Robust Control of Scalable Quantum Systems: Mathematical Foundations and Applications

Abstract. Fully harnessing the benefits of quantum technology confronts practical challenges in terms of robust design, control, and system identification. Quantum systems possess unique features necessitating the development of novel techniques to effectively bridge the gap from theory to application. However, lack of effective communication and collaboration between distinct communities in the physical sciences and applied engineering fields has blunted development of a *comprehensive* theory of quantum control. The programme aims to ameliorate this by bringing together researchers in the fields of control engineering, the physical sciences, and mathematics to create a common understanding of the *challenges in robust control* of quantum systems and facilitate collaboration toward resolution of these challenges. The main outcome is a *comprehensive framework* for robust control of open quantum systems that provides uncertainty bounds for the building blocks to inform the robustness of larger-scale quantum devices combining the operation of multiple gates or qubits. A secondary outcome is the establishment of criteria for the most efficient controller synthesis method based on system complexity. To achieve these ambitious goals, a distinctive *systems-theoretic and holistic approach* will be pursued, focusing on time-domain, non-linear control problems and novel aspects such as the definition of robustness in the context of *marginally stable systems*. This is timely in view of recent large-scale experimental efforts in quantum technology and metrology, which promise, for example, quantum sensors with vastly increased sensitivities, but the latter will not automatically translate into practical applications where signal-to-noise ratios matter, unless noise effects and robustness are critically addressed.

Outputs: The programme is expected to produce outputs such as a roadmap, datasets, methods and working papers. Cambridge University Press has tentatively agreed to publish these in the peer-reviewed journal *Research Directions in Quantum Technology.*

Duration: 4-Month Programme

1 Scientific Areas

- Quantum Technology and Control
- Dynamical Systems and Nonlinear Control
- Machine Learning and Differentiable Programming
- Differential Geometry and Topological Methods

2 Proposers' names and affiliations

- Sophie Shermer - (Lead Organizer) - Swansea University, UK
- Edmond Jonckheere - (Lead Organizer) - University of Southern California, USA
- Sean O'Neil (Lead Organizer / Early Career Researcher) - University of Southern California, USA
- Carrie Weidner (Equality, Diversity, and Inclusion Lead) - University of Bristol, UK
- Frank Langbein - Cardiff University, UK

3 Proposers' prior experience organizing events of this nature

Collectively the proposers have significant experience organizing programmes aimed at collaborative problem-solving in academia and government, ranging from facilitating network theory-based workshops with the Institute of Mathematics and Applications (IMA), the American Institute for Mathematics (AIM), the National Institute of Standards and Technology (NIST) and Nokia-Bell Labs to organizing defense-related conferences to guide military training plans in a resource-contained environment. The proposers have also organized summer schools, workshops, and outreach events under the QUAINT umbrella [1]. Jointly, several of the proposers have recently created a US-UK *Advanced Studies Institute in Robust Control of Quantum Networks* [2] and organized workshops and activities to equip early career researchers with the tools to contribute to the field of quantum control.

4 Outline of mathematical and scientific background

The scientific and mathematical focus addresses three major challenges in the development of a holistic robust quantum control theory. While system-specific control methods are common across the breadth of quantum applications, a comprehensive mathematical framework for robust control of quantum systems does not exist. Such a framework would serve to unify niche processes and translate the laboratory advances of the second quantum revolution into usable technology in the near and foreseeable future.

4.1 The non-classical dynamics of quantum systems

Closed quantum systems exhibit purely oscillatory dynamics that describe a marginally stable system, while open quantum systems display modes that decay to a steady-state but still retain eigenvalues on the imaginary axis [23, 24]. Both preclude the application of classical control techniques based on the synthesis of stabilizing controllers to modify the evolution of the system. Further, the concept of stability inherited from classical control generally implies the stabilization of a quantum system in a classical steady-state that nullifies the desirable features of coherence or entanglement [26]. This calls for the development of control strategies that are not simply a specialized application to niche quantum systems but a reworking of control theory to address systems with marginally stable dynamics where classical stabilization is not desired. Secondly, though some feedforward control algorithms effectively and successfully formulate the problem as a bilinear control problem, which modifies the natural Hamiltonian as $H = H_0 + \sum_k H_k(u(t))$, the controllability conditions based on the Lie Algebra of $\{H_0, H_k\}$ are weak in terms of providing detailed information beyond the binary answer of whether a system is controllable or not [13]. Conditions providing degrees of controllability would benefit the process of controller design and selection. Finally, in terms of observability, the challenge of quantum measurement prevents application of typical closed-loop observer techniques without accepting convergence of the quantum system to a steady-state that is often classical. Taken together, these issues call for a reformulation of control theory for marginally stable systems with weak controllability and highly restrictive observability conditions that this programme can address.

