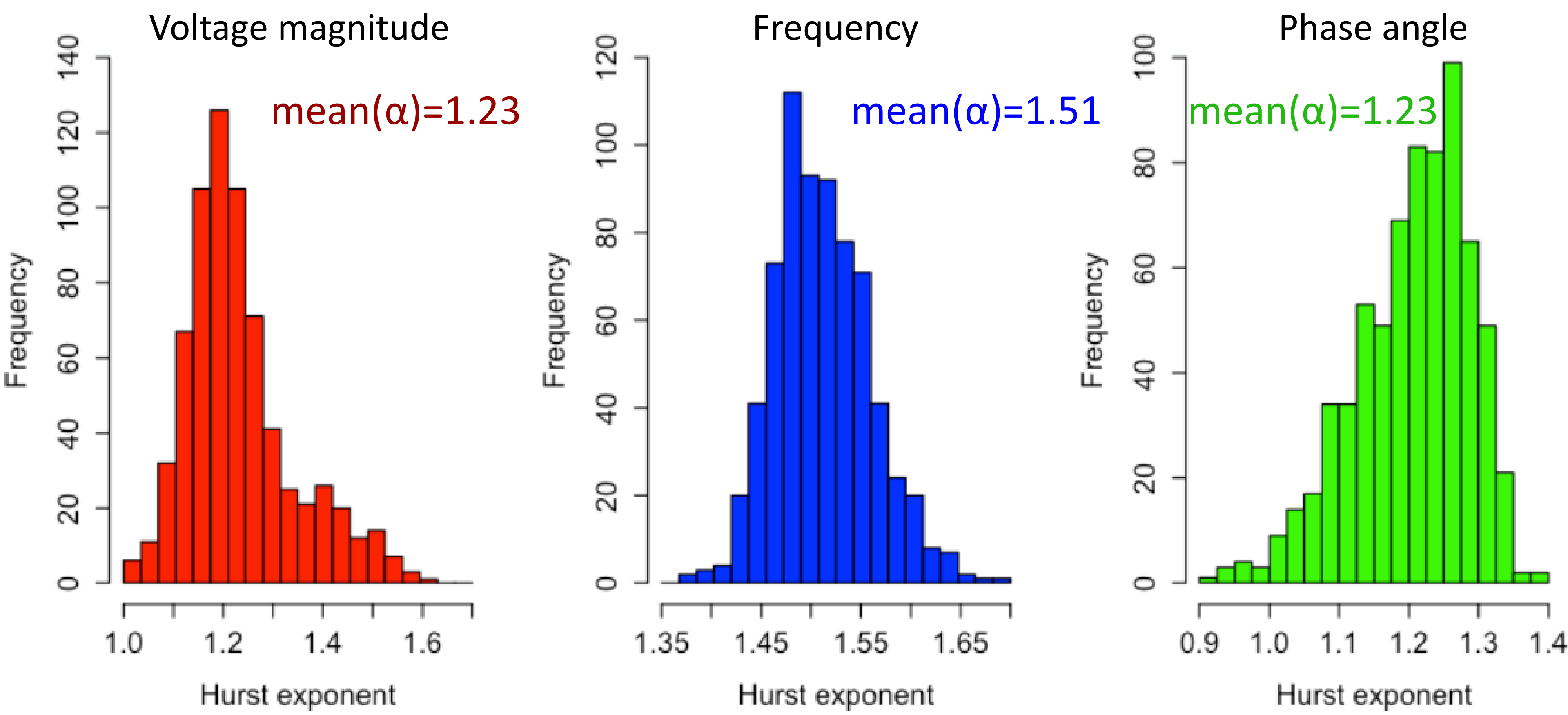


Hurst Exponents (EPFL)



Amazing consistency between the frequency α in Texas (1.54) and Switzerland (1.55)

Hurst Exponent Histograms (EPFL)



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Fractal Dimension

- Capacity (Box Counting)

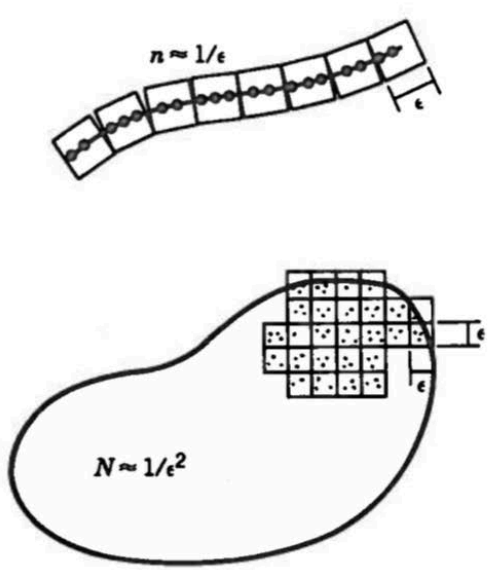


Figure 6-4 Covering procedure for linear and planar distributions of points.

$$d_C = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)}$$

where $N(\epsilon)$ - number of cubes to cover a surface
ϵ - cubes with sides of length ϵ

Said to be fractal for non-integer dimension d_C

Moon, F. C. Chaotic and Fractal Dynamics: An Introduction to Applied Scientist and Engineers. 1992.

Measures of Fractal Dimension

- **Pointwise Dimension**
  - Time-sample the trajectory to set of N points
  - Place a sphere of radius r at some point and count the number of points N(r) within sphere
  - Probability of finding a point in sphere of radius r

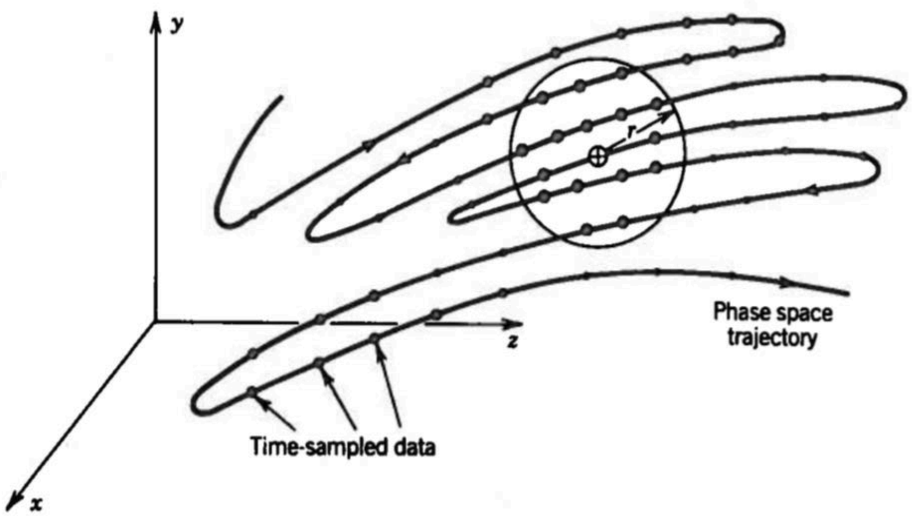
$$P(r) = \frac{N(r)}{N_0} \approx a r^{d_P}$$

Pointwise dimension

$$d_P = \lim_{r \to 0} \frac{\log P(r; x_i)}{\log r}$$

Averaged pointwise dimension

$$\hat{d}_P = \frac{1}{M} \sum_{i=1}^{M} d_P(x_i)$$



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Measures of Fractal Dimension

- **Correlation dimension (Grassberger and Proccacia, 1983)**
  - Discretizes trajectory to set of N points
    - One can also create a pseudo-phase-space
  - Calculates distances between pairs of points x_i and x_j $\rho(x_i, x_j) = |x_i - x_j|$

Correlation function:

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \begin{pmatrix} \text{number of pairs } (i,j) \\ \text{with distances } s_{ij} < r \end{pmatrix}$$

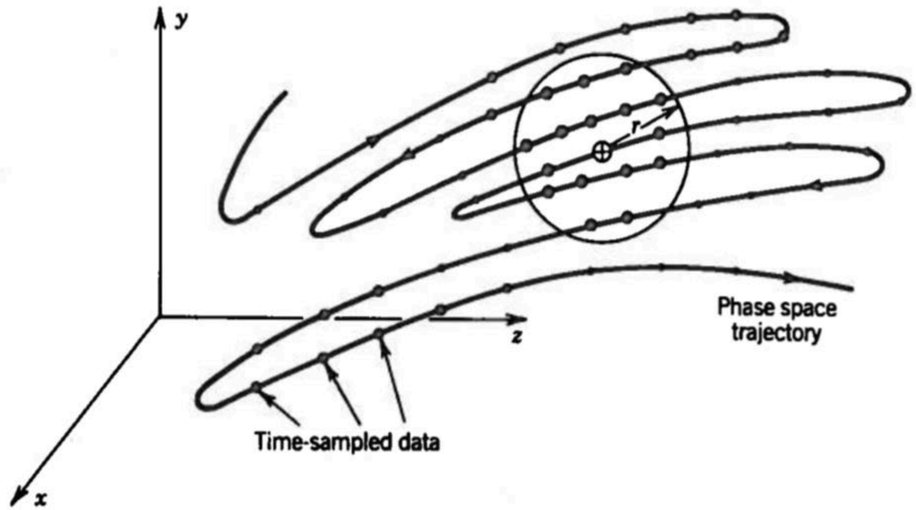
Power law dependence on r

$$\lim_{r\to 0} C(r) = ar^d$$

Fractal dimension:

$$d_G = \lim_{r\to 0} \frac{\log C(r)}{\log r}$$

*slope of the log C(R) vs log r curve



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Measures of Fractal Dimension

- **Effective implementation**

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \begin{pmatrix} \text{number of pairs } (i,j) \\ \text{with distances } s_{ij}<r \end{pmatrix}$$

$$C(r) = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} \sum_{j>i}^{N} H\left(r - \rho(x_i - x_j)\right)}{\frac{1}{2}N(N-1)}$$

Where:

Heaviside function: $$H(s) = \begin{cases} 1, & s \geq 0 \\ 0, & s < 0 \end{cases}$$

Distance: $$\rho(x_i, x_j) = |x_i - x_j|$$

Bounds:

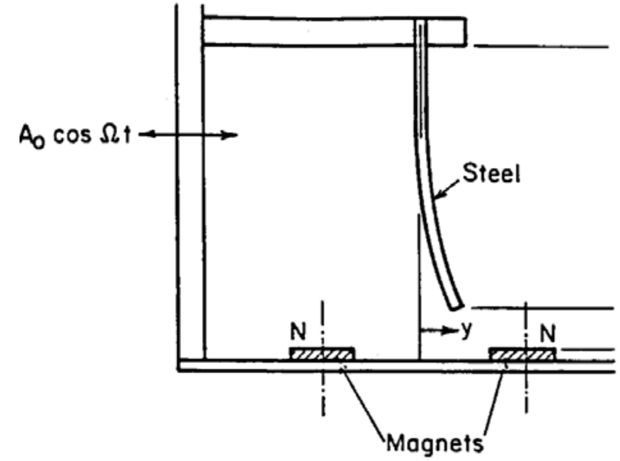
$$r_{max} = \max_{i,j} \rho(x_i, x_j) \qquad r_{min} = \max_{i,j} \rho(x_i, x_j)$$

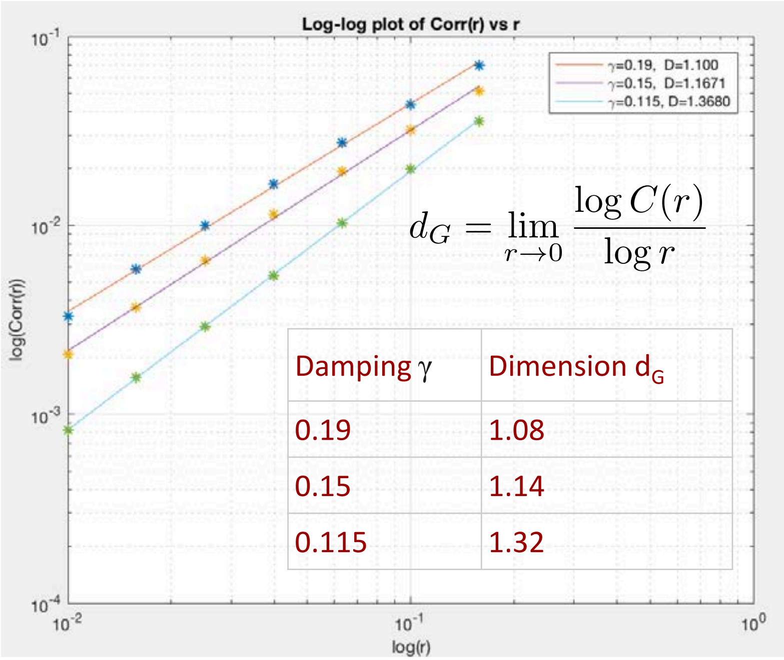
Only consider computations for C(r) within bounds $r_{min} \leq r \leq r_{max}$

Strange Attractor Example

- **Duffing Strange Attractor**
  - **Two-well potential strange attractor**

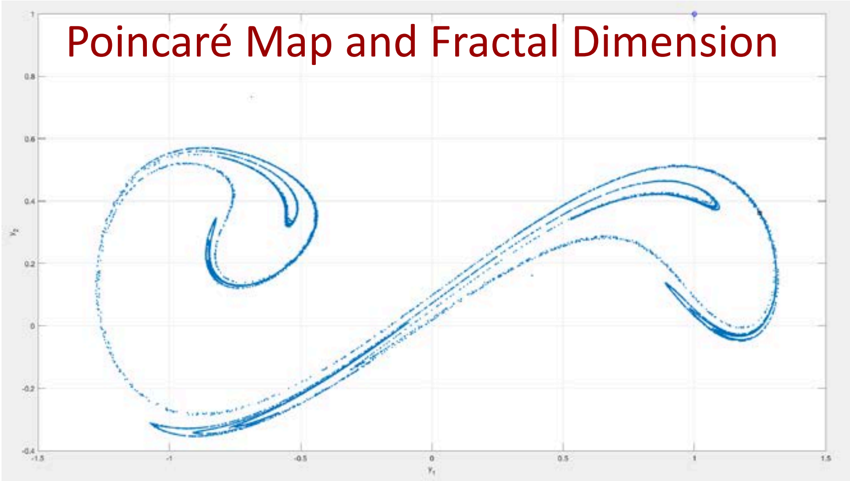
$$\ddot{y} + \gamma \dot{y} - \frac{1}{2}\left(1 - x^2\right)x = A_0 \cos \Omega t$$





$$d_G = \lim_{r \to 0} \frac{\log C(r)}{\log r}$$

Damping γ	Dimension d_G
0.19	1.08
0.15	1.14
0.115	1.32



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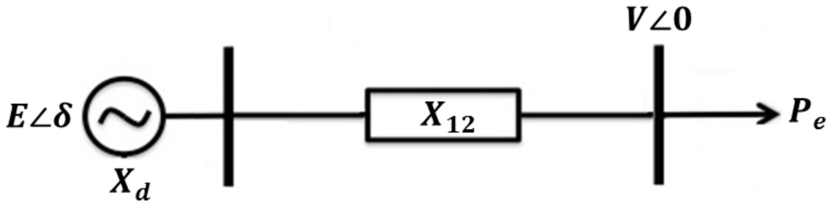
Swing Equation Model



$$P_m(t) = P_e(t) + D\left(1 + \dot{\delta}(t)\right) + M\ddot{\delta}(t)$$

$$P_e(t) = \frac{E_a V_2(t)}{X_d + X_{12}} \sin(\delta(t)), \quad V_2(t) = 1 + \mathcal{N}(0, \sigma_v)$$

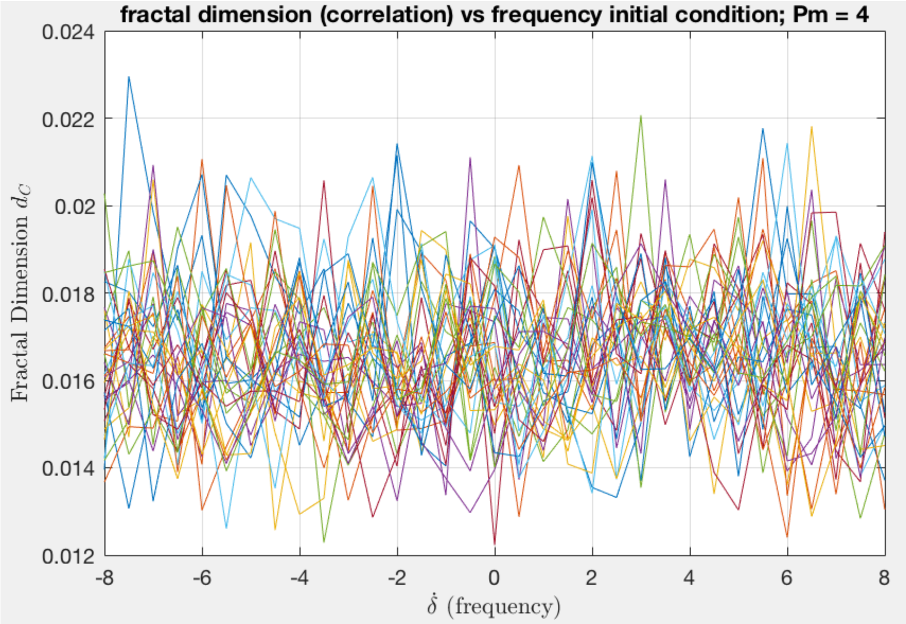
Noise perturbation at V_2

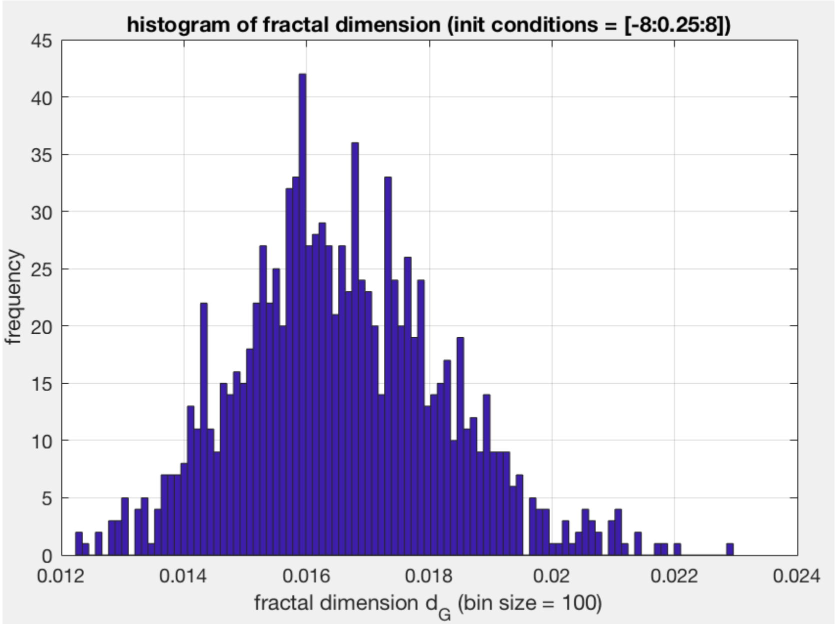
where

- P_m - Mechanical power
- P_e - Electrical power
- D - Damping coefficient
- M - Moment of inertia of the rotor
- δ - Phase angle of the rotor with respect to the rotating frame
- E_a - Generator voltage
- X_d - Internal resistance of generator
- X_{12} - Reactance of transmission line
- V_2 - Load bus voltage magnitude

Swing Equation Simulation Results

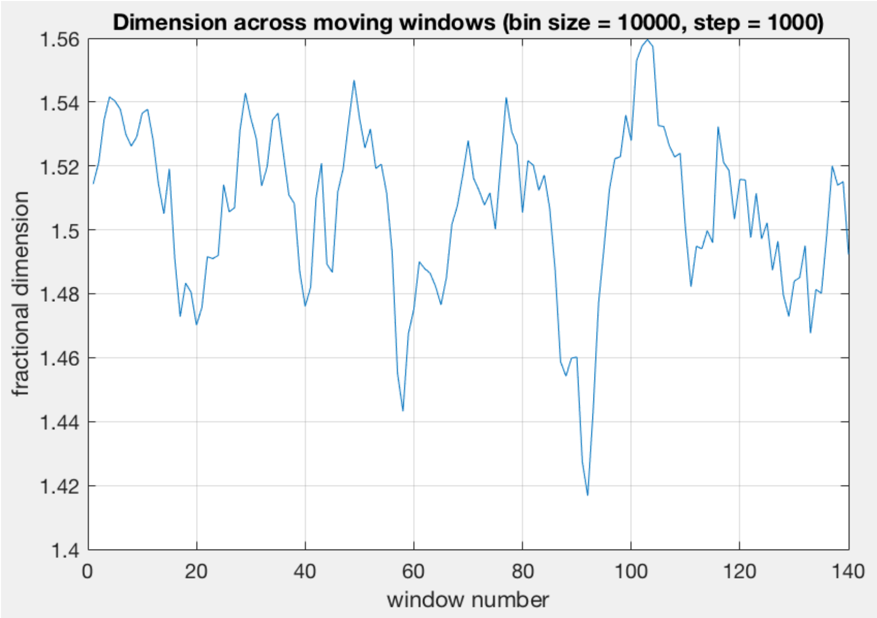
Noise added at V_2: $V_2(t) = 1 + N(0, \sigma_v)$, $\sigma_v = 0.01$

