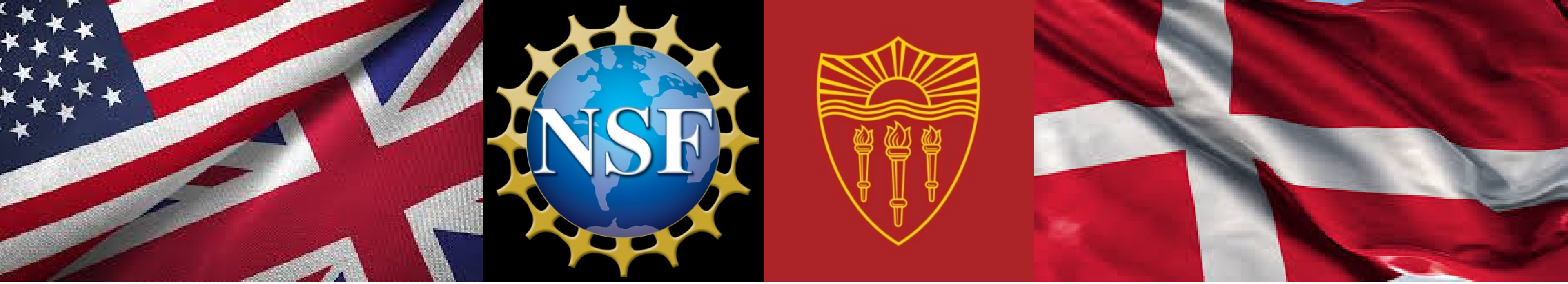

National Science Foundation (NSF)
Grant IRES 1829078
IRES TRACK II: US-UK international student research in robust control of quantum networks

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SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
1951

DARPA

US-UK international student research in robust control of quantum networks

- People (students, postdoc, faculty) involved
- Venue: Wales, United Kingdom
- First week program in Cardiff: spintronics control
- Second week program in Swansea: photonics
- Products
  - Classical versus quantum fundamental limitations
  - Structurally uncertain quantum network robustness
- Social (or aerospace?) activity: Wales Air show

NSF Grant IRES 1829078
Principal Investigators

Swansea University
Prifysgol Abertawe

CARDIFF
UNIVERSITY
PRIFYSGOL
CAERDYDD

Prof. Sophie Shermer

Prof. Edmond Jonckheere

Dr. Frank Langbein

US Students, postdoc participants

USC University of Southern California

RUTGERS

Washington University in St.Louis

Emily Reed,
Ph.D. student,
Univ. of South. Calif.

Benjamin Sheller
Postdoc, Rutgers Univ.,
Consultant, Iowa State Univ.

Jonathan Monroe
microelectronics physicist at
BOEING

Eliav Maas
System engineer,
startup

Carrie Weidner
Ph.D., U. Col., Boulder,
Postdoc, Aarhus Univ.,
Denmark

University of BRISTOL
Senior lecturer

"Carrie [Weidner] comes to Bristol with the ultimate goal of developing a quantum gas microscope with two orthogonal axes of high-resolution imaging for use in quantum simulation, but she plans to start with an ultracold atom setup that will be used (among other things) to study robust quantum control."

US Students personal statements

"I enjoyed the experience very much. The environment was very conducive for learning and exploring ideas, and especially the chance to engage with each other outside of the formal lecture environment was very useful. I would highly recommend the experience to others, or take advantage of it again if able. I am still in semi-regular contact with several of the other participants, and hope to work more with them in the future, as a result of the discussions we had at the time."

Benjamin Sheller,
Postdoc, Rutgers University

"I am grateful to have had the opportunity to attend the international robust quantum control workshop in the UK in 2019. Not only did I learn a lot about the foundations of robust quantum control, but I also was able to form relationships with new colleagues in this area of research. Out of this, we were able to write and submit a tutorial paper that outlines robust quantum control for new researchers and overviews the current challenges in this exciting area of study. I look forward to continuing to research in this area and collaborate with my colleagues."