4.2 Sensitivity Analysis and Robustness Guarantees

The efficacy of sensitivity analysis techniques to provide robustness guarantees remains an open question in the field of quantum control. Answers to this issue are essential to realizing the promised benefits of quantum technology beyond the laboratory. In contrast to the asymptotic performance measures of classical control's regulation and tracking problems, the performance measures of quantum systems, from the readout fidelity of a quantum router to the operation of quantum gates, are largely time-domain problems. This firstly begs the applicability of the frequency domain-based sensitivity techniques of classical control and highlights the need to synthesis new methods to assess both differential sensitivity and robustness to larger-scale uncertainty. In terms of differential sensitivity, nascent time-domain approaches [20] based on structured uncertainty require unification with more general topology-based techniques [16] capable of analyzing unstructured uncertainties. Likewise, statistically-based methods reliant on estimated probability distributions show promise in inferring the robustness and performance of quantum controllers [17]. However, these distinct methods lack a unifying mathematical underpinning that would equip researchers with the ability to determine the best tool for the analysis task at hand. More generally, only in limited cases do robustness guarantees exist in terms of largest allowable perturbations that destabilize a quantum state or violate performance measures [19, 21]. One goal of this programme is to provide a framework to develop robustness guarantees in the form of largest allowable parameter variations that would still permit a given level of performance in terms of non-linear correlations such as entanglement or coherence. These guarantees would translate directly to more efficient use of limited resources in the development of controllers by filtering those that exhibit unacceptable robustness bounds.

4.3 Controller Synthesis

Efficient design of robust and/or optimal controllers remains a significant challenge. Though a multitude of approaches exist from open-loop control to measurement-based feedback to coherent feedback [27], the choice of implementation is often based on the specific application or knowledge of control techniques from the relevant community (i.e. NMR, optics, quantum information). Further, the effectiveness of the control implemented is heavily influenced by the desired output function, be it error correction or state preparation [22]. Most quantum control designs also rely on dynamic control and are therefore not amenable to formulation as time-invariant control problems. Some techniques such as energy landscape shaping [24, 25] exist; these can be formulated as linear time-invariant control systems, do not require for measurement-based feedback and are capable of producing controllers with high fidelity and desirable robustness properties. However, these methods are not universally applicable and still require customized controller design. Traditional methods of designing these controllers are based in non-convex optimization in a complex landscape with multiple restarts to provide the best (highest fidelity) controllers. This raises the question of if non-convex optimization is the most efficient method for controller design. Are there more efficient methods based in differentiable programming or machine learning that have the potential to find equally effective controllers with less resource expenditure? Further, are there complexity measures inherent in the given system and performance indices that indicate whether one method is better suited to controller synthesis than another? As an output of the programme, an answer to this question would influence not just design of static controllers, but measurement-based feedback methods that have already shown compatibility with machine-learning design [12].

4.4 Proposed Workshops

We propose the following three workshops to set the stage, guide the program, and allow a venue for the dissemination of results:

1. Stage Setter - Fundamentals and Current State of Robust Control
2. Controller Synthesis - Techniques and Theory Development
3. Robustness - Analysis Methods and Guarantees

The workshops will be the busiest weeks of the programme and we encourage the maximum participation during these weeks. Each workshop will consist of a combination of morning seminars and afternoon discussion sessions. The Thursday afternoon discussion each week will be organized around poster presentations to allow early career researchers to present their work on the specific workshop topic or related research. The first workshop will provide the opportunity to baseline all participants on the theory and current state of robust quantum control, allow researchers to share best practices, and permit collaborative refinement of the outstanding problems the programme will tackle. The second workshop will center on the issue of controller synthesis, the range of techniques currently employed by experts, implementation of the most efficient methods for a given task, and, ideally, a road map for a theory that encompasses the range of techniques. The final workshop will focus on robustness of the control algorithms currently in use as well as any proposed techniques from the second workshop with the output of robustness guarantees for a given controller implementation. The placement of these workshops within the overall program structure is discussed in Section 7.

5 Why the programme is needed

There is considerable scope for disseminating research results online. Modern technology offers many excellent tools for collaboration between researchers across time zones and within virtual communities. Topical workshops and conferences offer the opportunity to learn about the latest advances in an area of interest. However, scientific programmes that bring together geographically distributed researchers with different areas of expertise in one place to collaborate over an extended period of time offer the opportunity to conduct an in-depth analysis of the state-of-the-art in the field, share expertise and ideas, identify broader challenges that need to be overcome for the field to progress and mature, and create a general framework to address these. In terms of technological advancement, the current state of research is ripe for robustness guarantees to improve state preparation and scalability in quantum dots, superconducting quantum circuits, and diamond color centers, and cold atoms and ion-based systems for use in applications from biological imaging to quantum computing [11, 15, 18]. The output of this programme has the potential to influence these and other initiatives in the present and near term.

5.1 Why Newton Institute

The proposed programme fits into the remit of applied mathematical sciences. In particular the prominent role of topological methods, differential geometry, and the mathematics of deep learning in the proposed programme coincides with themes of earlier workshops at the Newton Institute such as ‘Quantum control engineering: mathematical principles and applications’, ‘Mathematical challenges in quantum information’, ‘Mathematics of deep learning’, ensuring the institutional capacity to provide first-rate expertise to quantum researchers. Additionally, the infrastructure of the Newton Institute is conducive to effective collaboration of the longer-term research tracks described in Section 7 punctuated by shorter, intensive workshops.