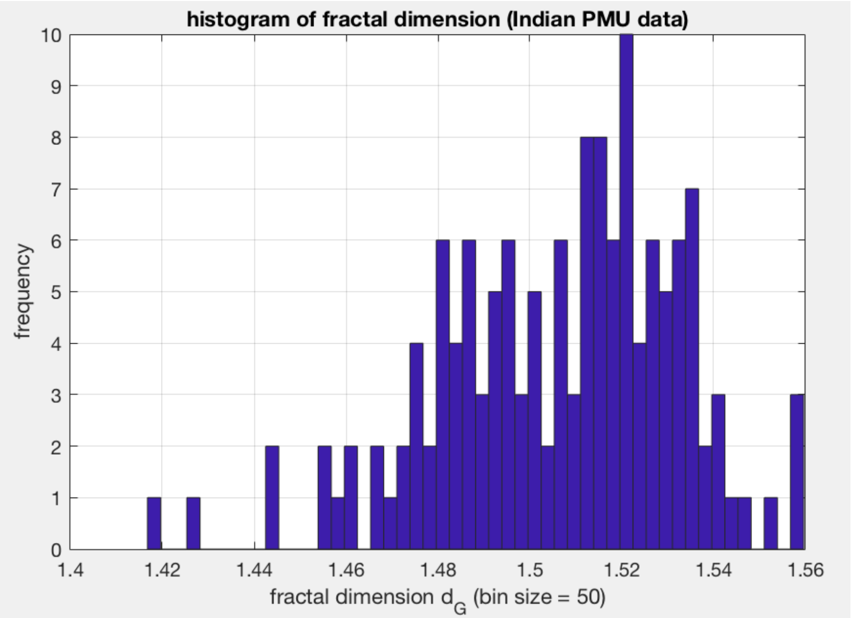




initial conditions	[-0.8:0.25:0.8]
N	10001
size(alpha)	1089
mean(d)	0.016572
std(d)	0.001675

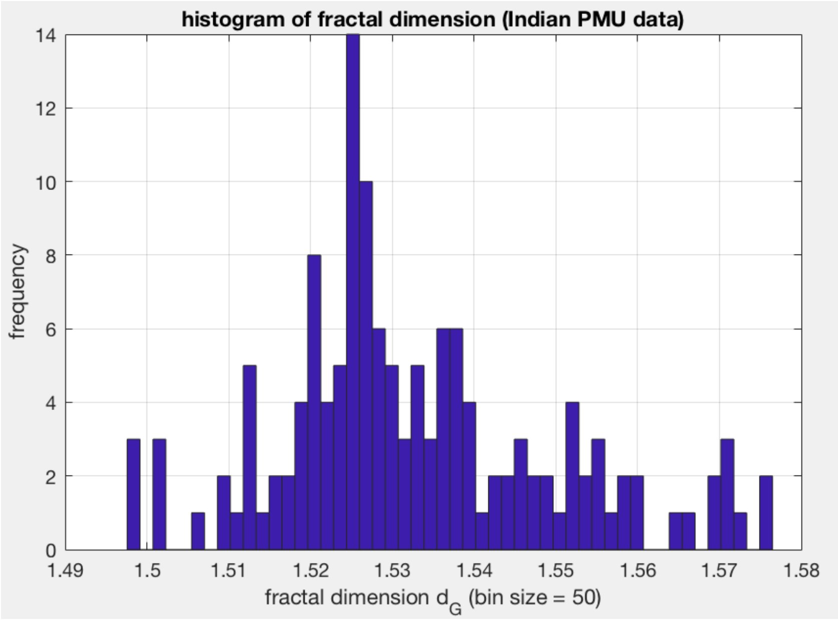
Indian Blackout PMU Time Series Data





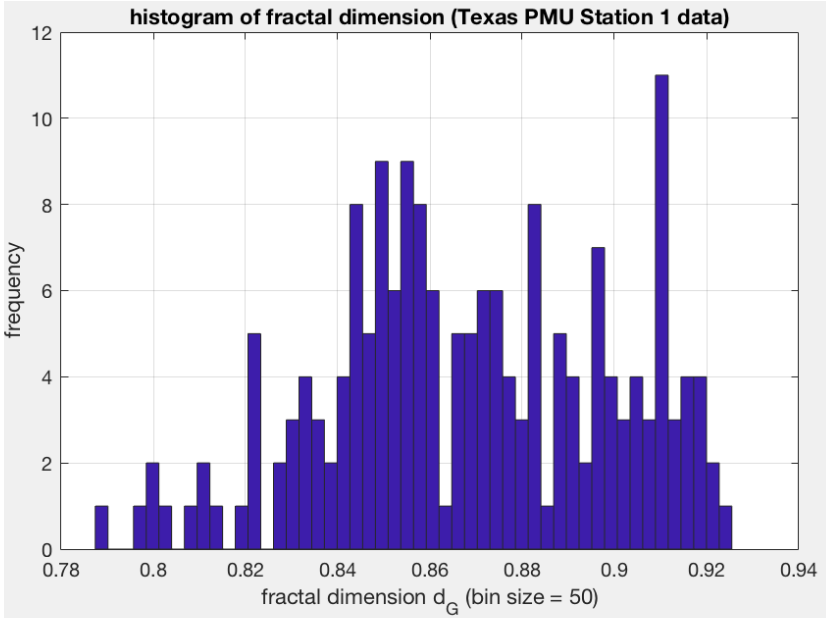
N	10000
size(alpha)	140
mean(d)	1.506059
std(d)	0.026241

Indian Blackout PMU Time Series Data



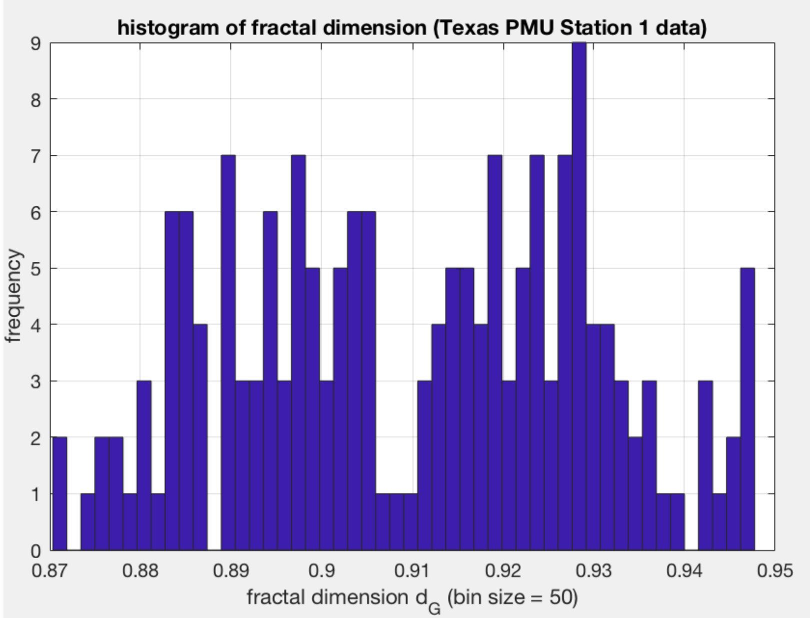
N	20000
size(alpha)	140
mean(d)	1.532877
std(d)	0.017066

Texas (Station 1) PMU Time Series Data



N	10000
size(alpha)	180
mean(d)	0.868504
std(d)	0.030948

Texas Station 1 PMU Time Series Data



N	20000
size(alpha)	176
mean(d)	0.910081
std(d)	0.019405

Statistical Test

- **Kolmogorov-Smirnov Test (Two-sampled)**
  - tries to determine if two datasets differ significantly
  - has the advantage of making no assumption about the distribution of data.

$$D_{n,m} = \sup_x |F_{1,n}(x) - F_{1,m}(x)|$$

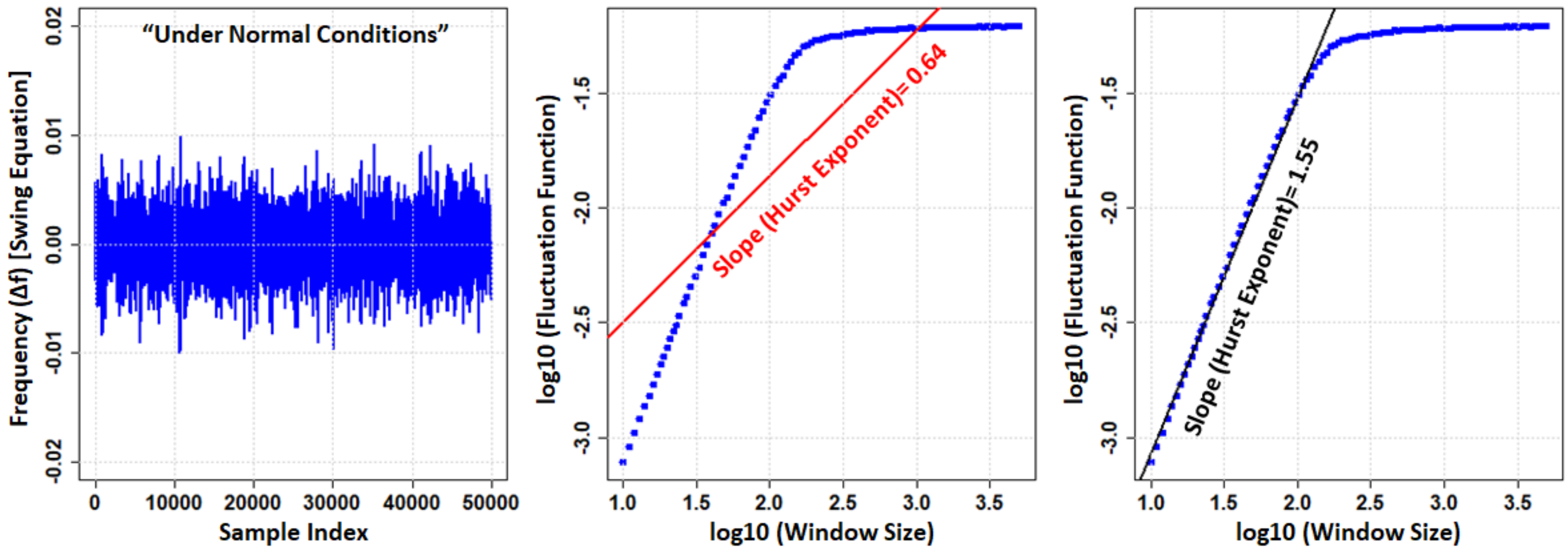
where: $F_{1,n}$, $F_{2,m}$ - empirical distributions with n and m sizes for the and second samples, respectively

Null hypothesis is rejected at level α

$$D_{n,m} > c(\alpha)\sqrt{\frac{(n+m)}{nm}}$$

- The K-S test was performed on the simulated swing equation data (with Gaussian noise (sigma = 0.01) vs. the PMU data (for both Indian blackout and Texas station 1)
- Both tests **reject** the null hypothesis (that the two sample sets are from the same distribution) at the 5% significance level

Hurst Exponent Analysis of the Swing Equation



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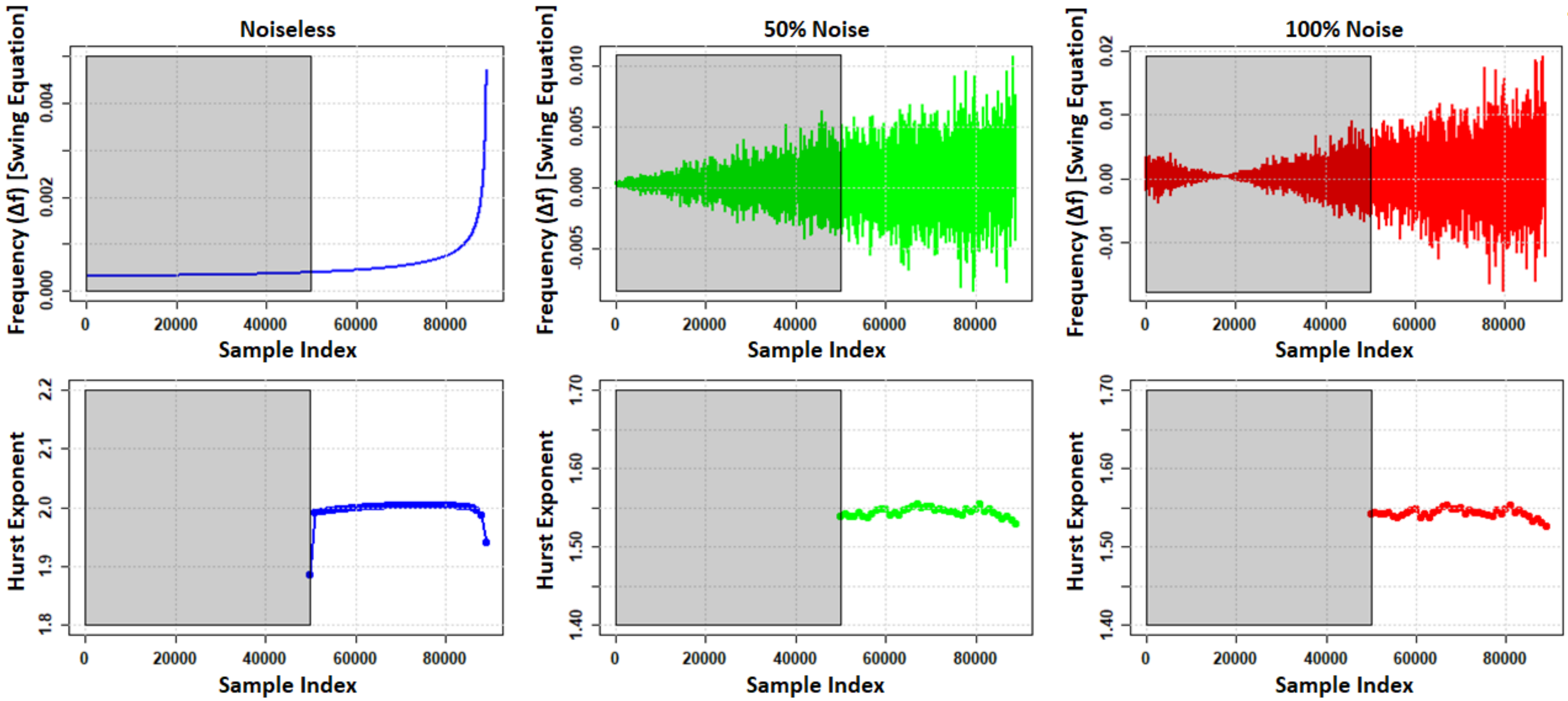
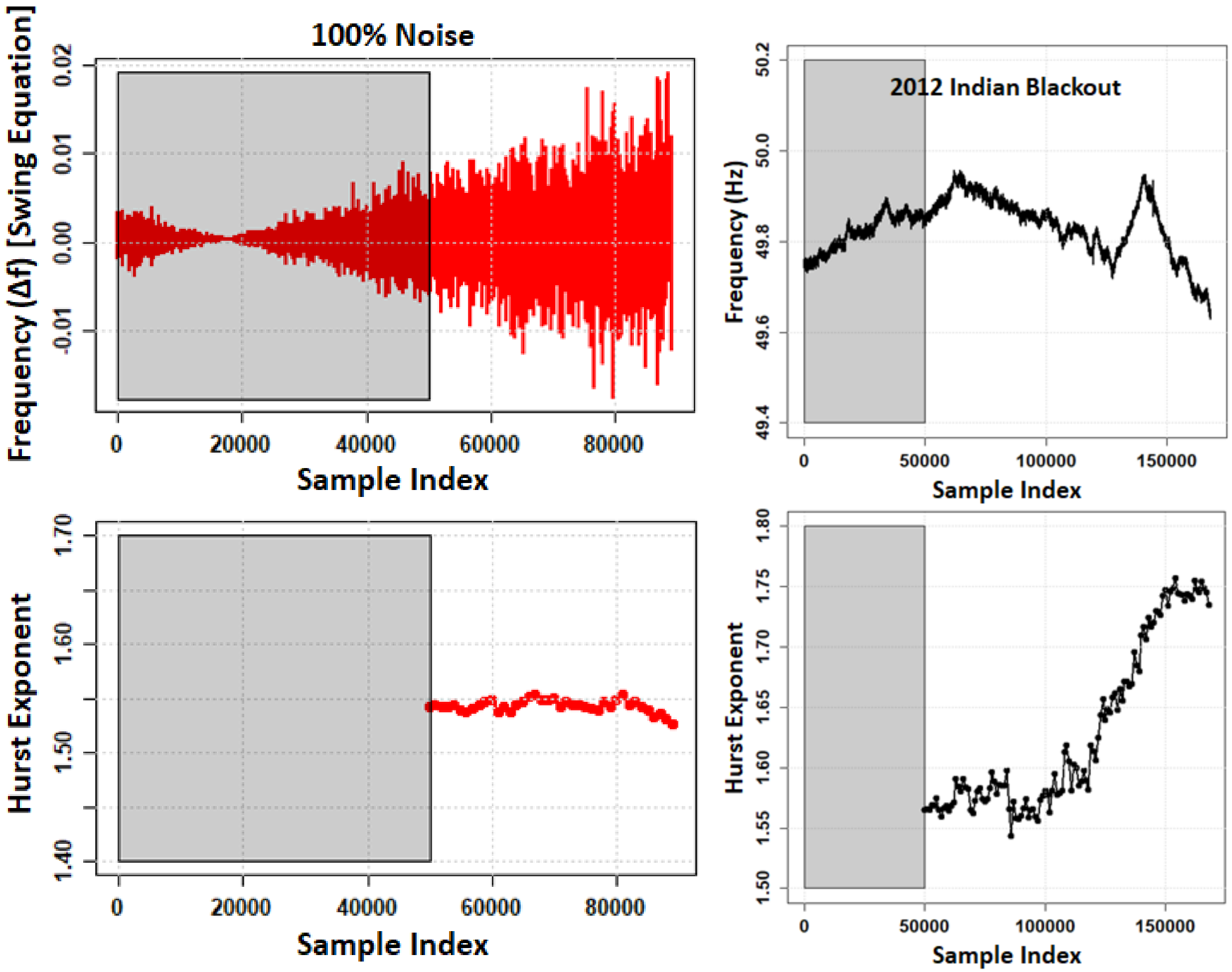
Hurst Exponent Analysis of the Swing Equation



- Hurst exponent of the frequency remains almost constant near the bifurcation.
- The Hurst exponent is equal to 2 for the noiseless frequency and approximately 1.55 for the frequency time series with 50% and 100% noise
- These results show that driving the swing equation to the unstable region by increasing Pm does not reproduce the increasing trend in Hurst exponent as in the 2012 Indian blackout.

Hurst Exponent Analysis of the Swing Equation



- These results show that driving the swing equation to the unstable region by increasing Pm does not reproduce the increasing trend in Hurst exponent as in the 2012 Indian blackout

- The swing equation with added noise do not show an increase in the Hurst exponent like the one in the Indian blackout.