Emily Reed,
Ph.D. student, Univ. of South. California

"The ASI was a phenomenal opportunity to learn from experts I would have otherwise not had the pleasure of meeting. The subjects we covered opened incredible opportunities in my current position, and I have benefitted immensely from the experience of learning alongside my new peers. The fact that we were able to publish a paper based on our discussions is an apt testimony to the depth of learning and productivity that I enjoyed at ASI."

Jonathan Monroe,
Microelectronics physicist at Boeing

US postdoc personal statement

"In many ways the ASI was a highlight of my postdoc. It was excellent to connect with students, postdocs, and professors in robust control, a field that I was (and still am) quite new to. I bonded immediately with my peers and truly enjoyed the balance of study and leisure that the program provided, because this facilitated discussion amongst all of us—even resulting in a paper under review. The collaboration that I have built up as a result of the ASI is, without a doubt, one of my most productive in terms of measurable research outputs and sheer enjoyment, and I am very grateful to have taken part."

Dr. Carrie Weidner
Lecturer, Bristol University

British student participants

Anastasia Ugaste,
Software engineer/
HP computing

Benedict Uttley,
Bioinformatician/
Computational Biologist

Chris Davis-Jenkins,
Postdoc
at Johns Hopkins
specializing in
Magnetic Resonance
spectroscopy

Max Chandler,
Site reliability engineer

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CITY AND COUNTY OF SWANSEA • DINAS A SIR ABERTAWE •

Swansea University
Prifysgol Abertawe

Anne Boden
Welch tech
entrepreneur

CARDIFF
UNIVERSITY
PRIFYSGOL
CAERDYDD

Iain Maxted,
Founder
Guardian Global
Technologies

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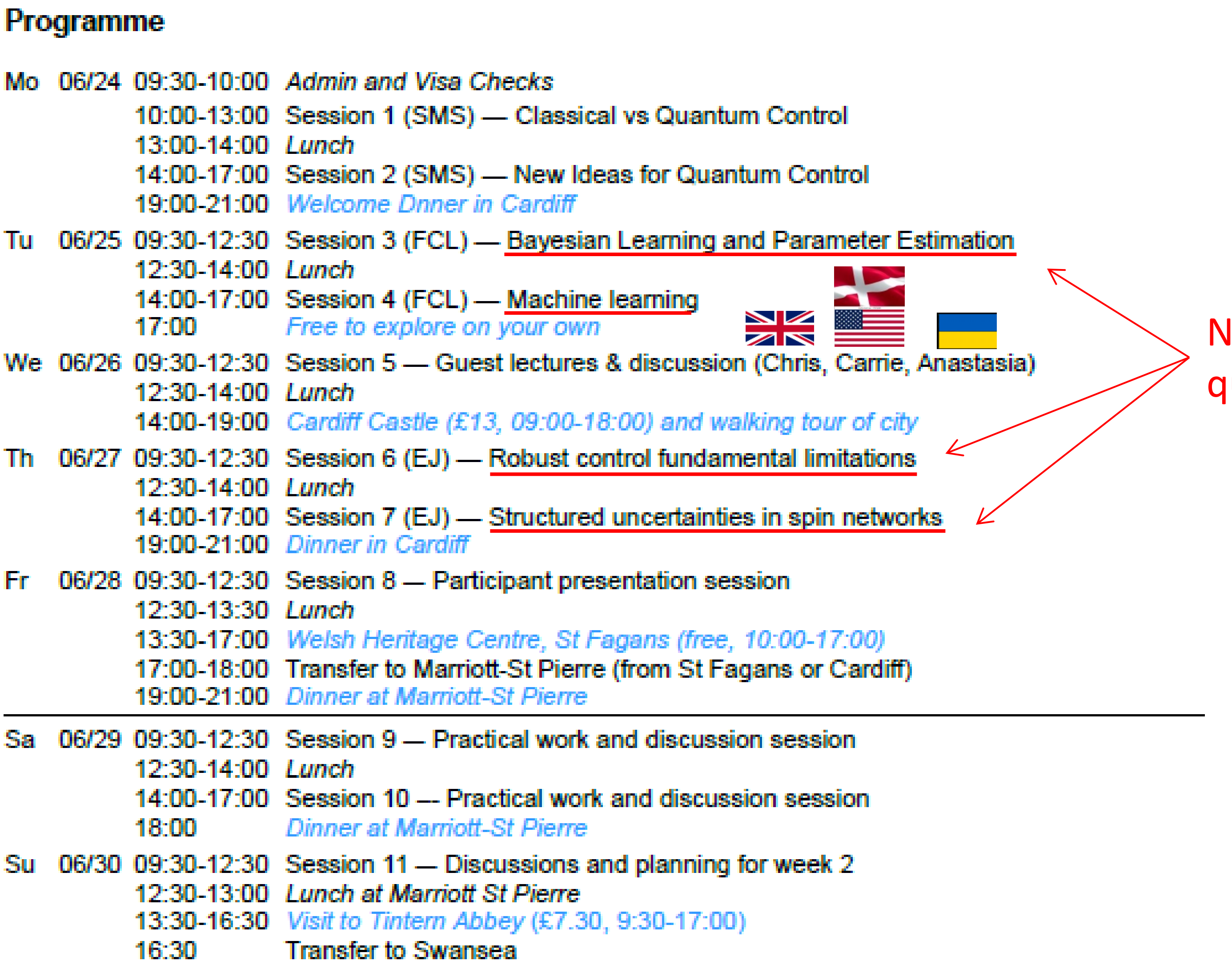
First Week Program in Cardiff