5.2 Why it would benefit the UK

The UK aims to be a world leader in the area of quantum technologies through the National Quantum Technologies Programme (NQTP). Other countries, the US in particular, follow suit. The proposed programme firmly nests with this initiative, will attract international talent, and inspire young UK and US researchers to pursue research in this area. Further, members of the hubs will be invited to participate in the programme, ensuring a unity of effort between the NQTP priorities and the output of the programme. The anticipated output of the programme will provide a mathematical system-theoretic foundation for robust control of quantum systems, applicable to the breadth of the four quantum technology hubs. By formulating challenges as theoretical problems in the language of dynamical systems, control theory, optimization and machine learning, etc, we can both harness knowledge and expertise in these areas and inspire researchers in mathematical sciences to study these problems and, ideally, develop a theoretical framework that is applicable to a wide range of physical realizations of quantum technology.

5.3 How we improve diversity

We aim to improve diversity in the field during our program in both the scope of the initial invitation and use of the Newton Institute's developed capability to accommodate special needs of underrepresented groups. In terms of our initial invitation to researchers, of 55 priority invites, 19% identify as female. Additionally, our potential participant listing covers 14 different countries encompassing Europe, East Asia, Southwest Asia, Oceania and the Western Hemisphere, ensuring an equitable representation of diverse geographical regions. In terms of accommodation, we plan to make full use of the child care services at the Newton Institute to ensure that researchers with children can still participate to the fullest extent. We also plan to take full advantage of the accessibility features of the Institute's infrastructure to ensure full access to all workshops and meetings for researchers with physical disabilities, ensuring their inclusion to participate fully. Finally, we will advertise the programme widely and encourage senior researchers to nominate Early Career Researchers (ECRs) who could benefit from the programme. We aim for least 25% of participants to be ECRs and can potentially assist in funding for those ECRs who would otherwise be unable to attend.

6 Potential Participants - See attached spreadsheet

7 Programme Structure

The structure of the programme is based on a 14-week model. The first week will be dedicated to conditions setting and apply mainly to the organizers and key speakers for the first workshop. The workshops as described in Section 4.4 will occur on Weeks 2, 8, and 14 of the programme. To guide progress between workshops while allowing the maximum collaborative potential, participants will be asked to organize themselves into various research tracks that bridge the workshop topics. These research tracks will be designed to especially engage Early Career Researchers who will be encouraged to remain on-site at the Newton Institute, taking advantage of the opportunity to collaboratively address the challenges. Participants who cannot remain on-site for the duration will have the opportunity for remote collaboration through virtual meetings. Specifically, bridging the stage setter and controller synthesis workshops between Weeks 3 and 7, we propose the research tracks:

1. Controllability and Closed Systems - Marginal Stability
2. Controllability and Open Systems - Decoherence and Dissipation
3. Numerical Methods of Controller Synthesis
4. Analytic Methods of Controller Synthesis

To complement these topics and build to the final workshop we propose the research tracks for Weeks 9 to 13:

1. Dynamics of entanglement as a performance measure
2. Quantum measurement issues
3. Topological methods in robustness analysis
4. Curvature, differential geometry, and sensitivity analysis

To capture the outputs of each research track, the research group composing the track will compose a short (conference-length) paper to summarize results. This same information will be presented in the workshops following the termination of each track. All workshop presentations and lectures will be recorded. The programme's output (proceedings, papers, and presentation videos) will be made available to all interested researchers in a public repository under the Cambridge University Press Research Directions: Quantum Technologies.

7.1 Related Meetings

Although there are have been many conferences and workshops on quantum technology and control, including recent and upcoming ones (e.g., [3, 4, 14]), there have not been many extended programmes dedicated to the mathematical aspects of quantum control for some time. The most relevant programmes took place at KITP in 2009 [5] and 2013 [6] and last relevant programme at the Newton Institute took place in 2014 [7]. While several KITP programs in the coming year are relevant to quantum technology [8, 9, 10], none are focused on resolution of the robustness challenges of the proposed programme.

7.2 External Funding

We have not currently applied for other funding for this activity.

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[2] URL: https://lw1660.wixsite.com/quantumcontrol/post/us-uk-advanced-studies-institute-in-robust-control-of-quantum-networks.

[3] Oct 6-13, 2021. URL: https://qusco-itn.eu/international_conference/.

[4] Aug 29-Sep 2, 2022. URL: http://www.fields.utoronto.ca/activities/22-23/CQIQC-IX.

[5] URL: https://www.kitp.ucsb.edu/activities/QCONTROL09.

[6] URL: https://www.kitp.ucsb.edu/activities/qcontrol-c13.

[7] URL: https://www.newton.ac.uk/event/qce/.

[8] URL: https://www.kitp.ucsb.edu/activities/manybody23.

[9] URL: https://www.kitp.ucsb.edu/activities/qmetro23.

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