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Static versus Dynamic Load Models

- <u>Static load model:</u>

$$P_L = K_p V_L^{p_v} \qquad Q_L = K_q V_L^{q_v}$$

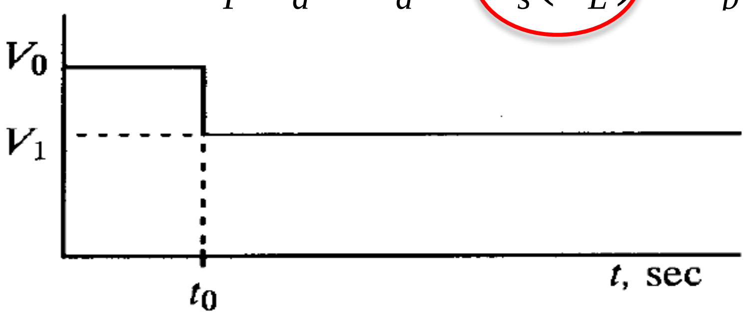
  - Constant Power ➡ $p_v = q_v = 0$
  - Constant Current ➡ $p_v = q_v = 1$
  - Constant Impedance ➡ $p_v = q_v = 2$

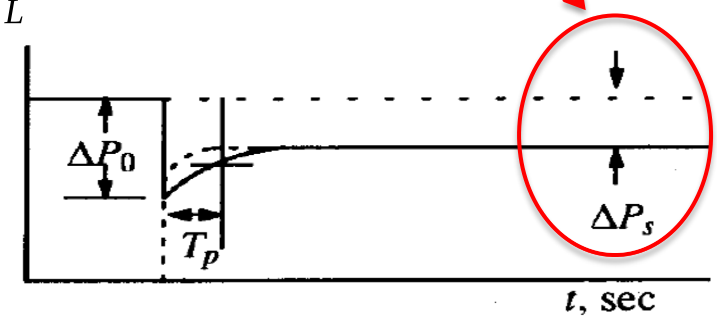
- <u>Dynamic load model (Hill):</u>

$$T_P \dot{P}_d + P_d = P_s(V_L) + k_p(V_L)\dot{V}_L$$

Should be

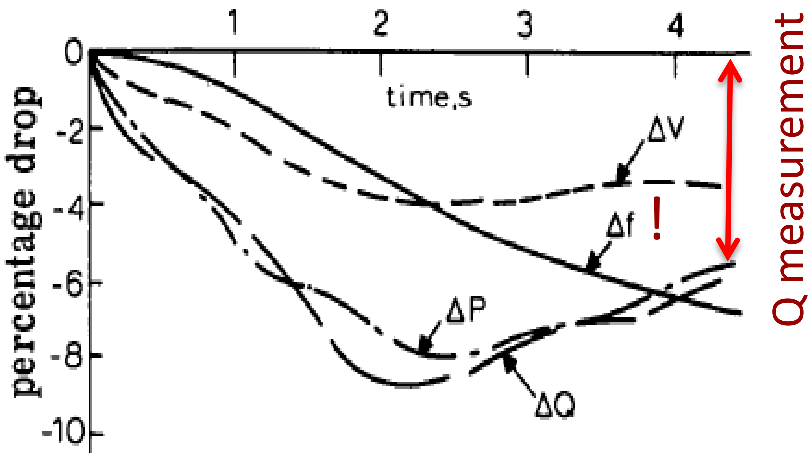
$$P_L(V_L, \omega)$$



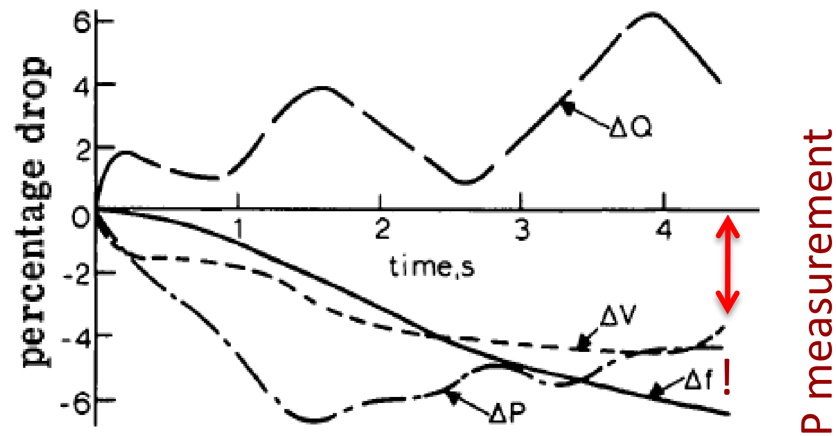


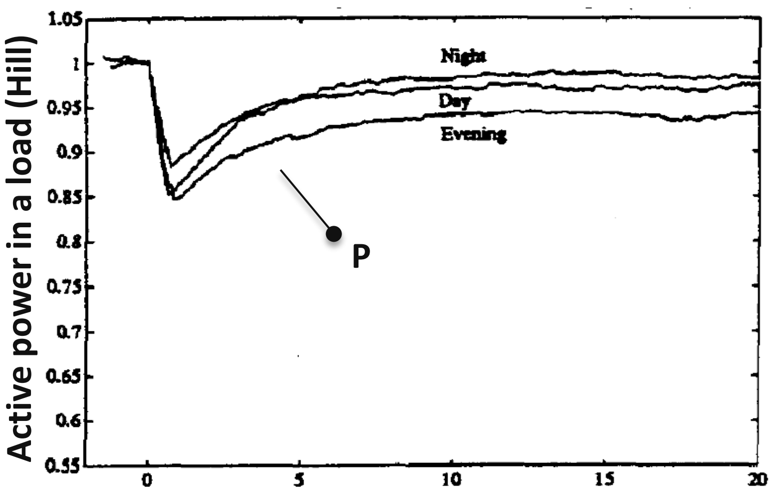
Berg Data-Driven Load Modeling Experiment
in a real microgrid

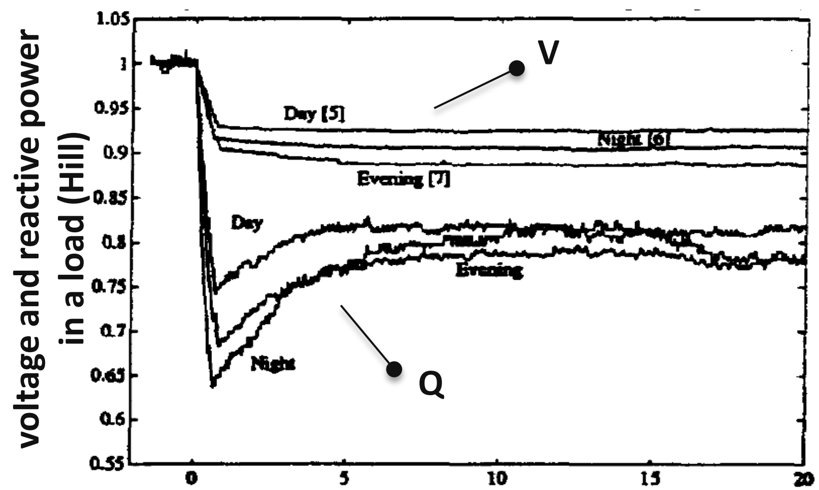
Electric Reduction Furnace (Berg)



Mechanical Plant (Berg)







Berg load model involves frequency to a noninteger exponent

$$\vec{S}_L = P_L + jQ_L \qquad P_L = K_P V_L^{p_v} \omega^{p_\omega} \qquad Q_L = K_Q V_L^{q_v} \omega^{q_\omega}$$

Load Type	p_v	p_ω	q_v	q_ω
Filament lamp	1.6	0	0	0
Fluorescent lamp	1.2	-1.0	3.0	-2.8
Heater	2.0	0	0	0
Induction motor (HL)	0.2	1.5	1.6	-0.3
Induction motor (FL)	0.1	2.8	0.6	1.8
Reduction furnace	1.9	-0.5	2.1	0
Aluminum plant	1.8	-0.3	2.2	0.6
Regulated aluminum plant	2.4	0.4	1.6	0.7

Impedance Describing Function

$$\vec{Z}_L = \frac{\vec{V}_L}{\vec{I}_L} = \frac{\vec{V}_L \vec{V}_L^{\,*}}{\vec{I}_L \vec{V}_L^{\,*}} = \frac{V_L^{\,2}}{\vec{S}_L^{\,*}} = \frac{V_L^{\,2}}{P_L - jQ_L} = \frac{1}{K_p V_L^{\,p_v-2}\omega^{p_\omega} - jK_q V_L^{\,q_v-2}\omega^{q_\omega}}$$

Load Type	Describing Function
Filament lamp	$(K_p V_L^{-0.4} - jK_q V_L^{-2})^{-1}$
Fluorescent lamp	$(K_p V_L^{-0.8}\omega^{-1} - jK_q V_L \omega^{-2.8})^{-1}$
Heater	$(K_p - jK_q V_L^{-2})^{-1}$
Induction motor (HL)	$(K_p V_L^{-1.8}\omega^{1.5} - jK_q V_L^{-0.4}\omega^{-0.3})^{-1}$
Induction motor (FL)	$(K_p V_L^{-1.9}\omega^{2.8} - jK_q V_L^{-1.4}\omega^{1.8})^{-1}$
Reduction furnace	$(K_p V_L^{-0.1}\omega^{-0.5} - jK_q V_L^{0.1})^{-1}$
Aluminum plant	$(K_p V_L^{-0.2}\omega^{-0.3} - jK_q V_L^{0.2}\omega^{0.6})^{-1}$
Regulated aluminum plant	$(K_p V_L^{0.4}\omega^{0.4} - jK_q V_L^{-0.4}\omega^{0.7})^{-1}$

Analytic Extension of Describing Function

$$Y_L = \frac{1}{Z_L} = L(V_L)\omega^p + jW(V_L)\omega^q$$

Crude way:
Leaves some coefficients complex, not completely in line with formal circuit theory

$$\omega \to \omega - j\sigma$$

Better way:
Coefficients are kept real, in line with formal circuit theory;
However, positive realness does not hold unless the load is a heater

$$Y_L \approx A(V_L)\times(j\omega)^\alpha + B(V_L)\times(j\omega)^\beta \xrightarrow{\text{extension}} A(V_L)s^\alpha + B(V_L)s^\beta$$

where $A(.)$ and $B(.)$ are real valued.

Can we replace s by $\frac{d}{dt}$???

Yes, but subject to correct interpretation:

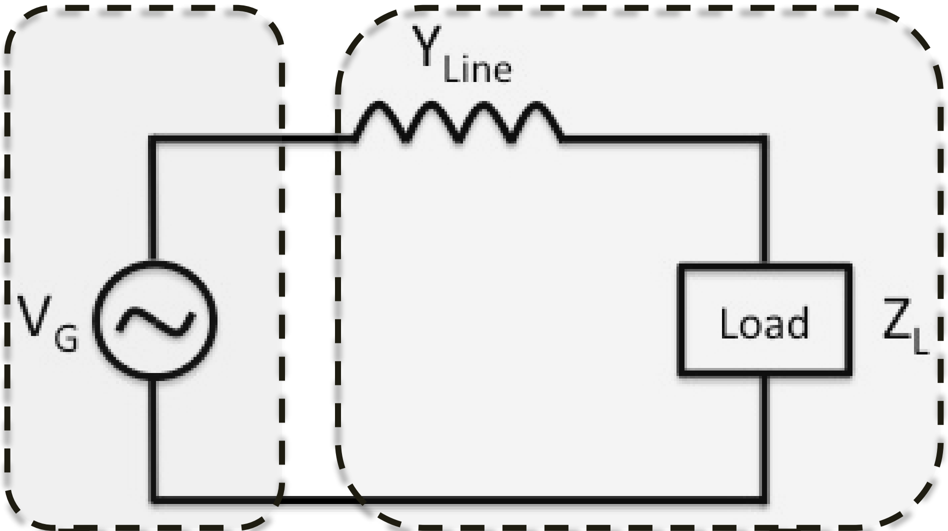
related {
- Caputo, $\boldsymbol{D_*}$ (initial conditions in terms of integer derivatives)
- Riemann-Liouville, $\boldsymbol{D}$ (initial conditions in terms of fractional derivatives)
- Grunwald-Leitnikov, $\boldsymbol{d}$ (close to ARFIMA model)

$$\underbrace{\begin{pmatrix} a_1 D_{(*)}^{\alpha_1} + b_1 D_{(*)}^{\beta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_n D_{(*)}^{\alpha_n} + b_n D_{(*)}^{\beta_n} \end{pmatrix}}_{\text{Distribution network}} \underbrace{\begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix}}_{\substack{\text{State} \\ \text{(load voltages)}}} = \underbrace{A(Y_{\text{Line}})}_{\text{Transmission network}} \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} + \underbrace{B(Y_{\text{Line}}) \begin{pmatrix} V_{G_1} \\ \vdots \\ V_{G_m} \end{pmatrix}}_{\text{Generation}}$$

Plan of Action

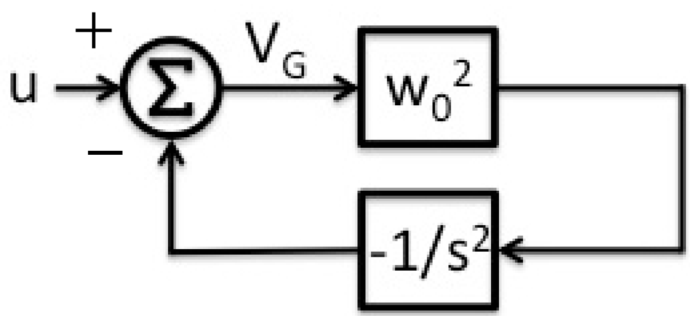
- The catalyst: Evidence of fractal PMU signals
  - Review of Detrented Fluctuation Analysis
  - Texas & EPFL (Switzerland) normal PMU data
- *Why* are PMU signals fractal???
  - Fractional dynamics load modeling
  - Load aggregation
- Voltage stability
  - The loads are the “villains”
- Early warning of imminent blackout
  - *Increase* of Hurst exponent before blackout
  - *Statistical confirmation by Kendall tau*

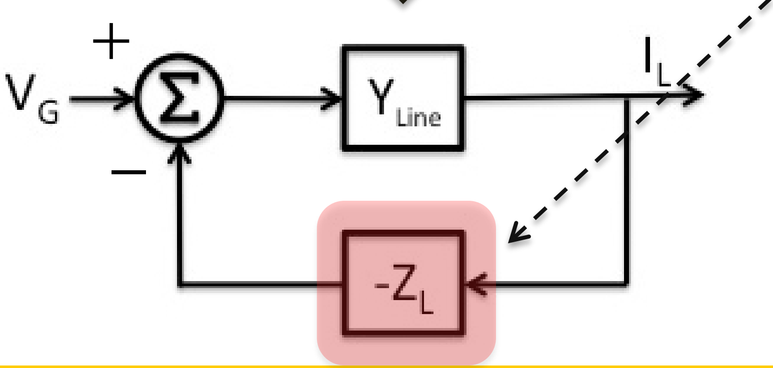
Hidden Feedback in Power Systems



Deliberately simplified model of the generator…

to put the load in the spotlight.





Feedback Model of Power System

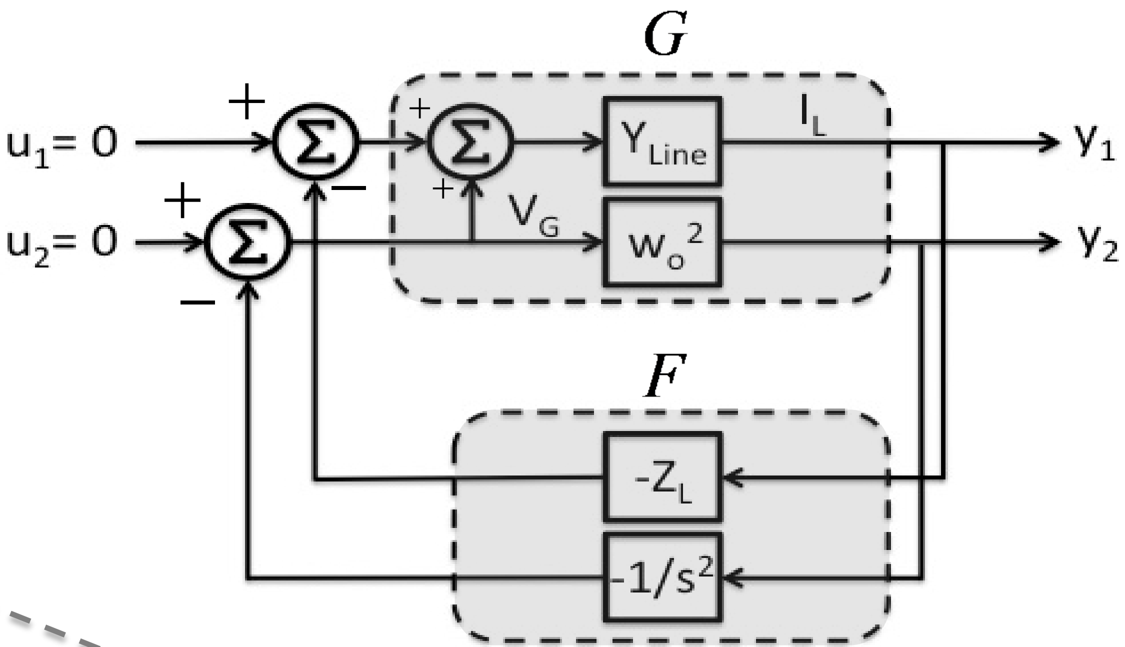
$$y = (I - GF)^{-1} G u$$

$$u = [u_1 \quad u_2]^t$$

$$y = [y_1 \quad y_2]^t$$

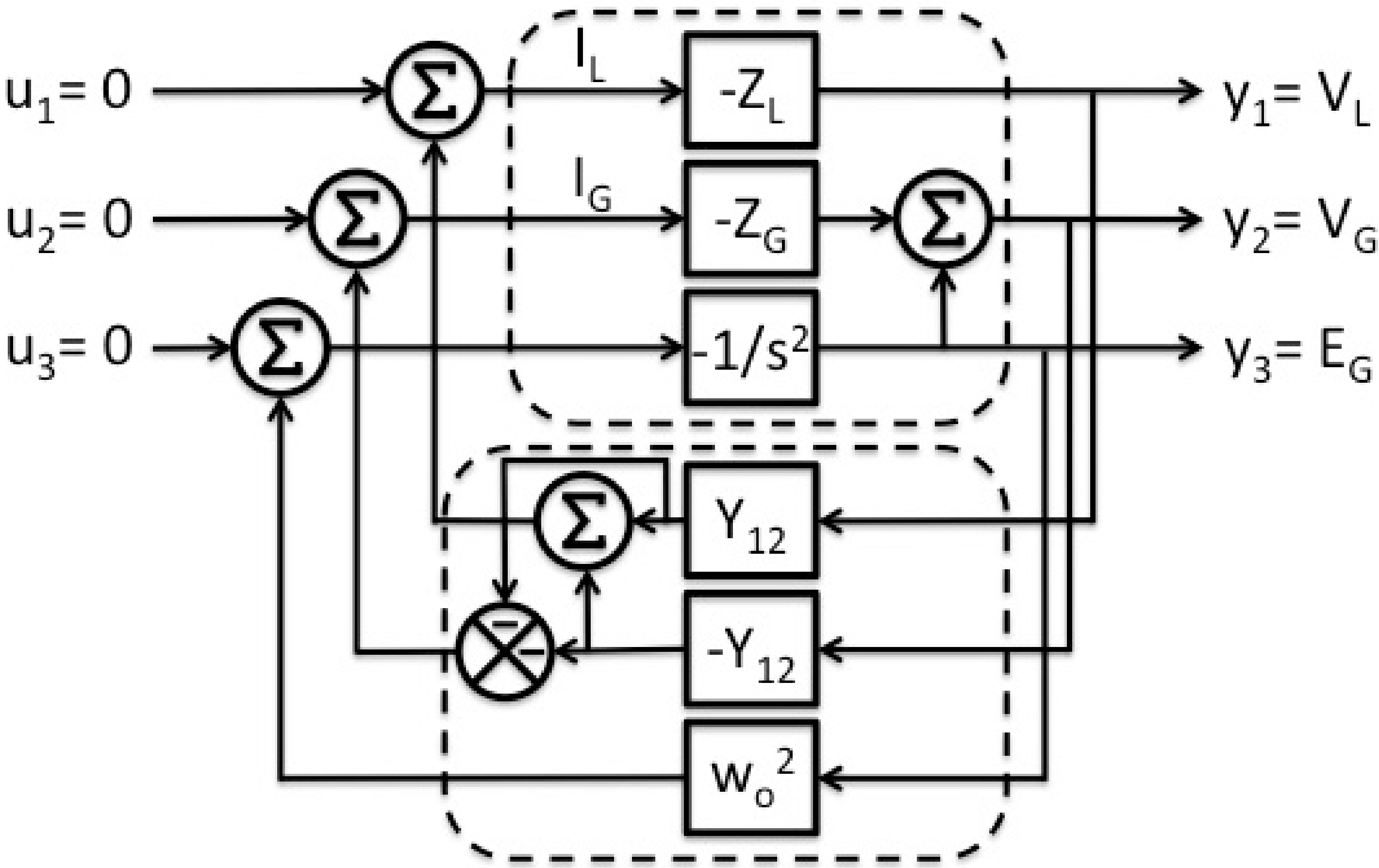
$$G = \begin{bmatrix} Y_{Line} & Y_{Line} \\ 0 & \omega_0^{\ 2} \end{bmatrix}$$