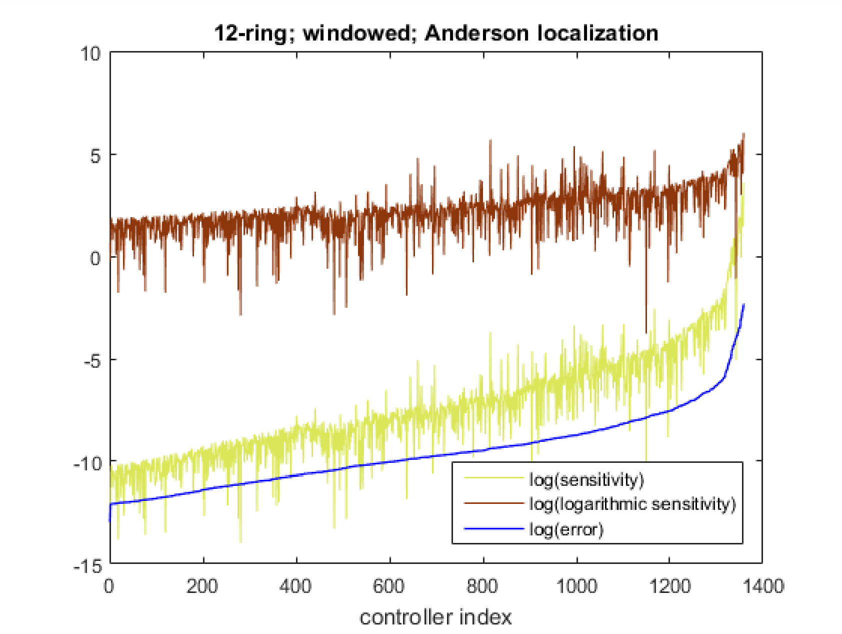


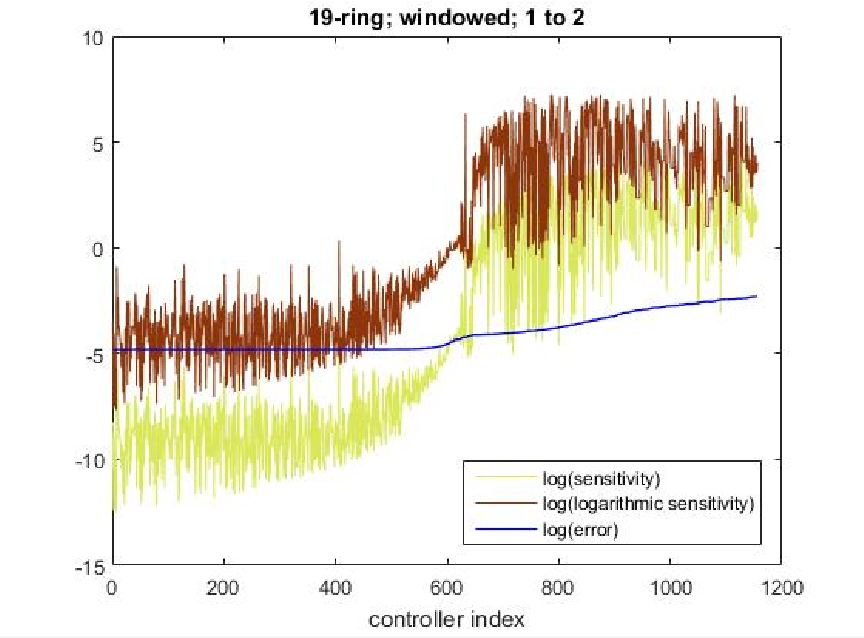
Programme

Mo	06/24	09:30-10:00	*Admin and Visa Checks*
		10:00-13:00	Session 1 (SMS) — Classical vs Quantum Control
		13:00-14:00	*Lunch*
		14:00-17:00	Session 2 (SMS) — New Ideas for Quantum Control
		19:00-21:00	*Welcome Dnner in Cardiff*
Tu	06/25	09:30-12:30	Session 3 (FCL) — Bayesian Learning and Parameter Estimation
		12:30-14:00	*Lunch*
		14:00-17:00	Session 4 (FCL) — Machine learning
		17:00	*Free to explore on your own*
We	06/26	09:30-12:30	Session 5 — Guest lectures & discussion (Chris, Carrie, Anastasia)
		12:30-14:00	*Lunch*
		14:00-19:00	*Cardiff Castle (£13, 09:00-18:00) and walking tour of city*
Th	06/27	09:30-12:30	Session 6 (EJ) — Robust control fundamental limitations
		12:30-14:00	*Lunch*
		14:00-17:00	Session 7 (EJ) — Structured uncertainties in spin networks
		19:00-21:00	*Dinner in Cardiff*
Fr	06/28	09:30-12:30	Session 8 — Participant presentation session
		12:30-13:30	*Lunch*
		13:30-17:00	*Welsh Heritage Centre, St Fagans (free, 10:00-17:00)*