$$F = \begin{bmatrix} -Z_L & 0 \\ 0 & -s^{-2} \end{bmatrix}$$

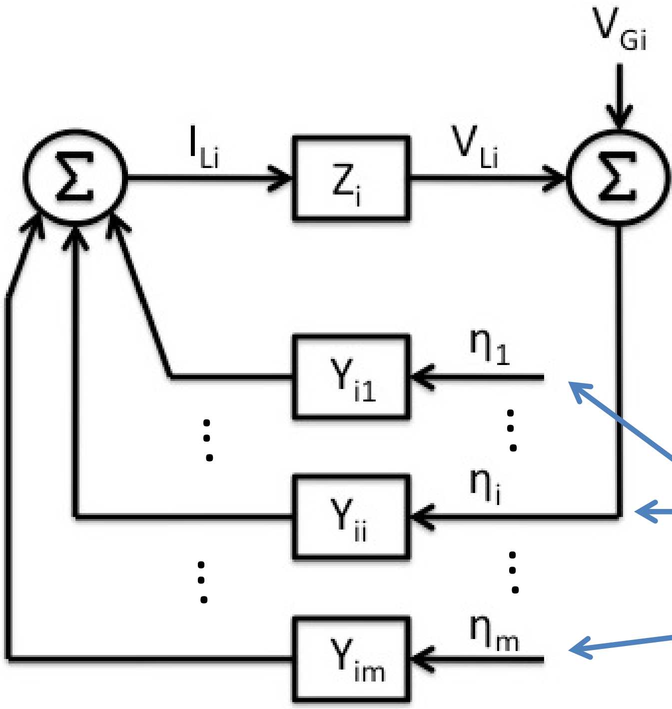


Simplification:
No back-action of the load to the generator

Towards more Complicated Feedback Models of Power System



Time to Conceptualize



714 IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS, VOL. CAS-23, NO. 12, DECEMBER 1976

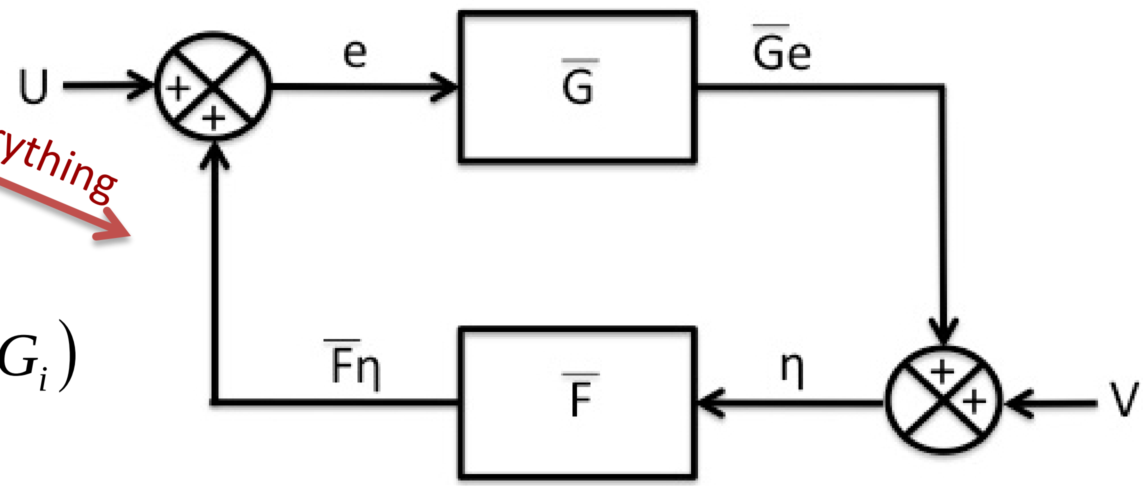
Input–Output Stability Theory of Interconnected Systems Using Decomposition Techniques

FRANK M. CALLIER, MEMBER, IEEE, WAN S. CHAN, STUDENT MEMBER, IEEE, AND
CHARLES A. DESOER, FELLOW, IEEE

Nominal impedance, line Z_i, Y_{ii}

Connecting lines 1, 2, ..., ≠i, ..., m

Lump everything

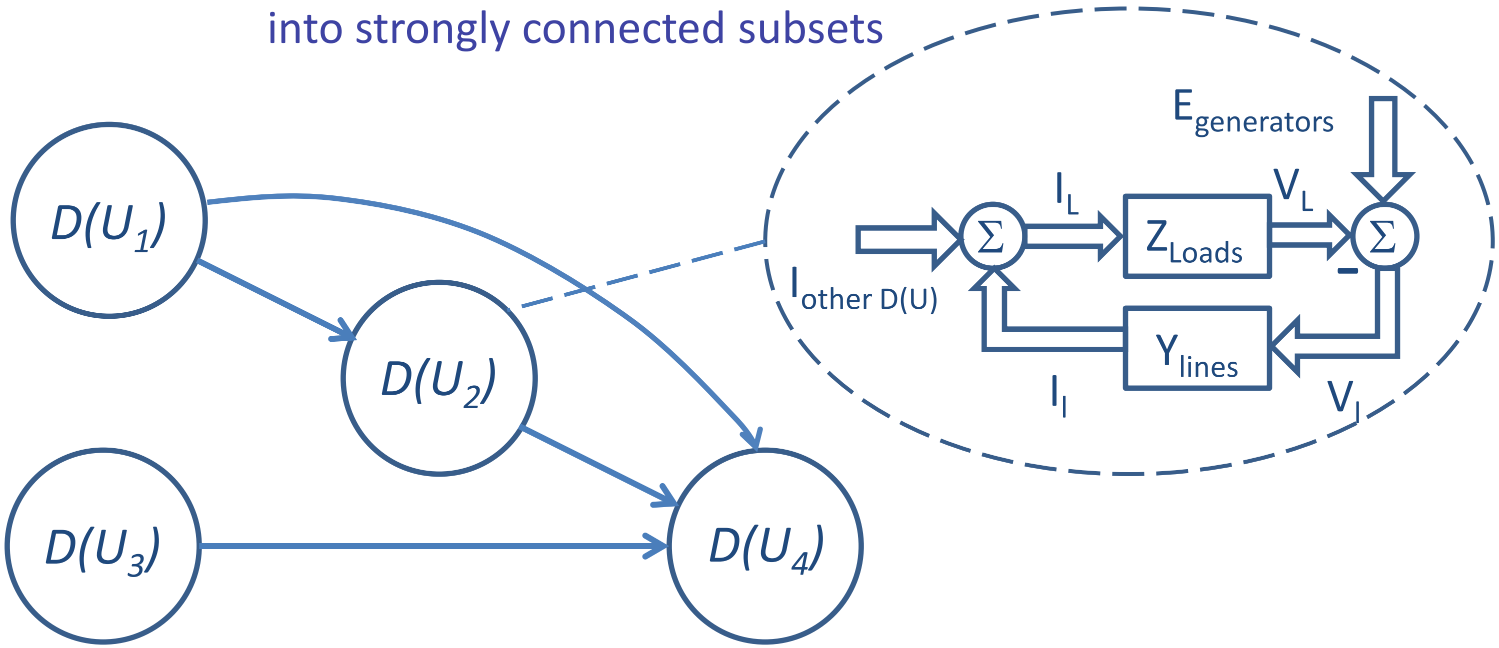


Are we sure that

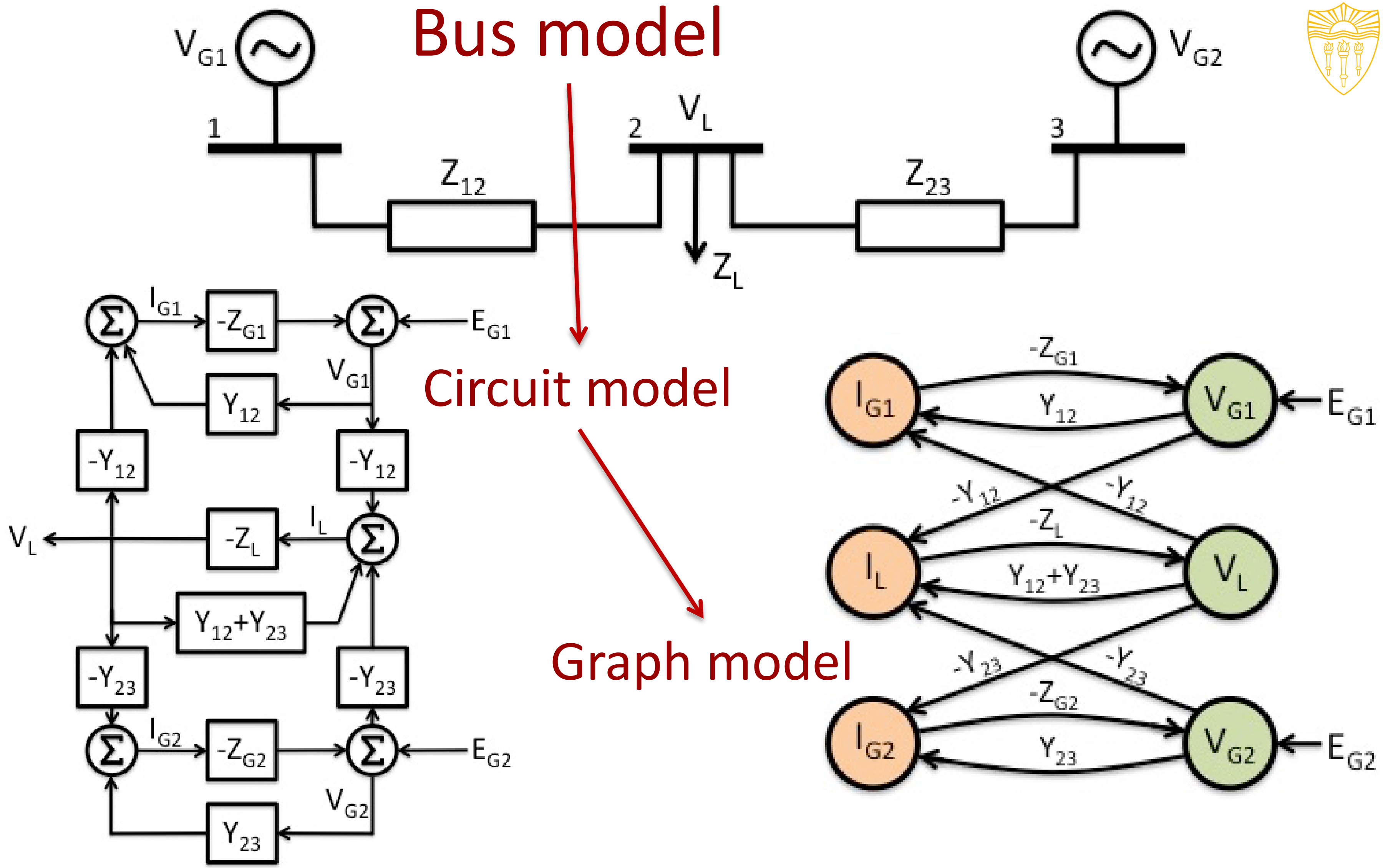
$$\det\left(I-\overline{F}\overline{G}\right)\neq \Pi_i \det\left(I-F_iG_i\right)$$

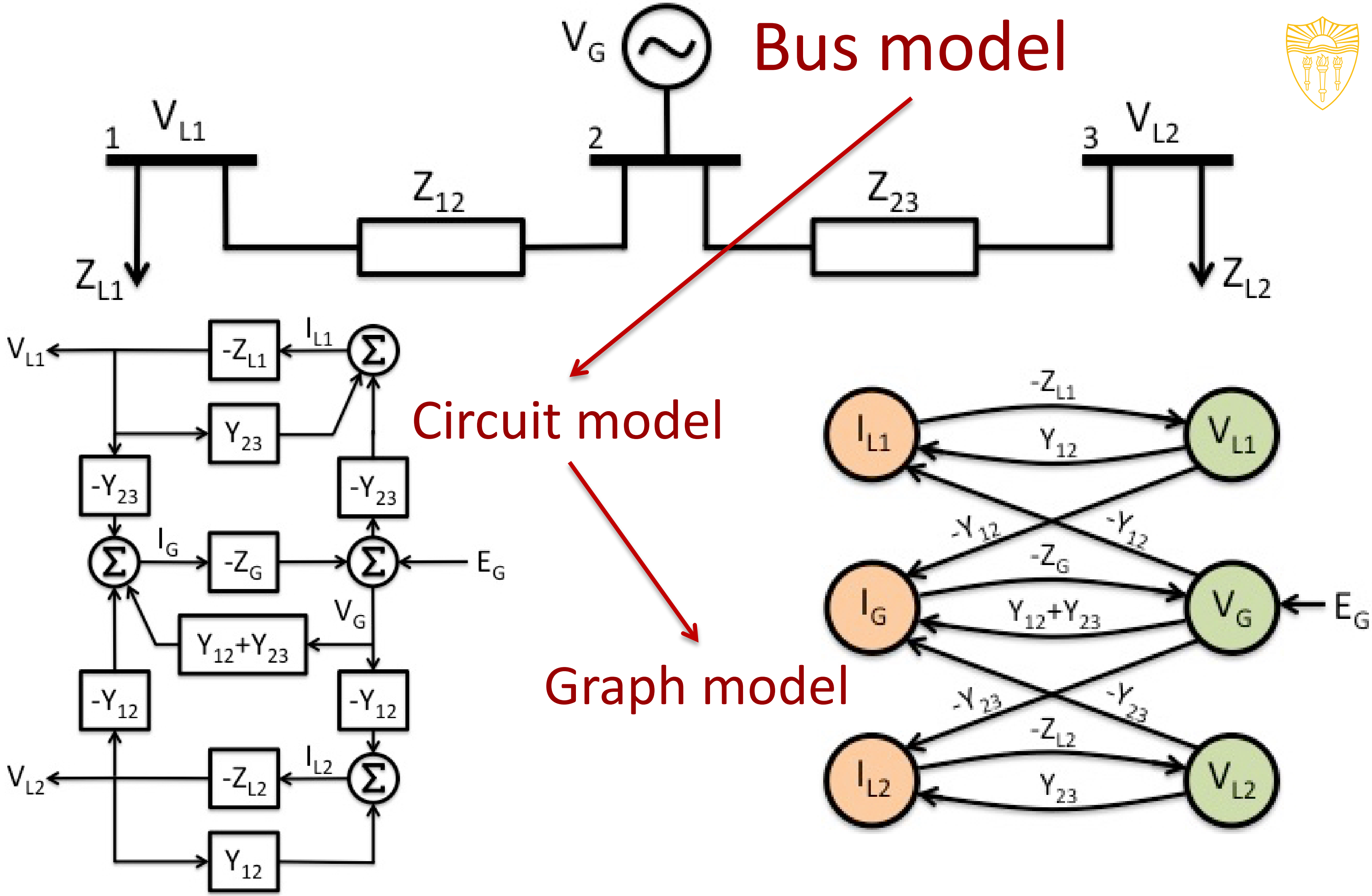
Decomposition of Digraph into Strongly Connected Components $D(U_i)$

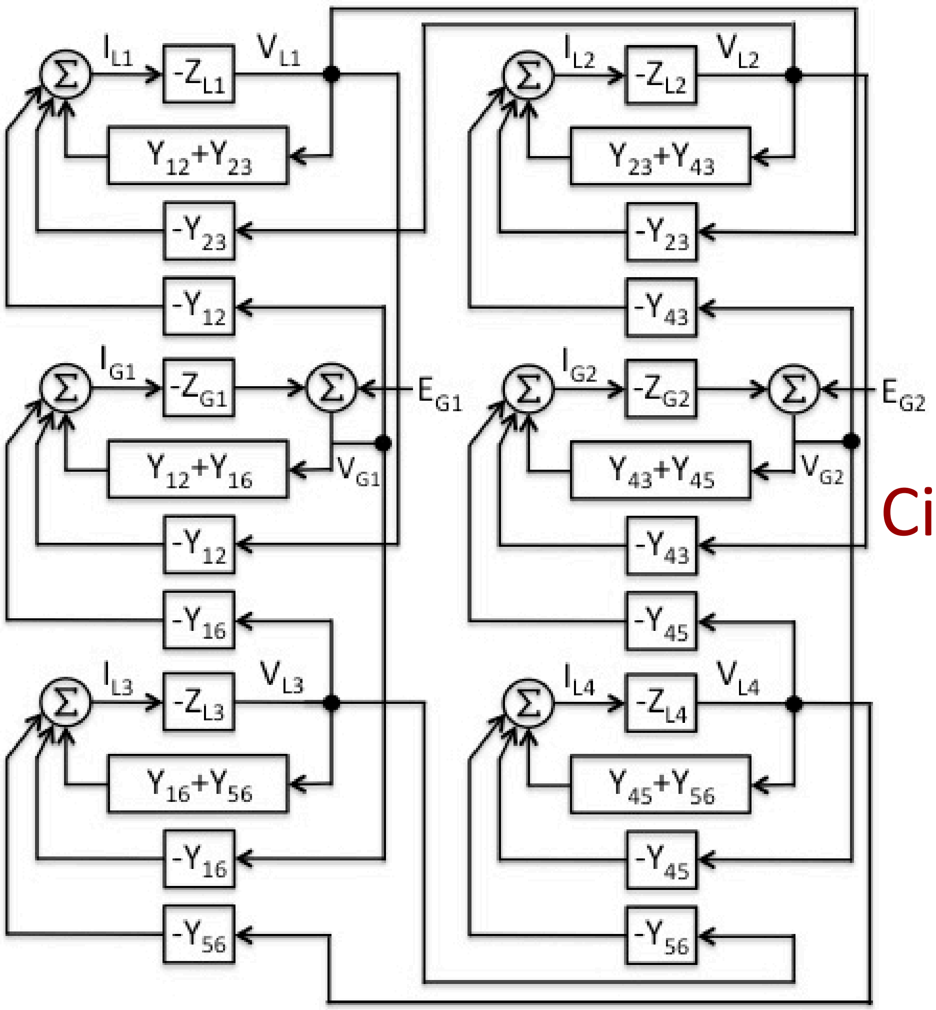
Feedback connections, if any, are lumped into strongly connected subsets



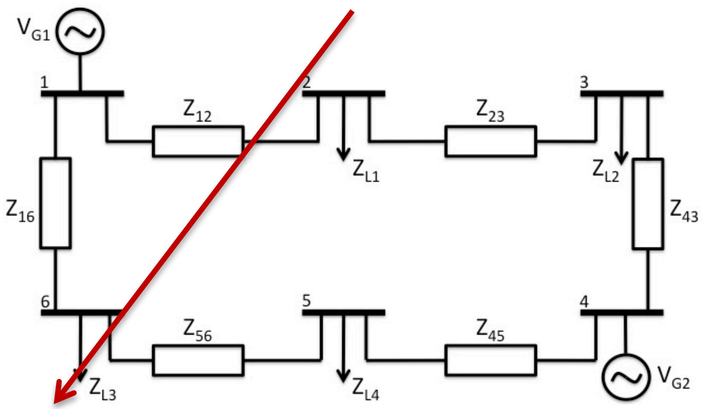
No large scale feedback connections at the large scale of the structure graph


Bus model
Circuit model
Graph model
V_G1
V_G2
1
2
3
V_L
Z_12
Z_23
Z_L
I_G1
-Z_G1
E_G1
V_G1
Y_12
-Y_12
-Y_12
V_L
-Z_L
I_L
Y_12+Y_23
-Y_23
-Y_23
I_G2
-Z_G2
E_G2
V_G2
Y_23
-Z_G1
Y_12
I_G1
V_G1
E_G1
-Y_12
-Y_12
-Z_L
I_L
Y_12+Y_23
V_L
-Y_23
-Y_23
-Z_G2
I_G2
Y_23
V_G2
E_G2

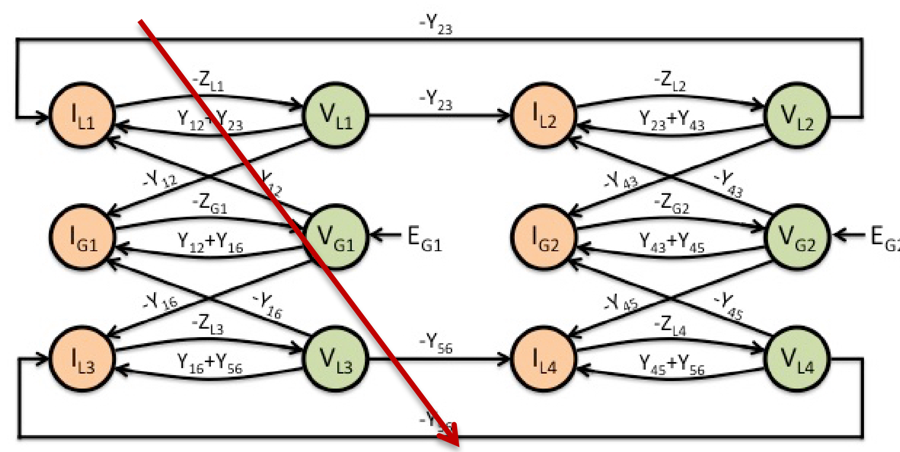

Bus model
Circuit model
Graph model
V_G
1
V_L1
2
3
V_L2
Z_12
Z_23
Z_L1
Z_L2
V_L1
-Z_L1
I_L1
Σ
Y_23
-Y_23
-Y_23
I_G
-Z_G
E_G
V_G
Y_12+Y_23
-Y_12
-Y_12
V_L2
-Z_L2
I_L2
Y_12
I_L1
V_L1
-Z_L1
Y_12
-Y_12
-Y_12
I_G
V_G
-Z_G
Y_12+Y_23
E_G
-Y_23
-Y_23
I_L2
V_L2
-Z_L2
Y_23



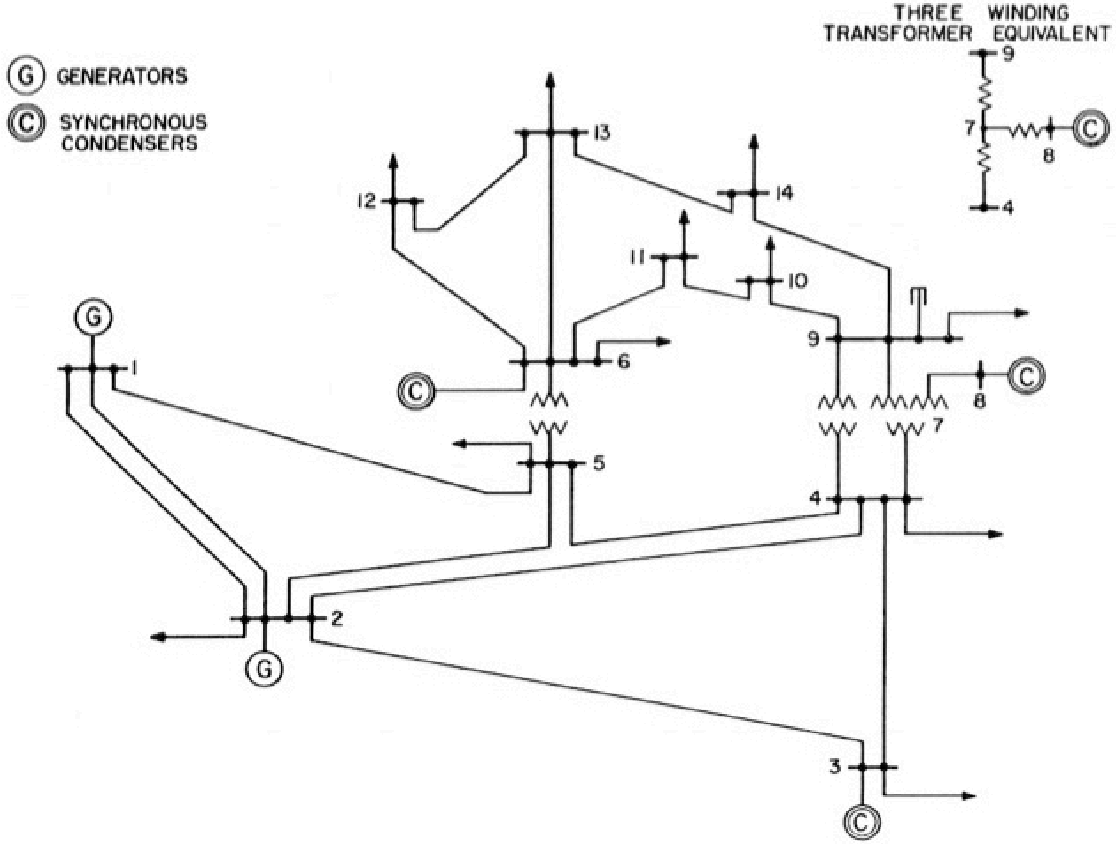
Bus model



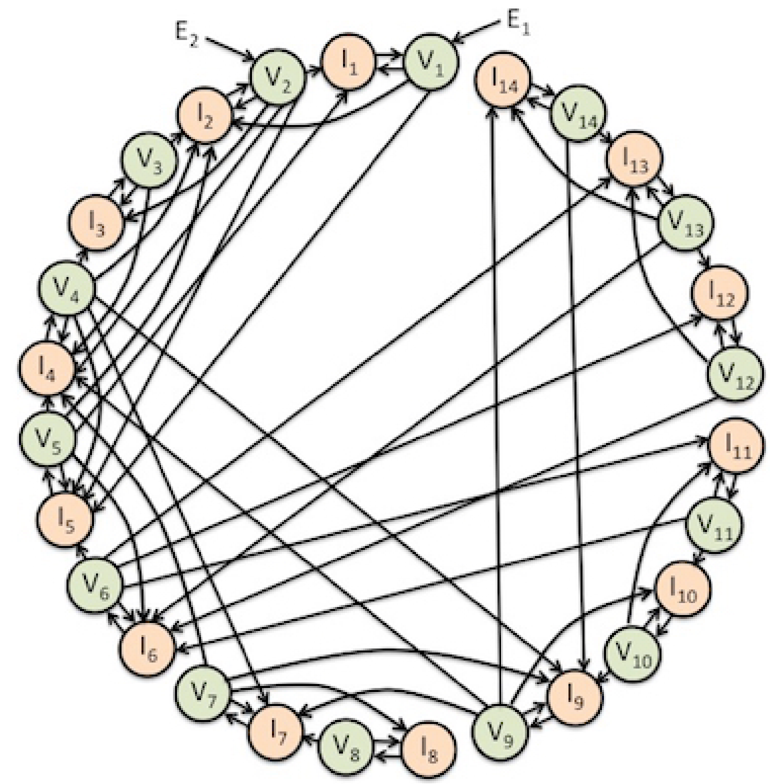
Circuit model



Graph model

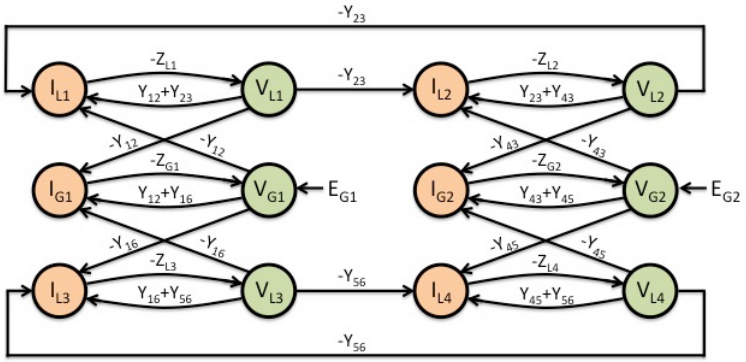


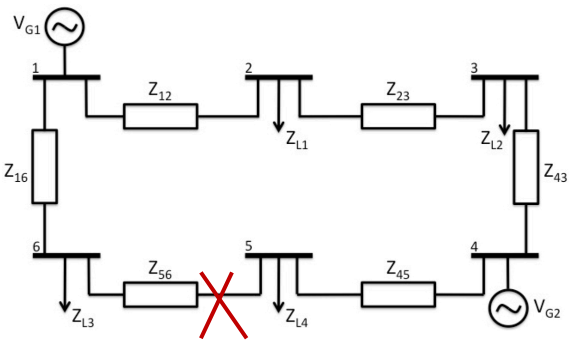
GENERATORS
SYNCHRONOUS CONDENSERS
THREE WINDING TRANSFORMER EQUIVALENT



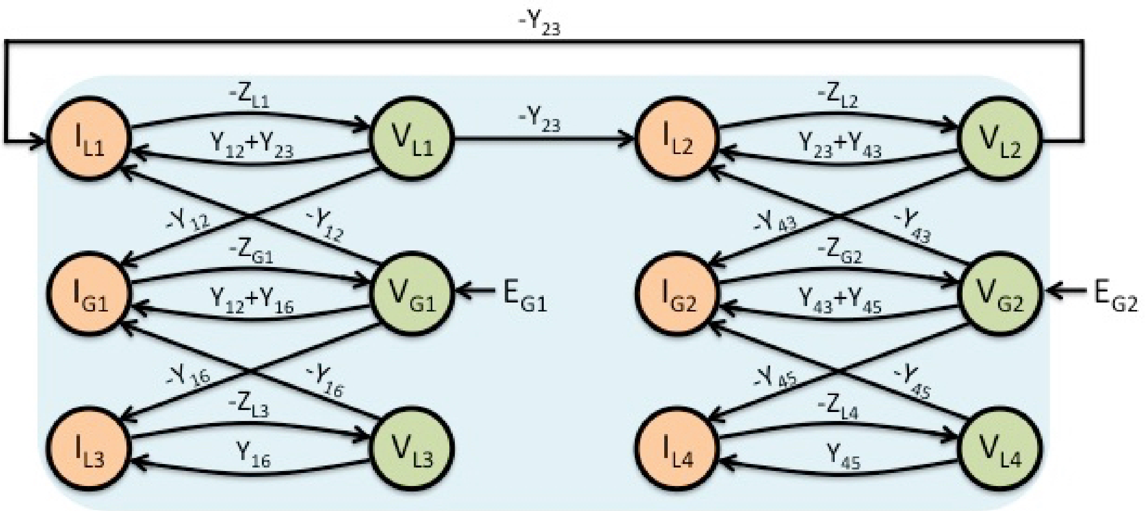
E1
E2

Effect of Single Contingency



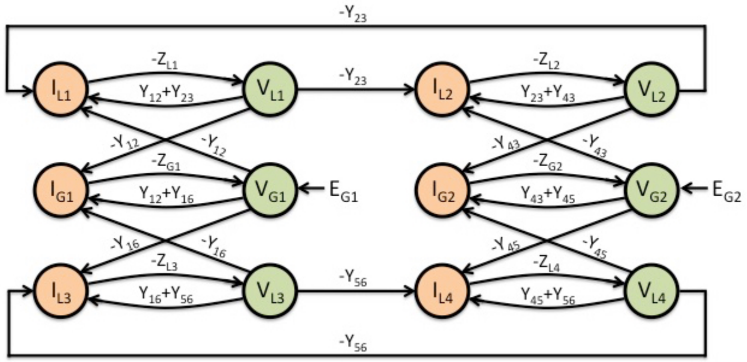


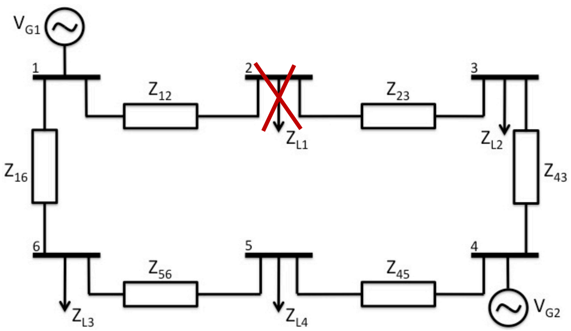
Single transmission line 5-6 tripping:



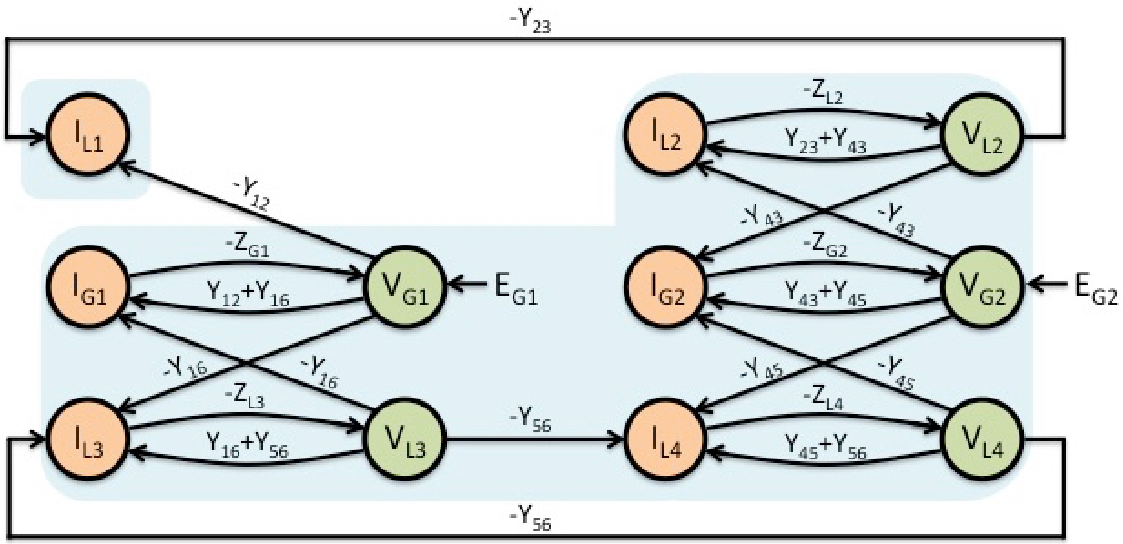
No loss of strong connectivity!

Effect of Single Contingency



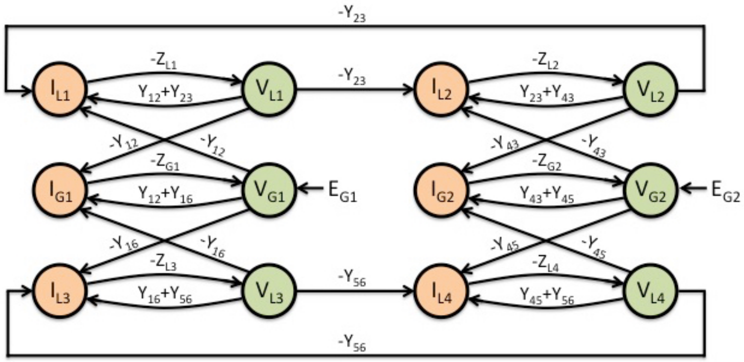


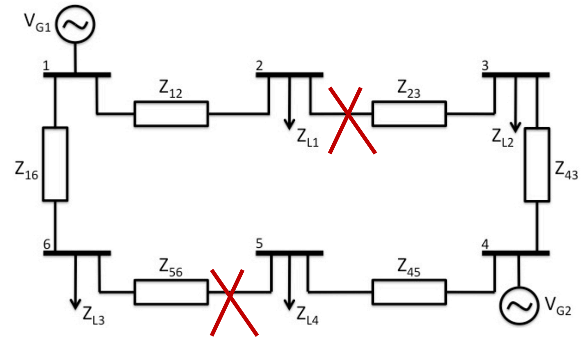
Three-phase fault at Load 1:



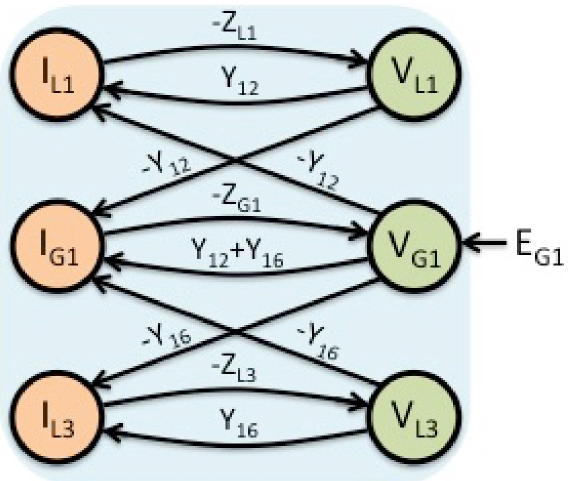
Loss of strong connectivity: two strongly connected components!

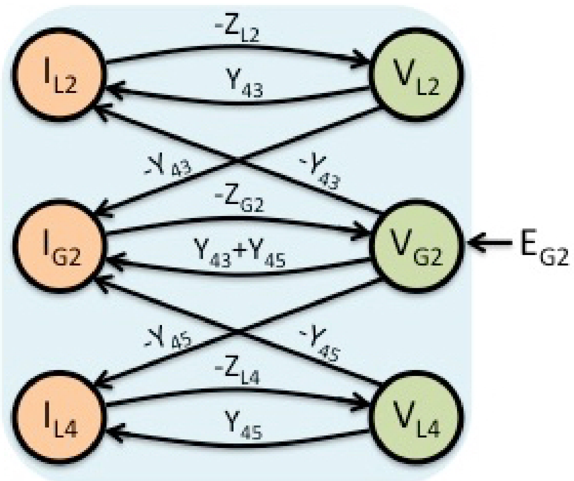
Effect of Double Contingency





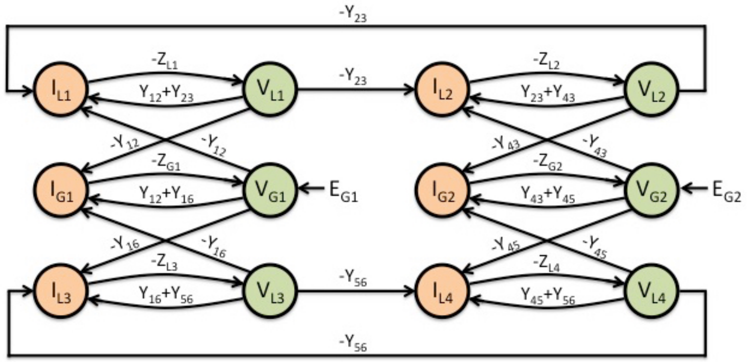
Double transmission line 5-6, 2-3 tripping:

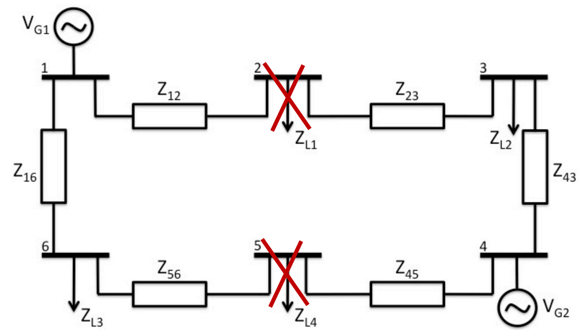




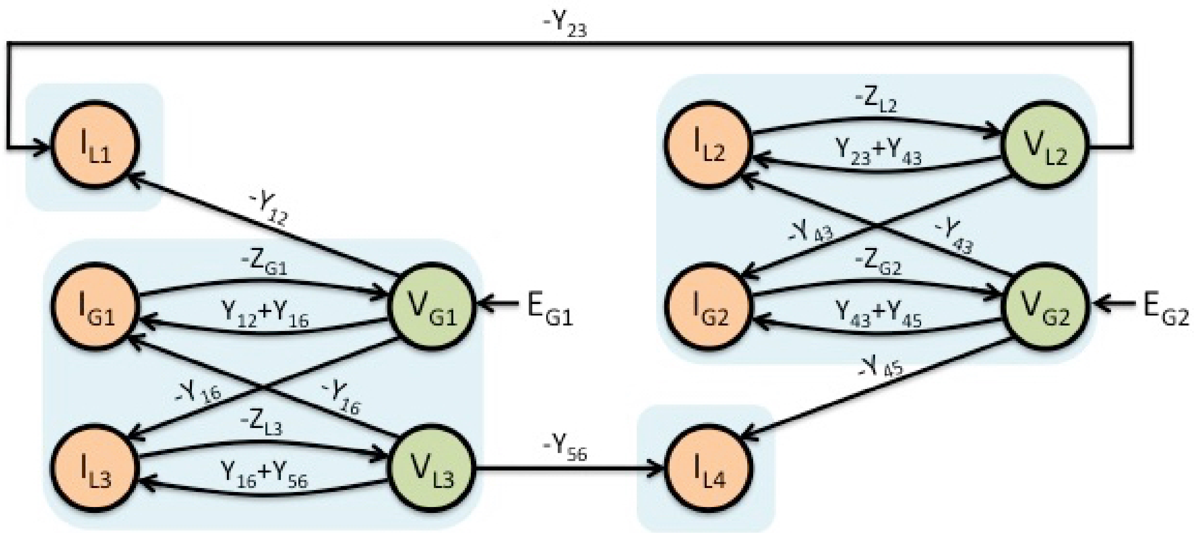
Loss of connectivity: two connected components!