		17:00-18:00	Transfer to Marriott-St Pierre (from St Fagans or Cardiff)
		19:00-21:00	*Dinner at Marriott-St Pierre*
Sa	06/29	09:30-12:30	Session 9 — Practical work and discussion session
		12:30-14:00	*Lunch*
		14:00-17:00	Session 10 –- Practical work and discussion session
		18:00	*Dinner at Marriott-St Pierre*
Su	06/30	09:30-12:30	Session 11 — Discussions and planning for week 2
		12:30-13:00	*Lunch at Marriott St Pierre*
		13:30-16:30	*Visit to Tintern Abbey* (£7.30, 9:30-17:00)
		16:30	Transfer to Swansea

New pitch in quantum control

Spin coupling uncertainty: Mostly anti-classical behavior, especially Anderson localization





The hint for this apparently aberrant behavior is a discrepancy between the classical and the quantum concept of error:

$$\left\| e_{\mathrm{proj}}(t) \right\| := \min_{\varphi(t)} \left\| |\mathrm{OUT}\rangle - e^{\varphi(t)} |\Psi(t)\rangle \right\| << \left\| |\mathrm{OUT}\rangle - |\Psi(t)\rangle \right\|$$

F. C. Langbein, S. G. Schirmer and E. Jonckheere, “Static bias controllers for XX spin-1/2 rings,” Data set, figshare, DOI:10.6084/m9.figshare.3485240.v1, July 3, 2016.