Effect of Double Contingency





Two three-phase faults at Loads 1 and 4:



Loss of strong connectivity: four strongly connected components!

Main Theorem

Theorem: Under the conditions that

- the bus system is connected,
- all generators have nonvanishing internal impedance,

and the contingencies are restricted to

- single transmission line tripping,

the graph model is strongly connected.

Plan of Action

- The catalyst: Evidence of fractal PMU signals
  - Review of Detrented Fluctuation Analysis
  - Texas & EPFL (Switzerland) normal PMU data
- *Why* are PMU signals fractal???
  - Fractional dynamics load modeling
  - Load aggregation
- Voltage stability
  - The loads are the "villains"
- Early warning of imminent blackout
  - *Increase* of Hurst exponent before blackout
  - *Statistical confirmation by Kendall tau*

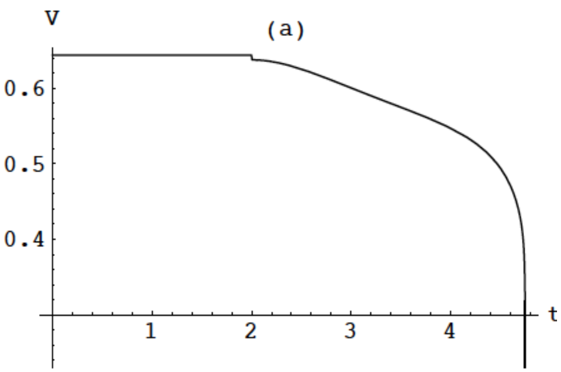
Voltage Collapse

<u>Definition:</u>

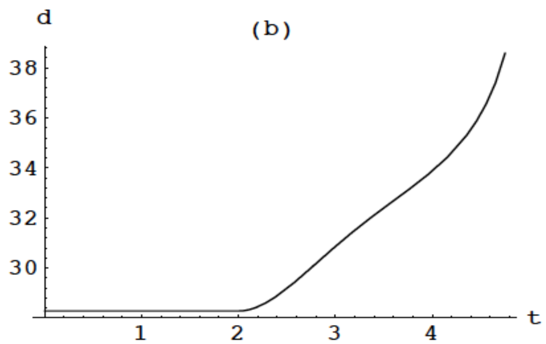
Voltage collapse is critical phenomena that threatens the power infrastructure, and that manifests itself by a sudden and fast collapse of the system voltage.

<u>Source of problem:</u>

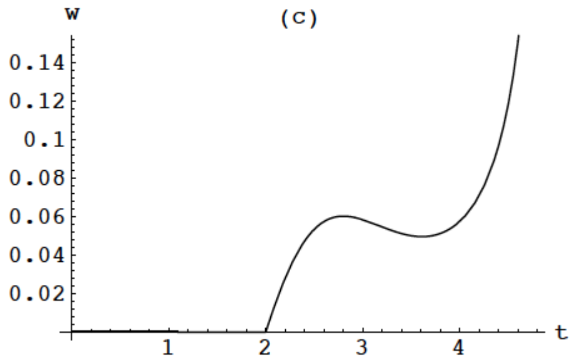
Traditionally, it is blamed on a supply-demand imbalance…



Voltage collapses



Damping increases



Frequency is disrupted!!!

The Frequency Dependence Debate

''The differences in time constants have led many researchers to only consider voltage dynamics for the analysis of bifurcations problems, ignoring frequency dynamics. However, the previous example clearly shows that this assumption is not completely justifiable''

Prof. Claudio Cañizares

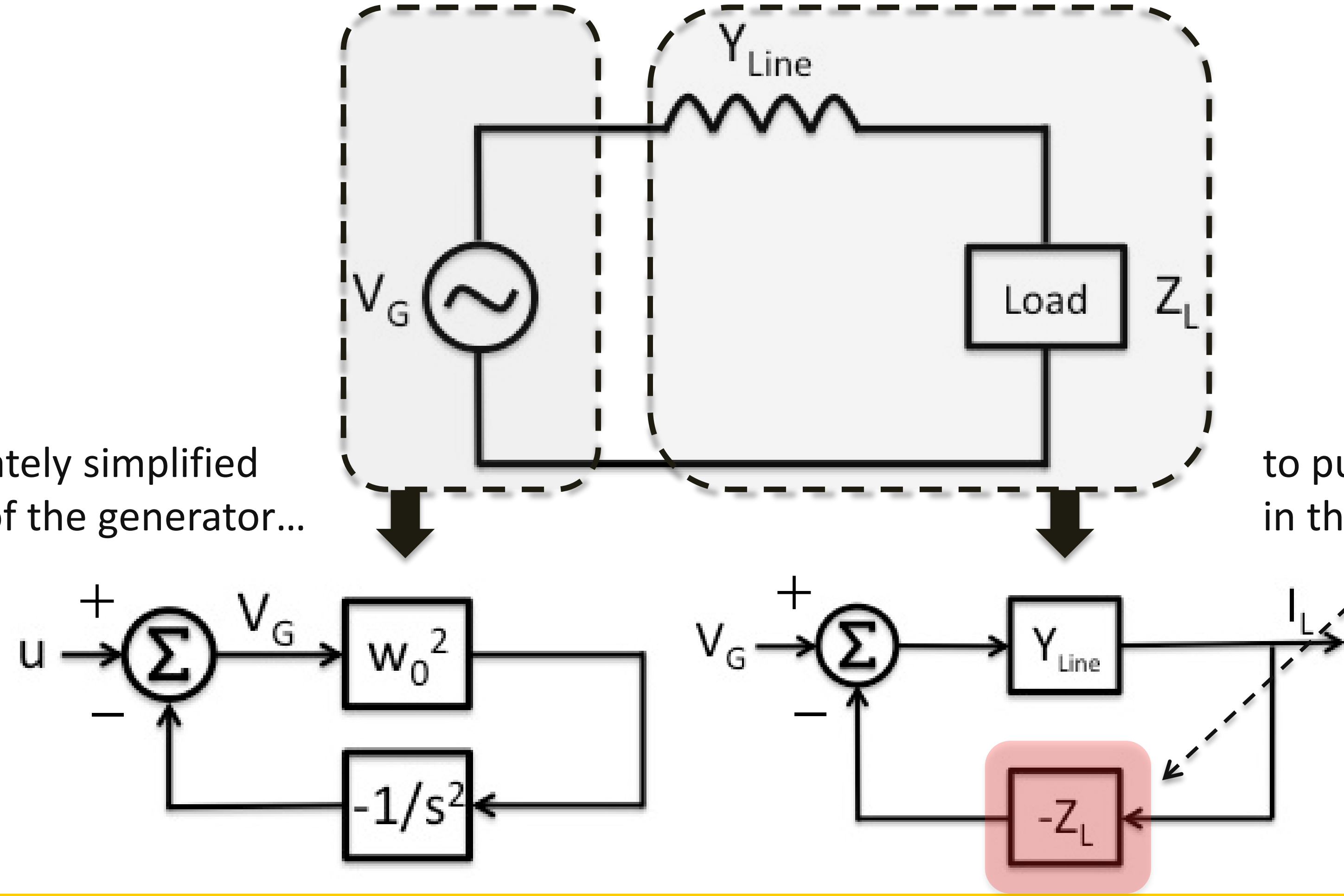
''This model was motivated by voltage stability studies; frequency dependence of the load has not been considered''

Prof. David Hill

"Wehenkel stated that better modeling of loads and demand is also needed; specifically, better dynamic models that respond to voltage/frequency variations over shorter time periods (seconds and minutes) are needed for stability analysis"

Government Report

Feedback Model of Power System



Deliberately simplified model of the generator…

to put the load in the spotlight.

Feedback Model of Power System

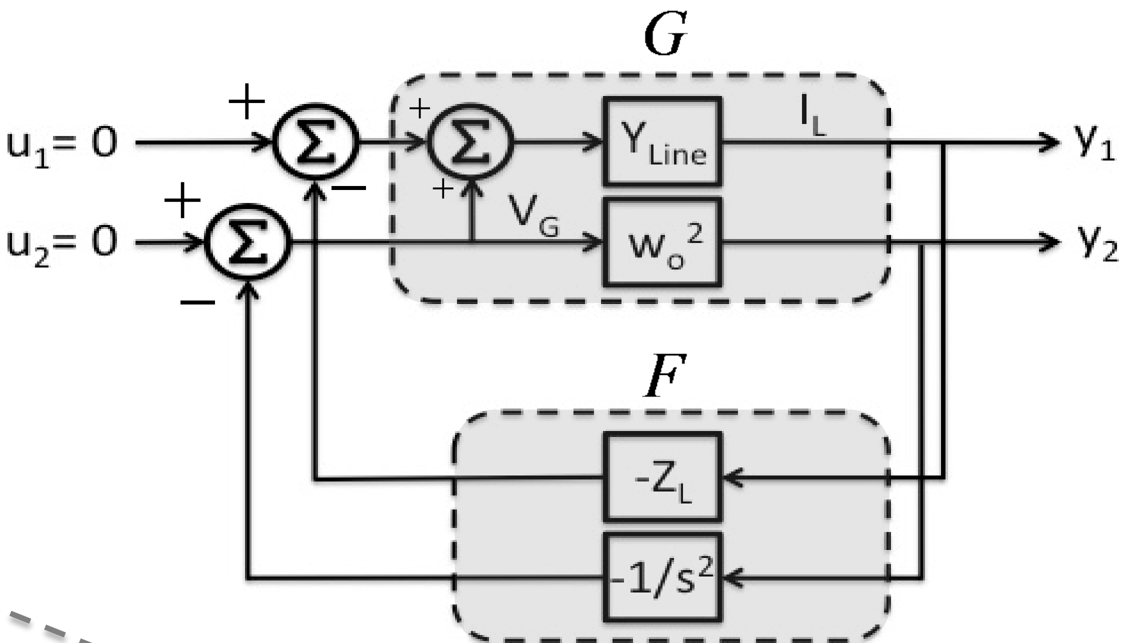
$$y = (I - GF)^{-1} Gu$$

$$u = [u_1 \quad u_2]^t$$

$$y = [y_1 \quad y_2]^t$$

$$G = \begin{bmatrix} Y_{Line} & Y_{Line} \\ 0 & \omega_0^{\ 2} \end{bmatrix}$$

$$F = \begin{bmatrix} -Z_L & 0 \\ 0 & -s^{-2} \end{bmatrix}$$



Simplification:
No back-action of the load to the generator

Voltage Collapse Solution

- Power system represented by the feedback model has a solution if

$$\left|(I-GF)^{-1}G\right| = \left(1+Z_L Y_{Line}\right)\left(1+\omega_0{}^2/s^2\right)=0$$

  - $\left(1+\omega_0{}^2/s^2\right)=0$ ➡ Purely harmonic solution $V_L cos(\omega_0 t)$
  - $\left(1+Z_L Y_{Line}\right)=0$ ➡ Voltage collapsing solution $V_L e^{\sigma t} cos(\omega t)$

- The voltage collapsing solution exists if

➢ $1+Z_L Y_{Line} = 0$

➢ $Y_L\left(V_L, \omega - j\sigma\right)+Y_{Line}\left(\omega - j\sigma\right)=0$

➢ $K_p V_L{}^{p_v-2}\left(\left(\omega - j\sigma\right)/\omega_0\right)_{p_\omega} - jK_q V_L{}^{q_v-2}\left(\left(\omega - j\sigma\right)/\omega_0\right)_{q_\omega} + K_{Line}/(\sigma + j\omega) = 0$

➢ $K_p(-j/\omega_0)^{p_\omega} V_L{}^{p_v-2} s^{p_\omega+1} - jK_q(-j/\omega_0)^{q_\omega} V_L{}^{q_v-2} s^{q_\omega+1} + K_{Line} = 0$

Voltage Collapse Solution - Special Case

- The voltage collapse solution exists in case of special loads ($p_v = q_v$ and $p_\omega = q_\omega$) if

$$s = \sigma + j\omega = \alpha V_L^{\beta}$$

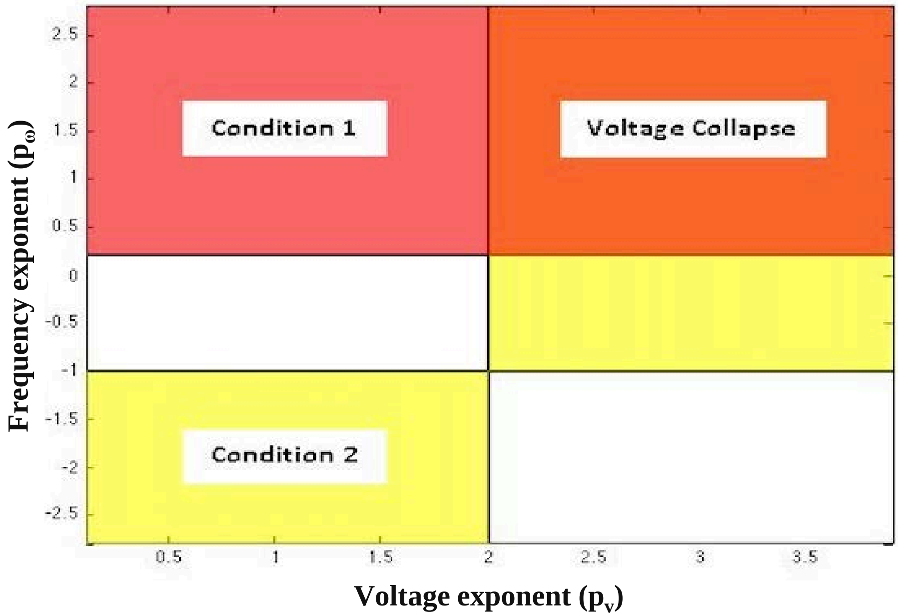
$$\alpha = (-K_{Line} / ((-j/\omega_0)^{p_\omega} (K_p - jK_q)))$$

$$\beta = (2 - p_v)/(p_\omega - 1)$$

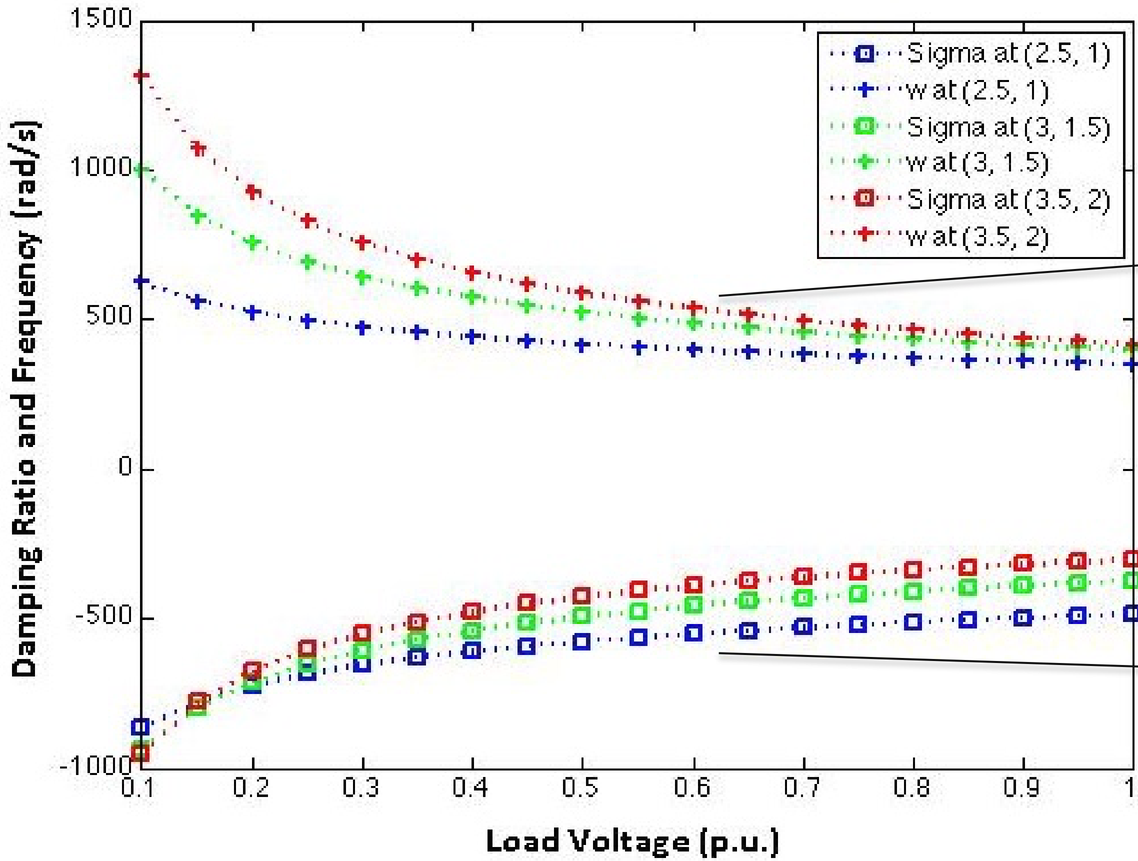
- Voltage collapse conditions:

1) $\Re(\alpha) < 0$ and $\Im(\alpha) > 0$

2) $\beta < 0$



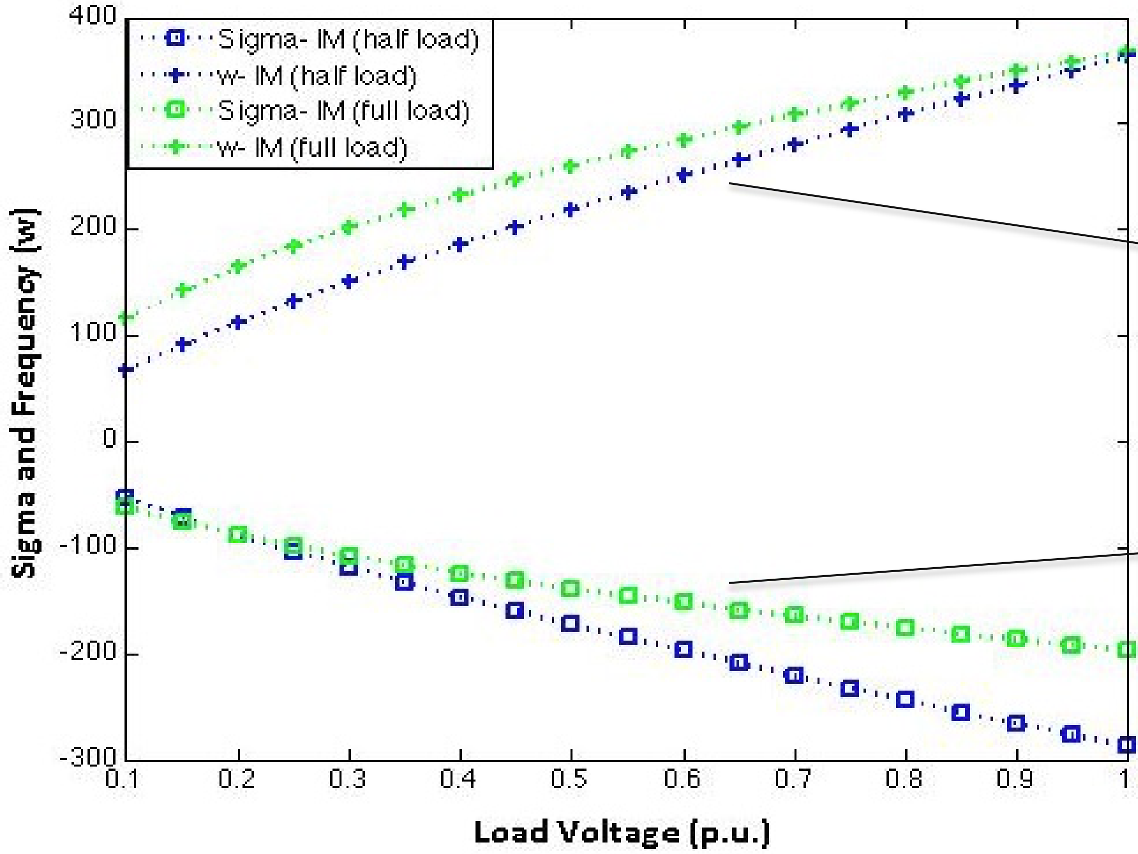
Sigma (σ) and Frequency (ω) for Different Special Loads



Frequency is disrupted as well

Voltage decreases and damping increases

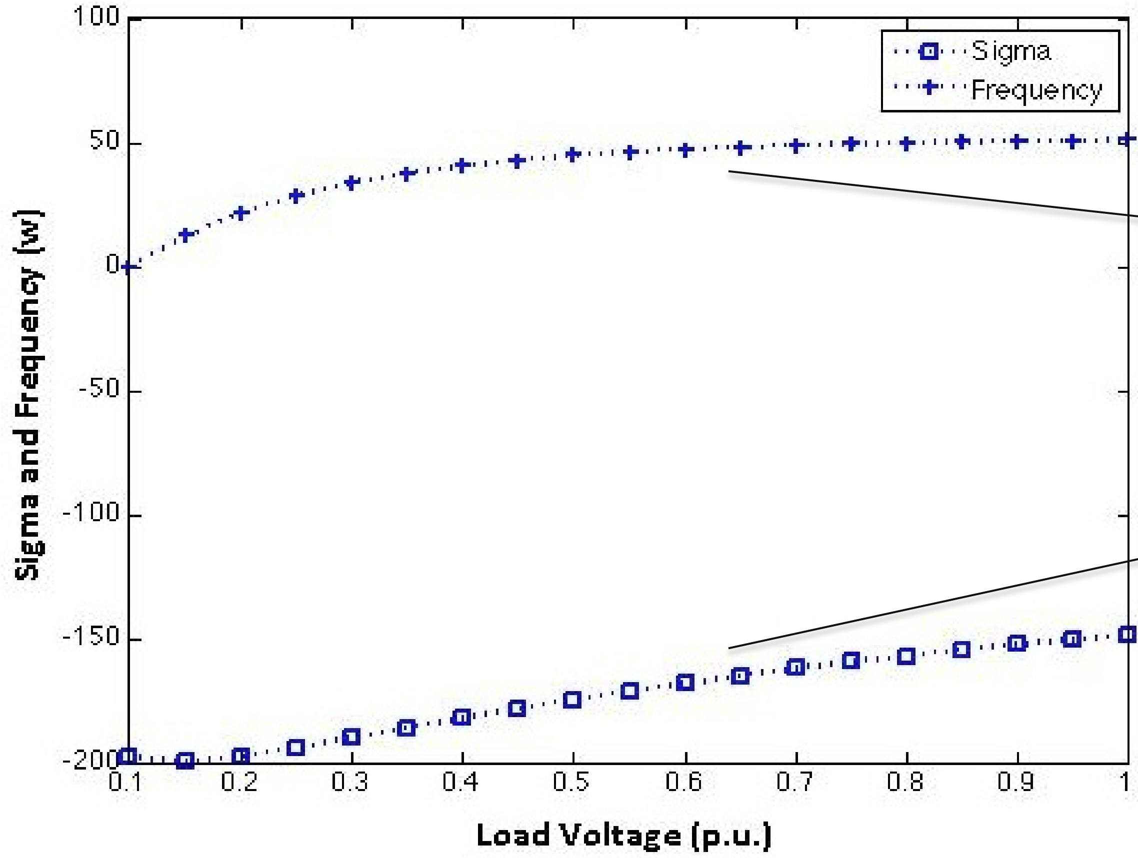
Sigma (σ) and Frequency (ω) for Induction Motor (Stable)



Frequency is disrupted as well

Voltage decreases and damping decreases

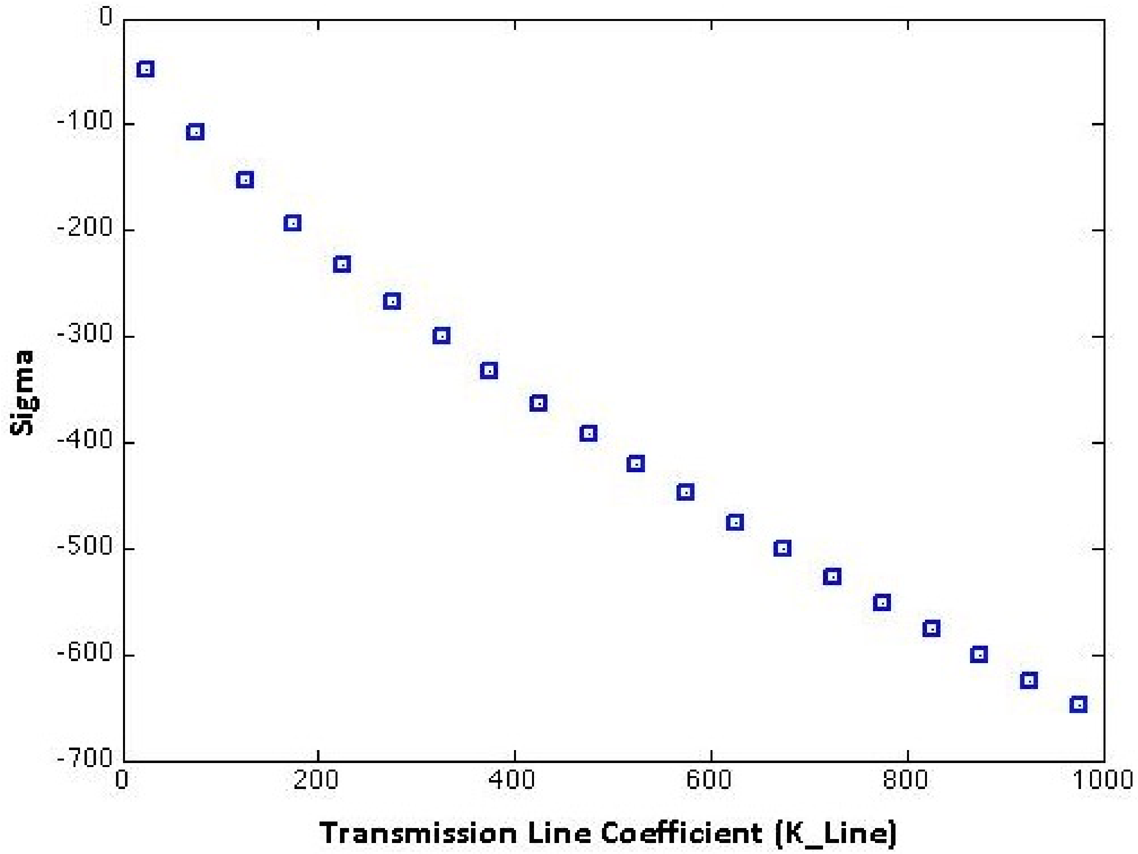
Sigma (σ) and Frequency (ω) for Regulated Aluminum Plant (Unstable)



Frequency is disrupted as well

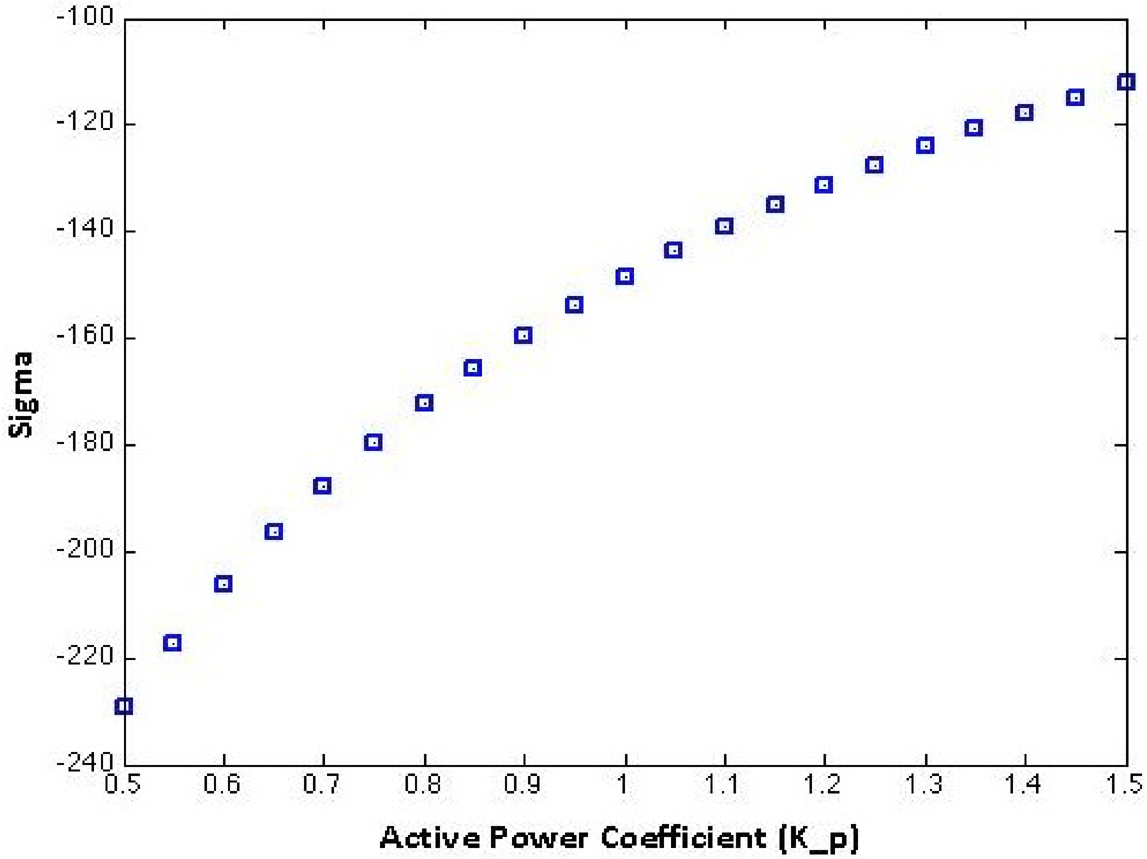
Voltage decreases and damping increases

The Relationship Between Transmission Line Coefficient (K_{Line}) and Sigma (σ)

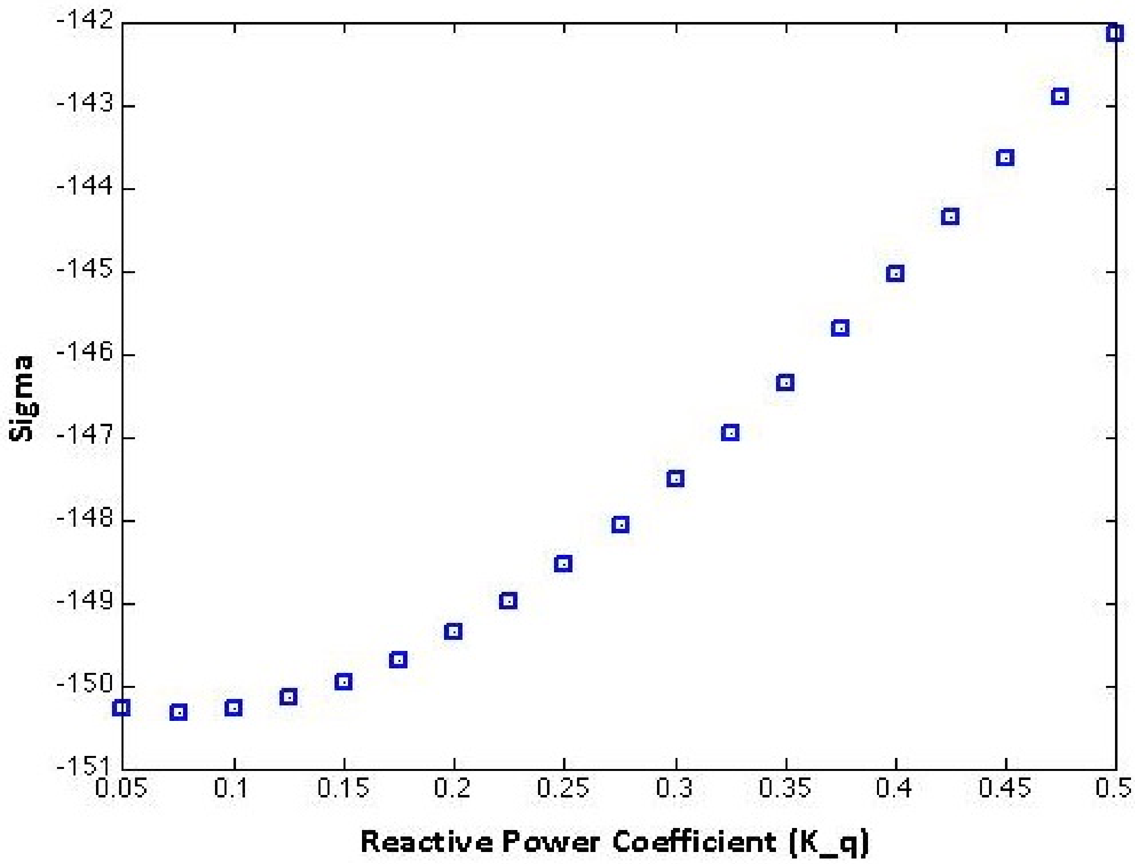


K_{line} is directly proportional to maximum power transfer

The Relationship Between Active Power Coefficient (K_p) and Sigma (σ)



The relationship Between Reactive Power Coefficient(K_q) and Sigma (σ)



Conclusions and Future Work

- While voltage collapse is usually blamed on a generation-load imbalance, it is shown here that a more subtle phenomenon could contribute.
- This subtle phenomenon is a nonlinear feedback effect creating an increasing damping when the load voltage decreases.
- By the same token, we provide a theoretical explanation of the frequency dependence in the voltage collapse.
- The Berg model involves noninteger exponents of the frequency, which can be reinterpreted as fractional derivatives, leading to

$$\underbrace{\begin{pmatrix} a_1 D_{(*)}^{\alpha_1} + b_1 D_{(*)}^{\beta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_n D_{(*)}^{\alpha_n} + b_n D_{(*)}^{\beta_n} \end{pmatrix}}_{\text{Distribution network}} \underbrace{\begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix}}_{\substack{\text{State} \\ \text{(load voltages)}}} = \underbrace{A(Y_{\text{Line}})}_{\text{Transmission network}} \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} + \underbrace{B(Y_{\text{Line}}) \begin{pmatrix} V_{G_1} \\ \vdots \\ V_{G_m} \end{pmatrix}}_{\text{Generation}}$$

- This new state space model involving fractional derivatives is corroborated by PMU signal analysis, showing long range dependence [Power and Energy Society General Meeting (PESGM), Boston, 2016].

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  - Review of Detrented Fluctuation Analysis
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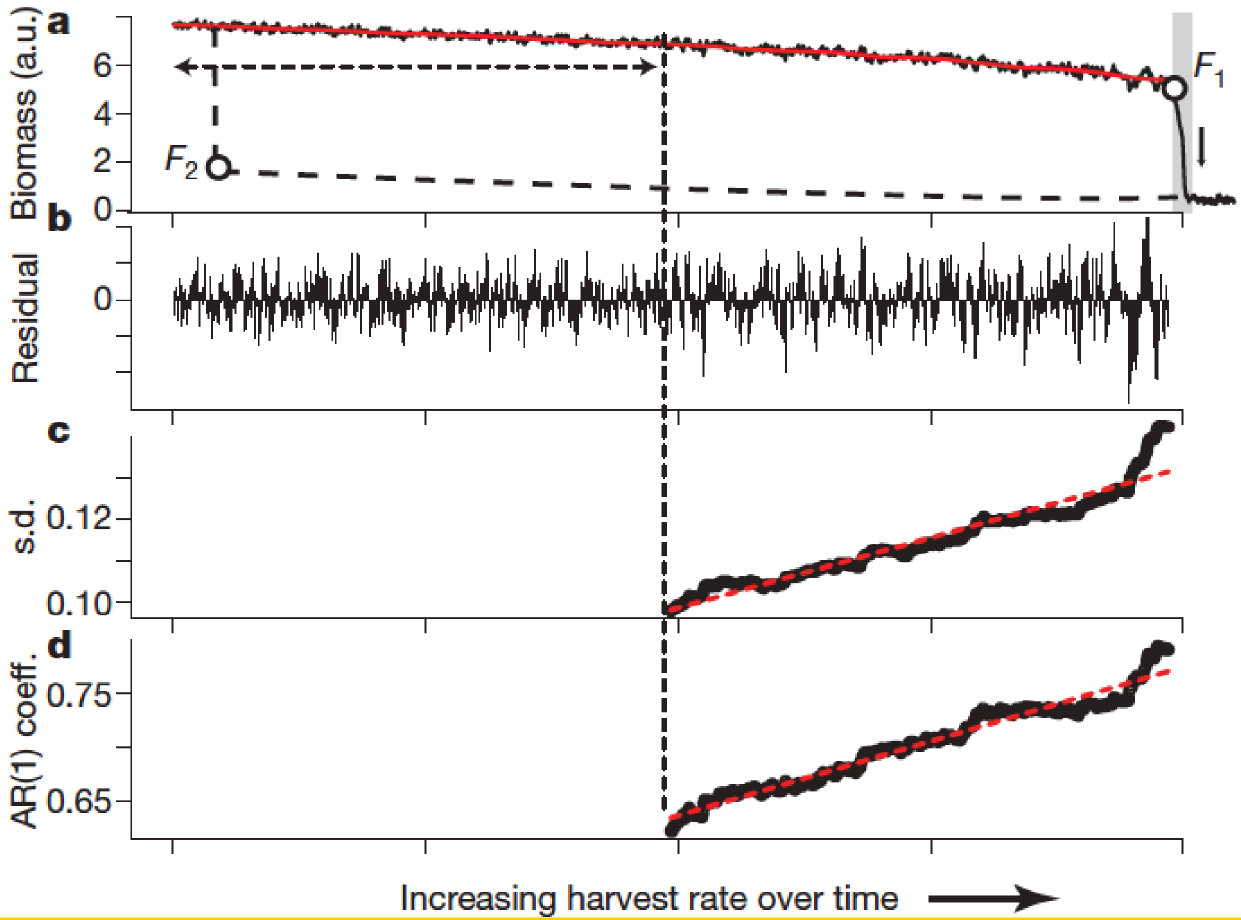
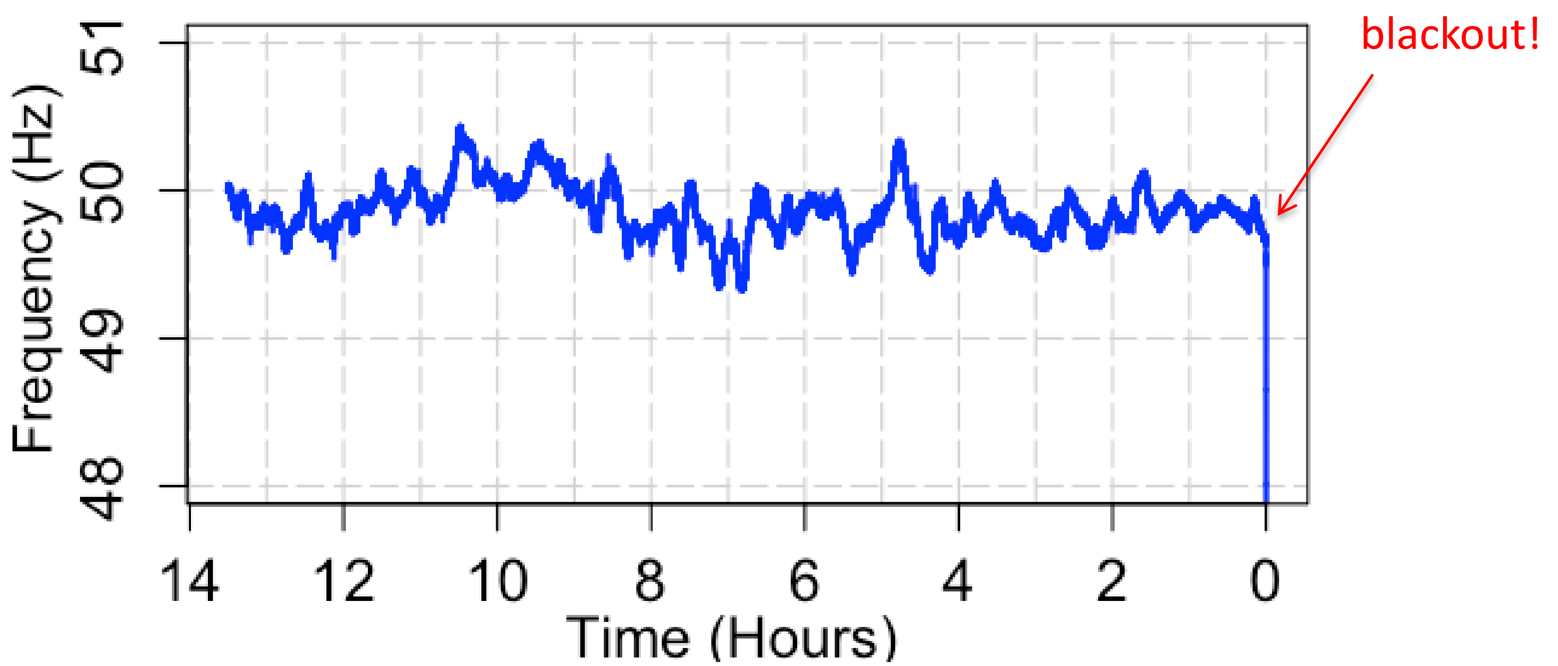
- *Why* are PMU signals fractal???
  - Fractional dynamics load modeling
  - Load aggregation
- Voltage stability
  - The loads are the "villains"
- Early warning of imminent blackout
  - *Increase* of Hurst exponent before blackout
  - *Statistical confirmation by Kendall tau*