E. Jonckheere, S. Schirmer and F. Langbein , “Jonckheere-Terpstra test for nonclassical error versus log-sensitivity relationship of quantum spin network controllers, *International Journal of Robust and Nonlinear Control,* **28**, pp. 2383-2403, 2018. arXiv:1612.02784 [math.OC].

S. Schirmer and E. Jonckheere and F. Langbein, “Design of feedback control laws for spintronics networks,” IEEE AC, 2018. arXiv:1607.05294.

British and American students interacting during a coffee break in Cardiff University

Professor Schirmer lecturing

End of First Week Program in Chepstow

Programme

Mo	06/24	09:30-10:00	*Admin and Visa Checks*
		10:00-13:00	Session 1 (SMS) — Classical vs Quantum Control
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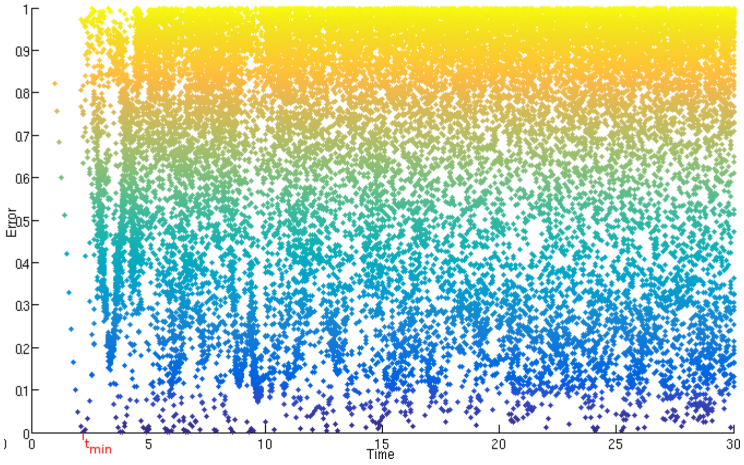

Quantum dynamics is ergodic (there is Poincare recurrence of the minimum fidelity error.)
Is it (exponentially?) mixing??? What is the distribution of the return time? Erlang?

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Second Week Program in Swansea

Quantum Light Sources using InAs quantum dots

David Ritchie[1,4]

Mark Stevenson[2], Robert Young[1,2], Andy Hudson[1,2], Cameron Salter[1,2], Raj Patel[1,2], Antoine Boyer de la Giroday[1,2], Matt Pooley[1,2], David Ellis[1,2], Andre Schwagemann[1,2], Christiana Varnava[1,2], Anthony Bennett[2], Martin Ward[2], Joanna Skiba-Syzmanska[2], Paola Atkinson[1], Ken Cooper[1], Ayesha Jamil[1], Christine Nicoll[1], Ian Farrer[1,3], Peter Spencer[1], Andrey Krysa[3], Jon Heffernan[3], Andrew Shields[2]

[1] Cavendish Laboratory, University of Cambridge
[2] Quantum Information Group, Toshiba Research Europe
[3] University of Sheffield
[4] Swansea University