Critical Transition in Harvested Population

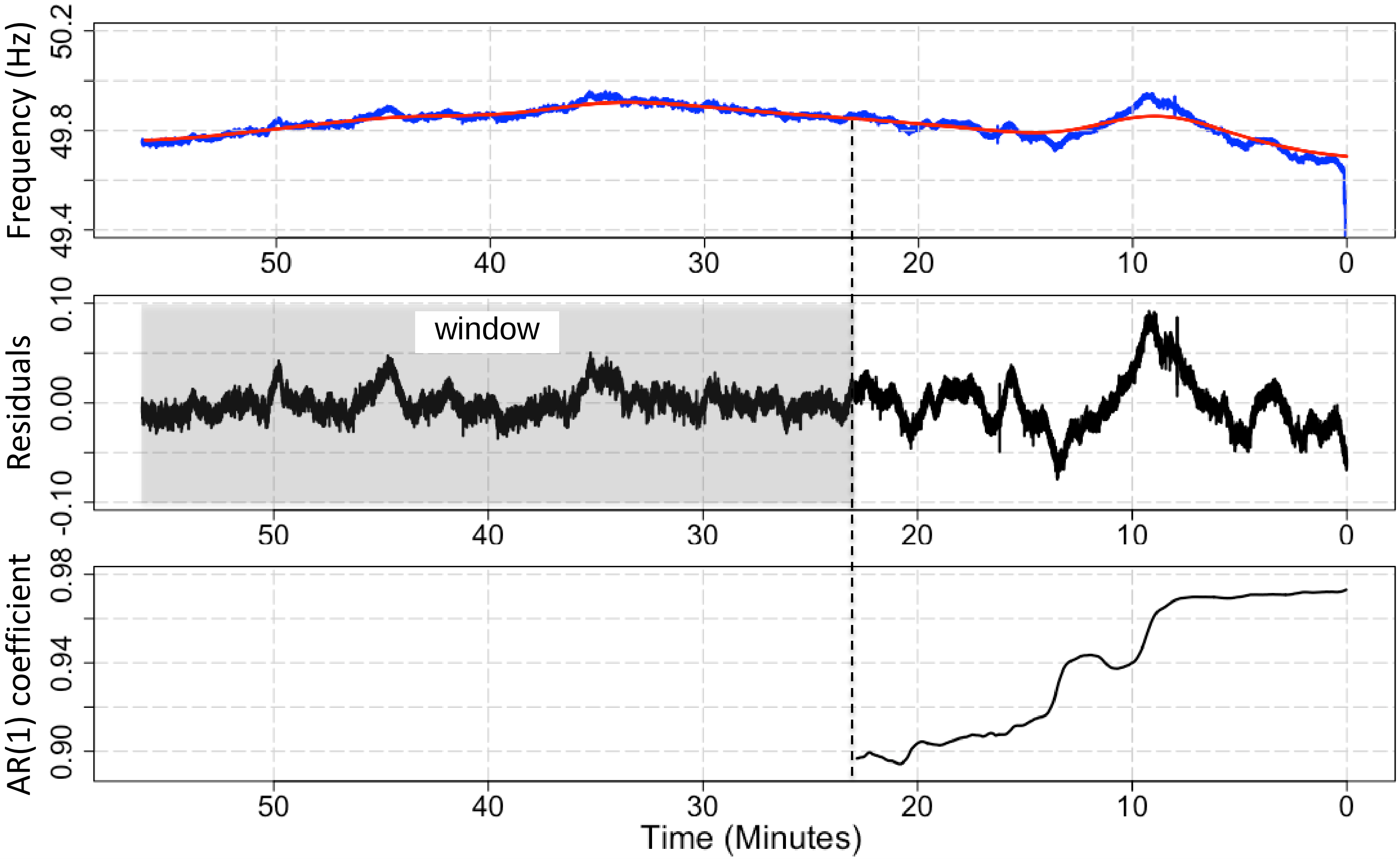


2012 Indian Blackout

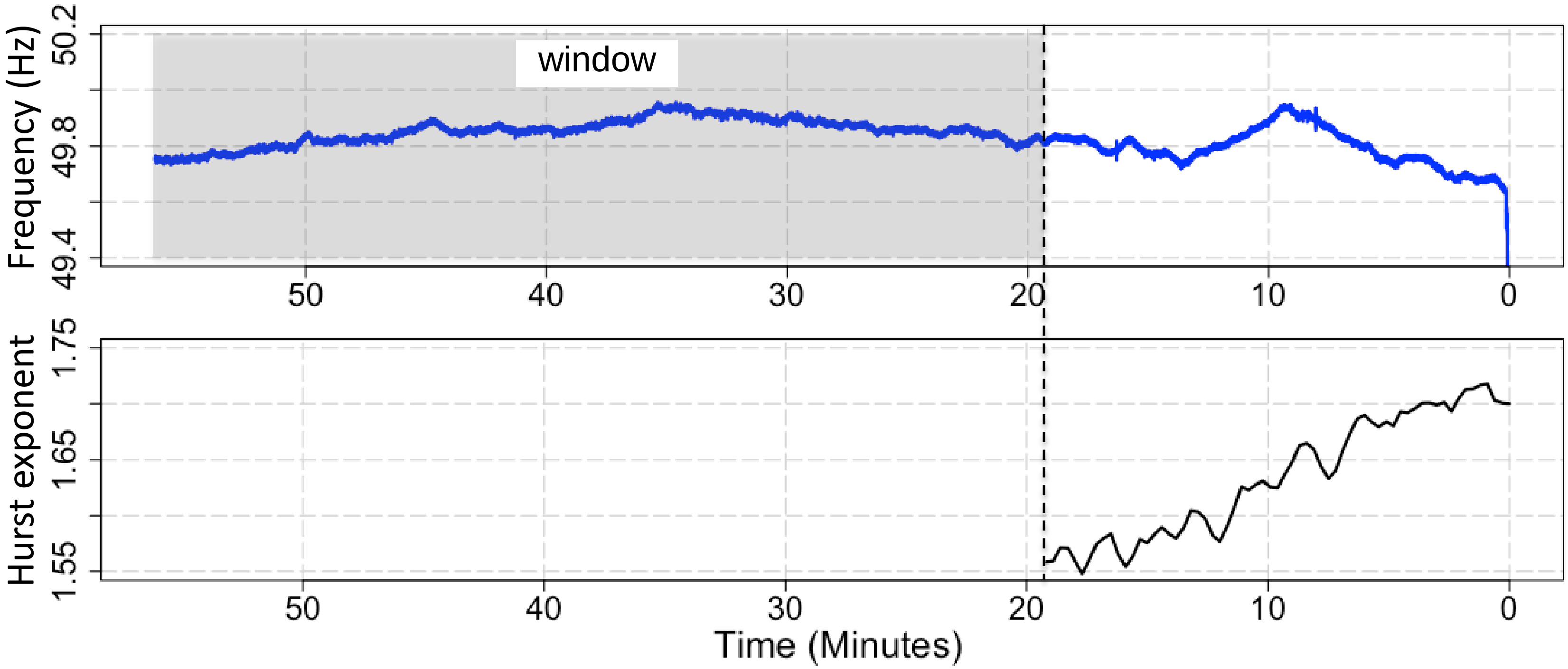
- The blackout occurred on July 30, 2012 and affected more than 300 million people living in Northern India.



Increase in Autoregressive Coefficient before Blackout



Increase in Hurst Exponent before Blackout



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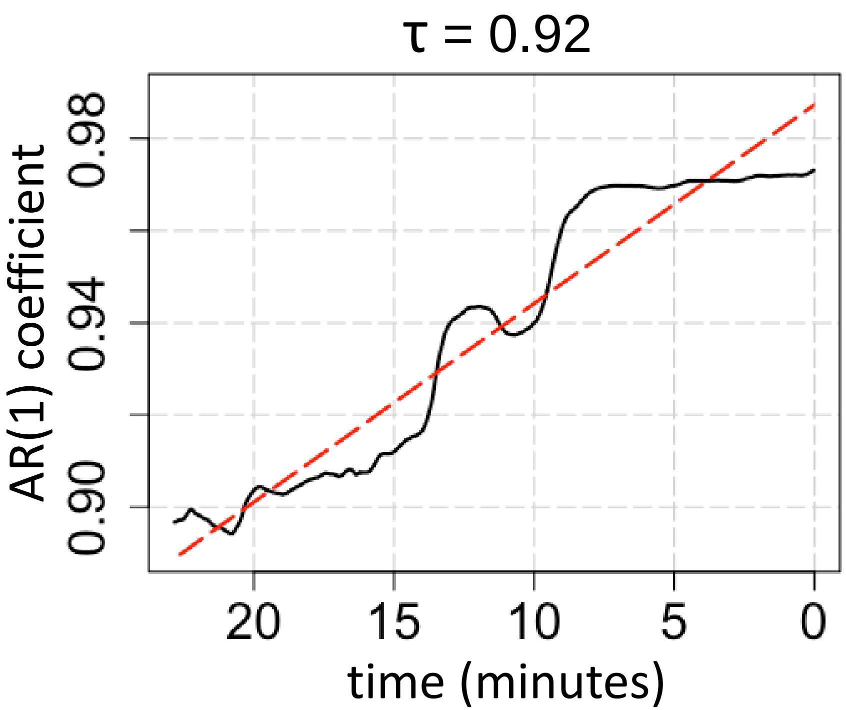
Kendall's tau

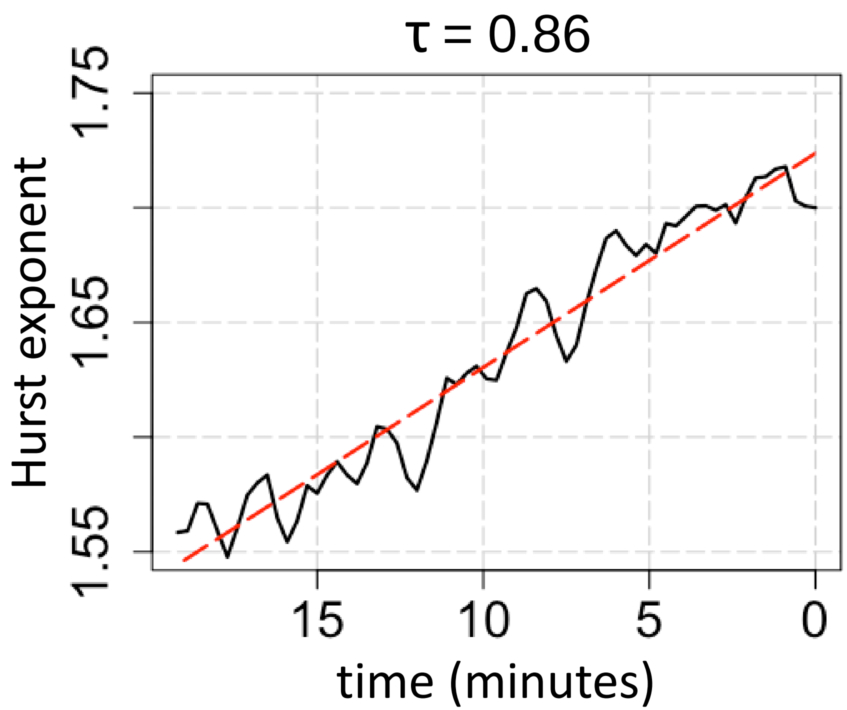
- Kendall's tau is a rank correlation coefficient that is used to measure—*in a statistically meaningful sense*—the ordinal association between two datasets, $\{(t_i,\alpha_i)\}$.
- Assuming that we have n pairs of x and y data
  - $((x_1,y_1); (x_2,y_2); \ldots; (x_n,y_n))$,
  - Kendall's tau is defined as

$$\tau = \frac{\#\text{ of concordant pairs} - \#\text{ of discordant pairs}}{n(n-1)/2}$$

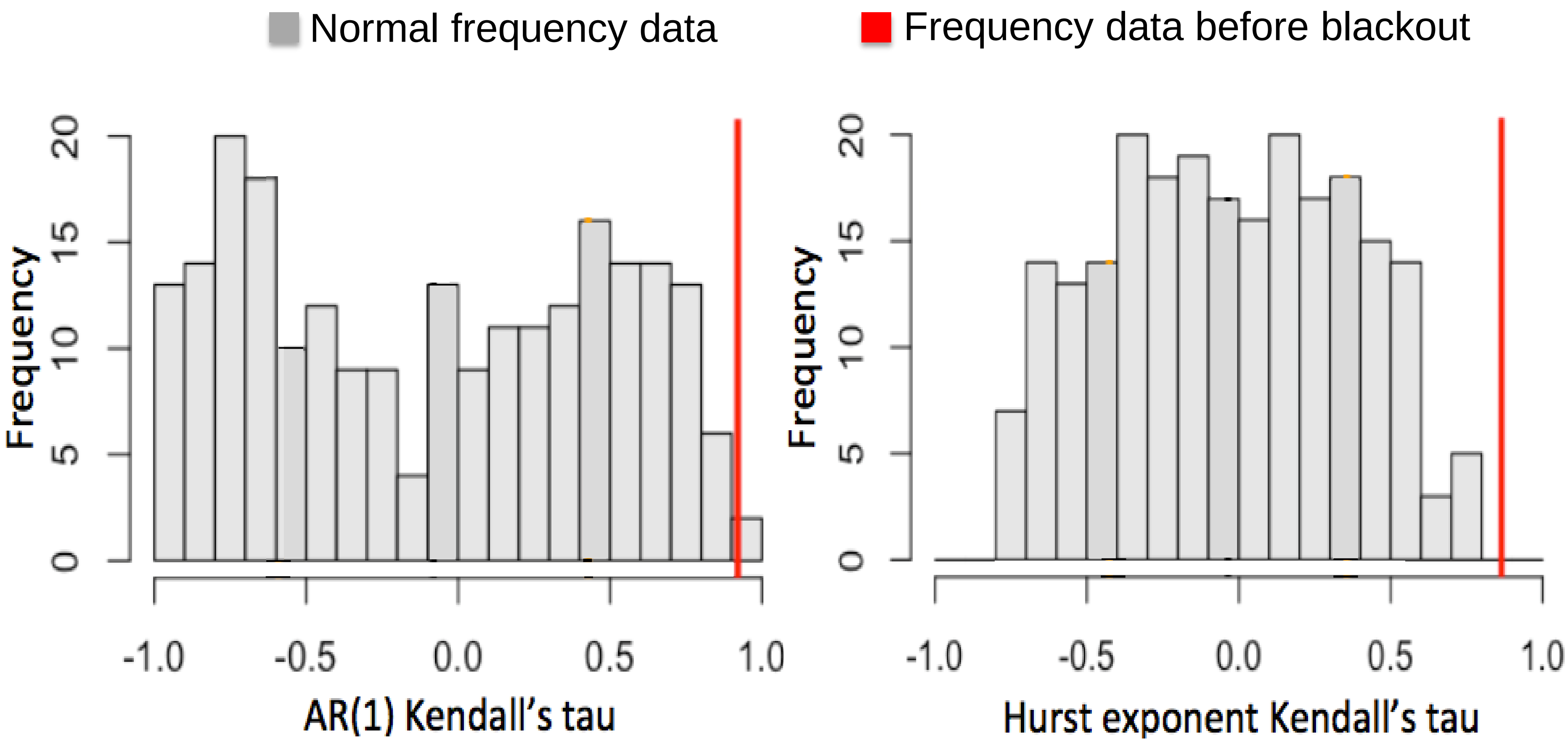
- Concordant pair ➡ $x_i > x_j$ & $y_i > y_j$ or $x_i < x_j$ & $y_i < y_j$
- Discordant pair ➡ $x_i > x_j$ & $y_i < y_j$ or $x_i < x_j$ & $y_i > y_j$

Kendall's Tau of AR(1) Coefficient versus Hurst Exponent



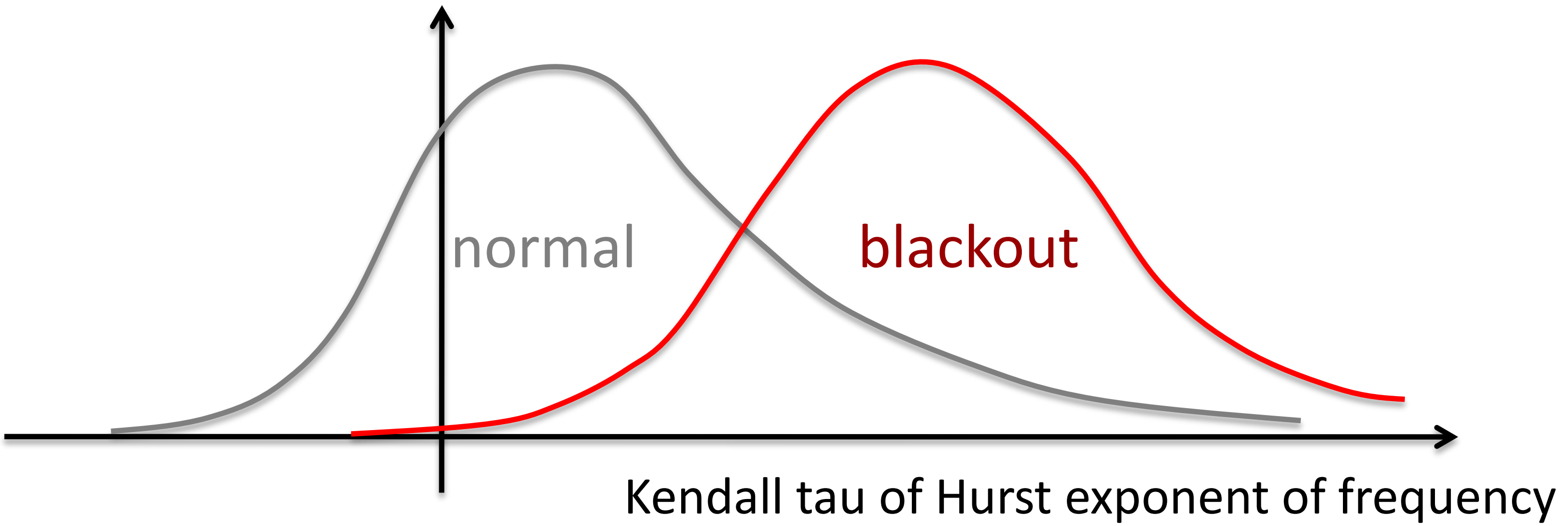


AR(1) versus Hurst Exponent Sample Distributions



Future Work

- With more blackout data points, we hope to demonstrate—with enough confidence—that the empirical distributions of the normal and blackout Hurst frequency data *are random draws from different distributions.*



Conclusions

- The fractal behavior of the PMU signals is puzzling…
- Its potential for anticipating black-out and/or cyber attacks has been demonstrated.
- So, it is of paramount importance to understand *why the PMU signals are fractal.*
  - The Berg load models provide a clue with their fractional exponents of ω.
  - In the Berg experiment, the load is modeled in its microgrid environment.
  - The aggregation of the loads combines a great many lumped parameter circuit elements *to make distributed parameter elements.*