Engineering and Physical Sciences Research Council

Mo	07/01	09:30-12:30	Session 12
		12:30-14:00	*Lunch*
		14:00-17:00	Session 13
		18:00	*Swansea Marina and Dinner (Meridian Tower)*
Tu	07/02	09:30-12:30	Session 14 — Guest lectures, Lab Tours, Discussion
		12:30-14:00	*Lunch*
		14:00-19:00	*Brecon Beacons or Pembrokeshire Trip*
We	07/03	09:30-12:30	Session 15
		12:30-14:00	*Lunch*
		14:00-17:00	Session 16
		19:00-21:00	*Dinner at Mumbles*
Th	07/04	09:30-12:30	Session 17 — Guest lectures, Lab Tours, Discussion
		12:30-14:00	*Lunch*
		14:00-19:00	*Gower trip (Arrows, Talons and Tea at Perriswood, Dinner at Wormshead)*
Fr	07/05	09:30-12:30	Session 18
		12:30-14:00	*Lunch*
		14:00-17:00	Session 19 — Participant final presentations
		17:00	*Free time*
Sa	07/06	*Welsh National Airshow (Swansea)*	
Su	07/07	*Welsh National Airshow (Swansea)*	

Second Week Program in Swansea

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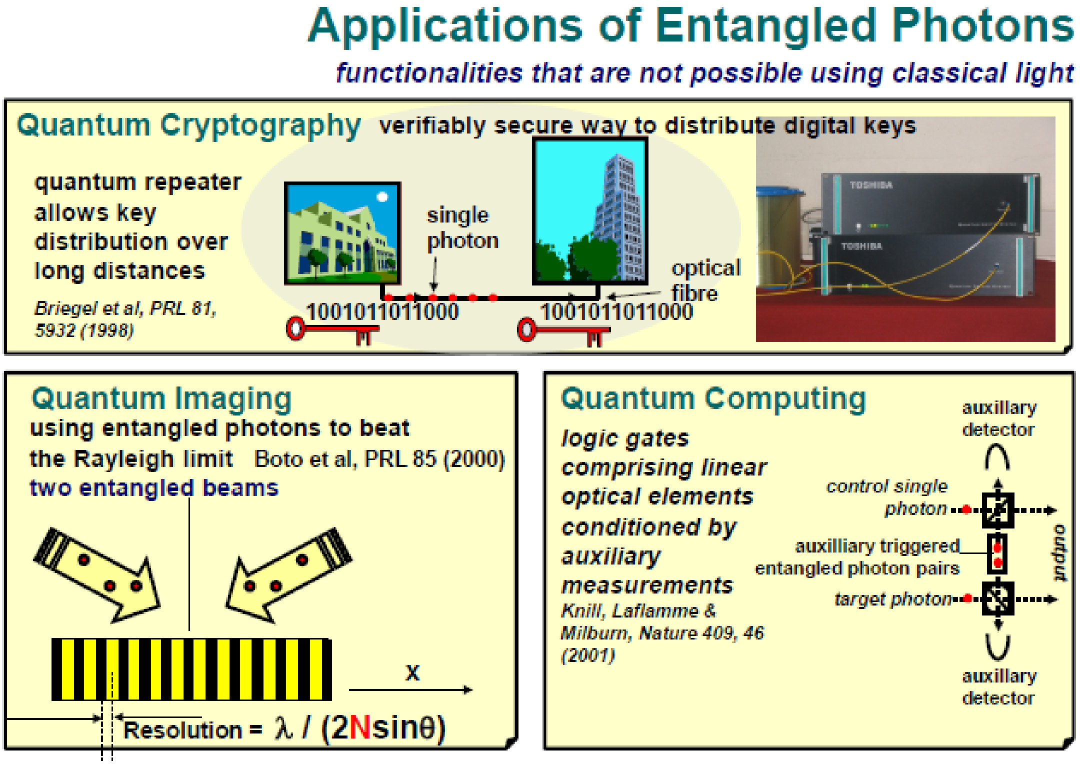

Applications of Entangled Photons

functionalities that are not possible using classical light

Quantum Cryptography verifiably secure way to distribute digital keys

quantum repeater allows key distribution over long distances

Briegel et al, PRL 81, 5932 (1998)

Quantum Imaging

using entangled photons to beat the Rayleigh limit Boto et al, PRL 85 (2000)

two entangled beams

Resolution = $\lambda / (2N\sin\theta)$

Quantum Computing

logic gates comprising linear optical elements conditioned by auxiliary measurements

Knill, Laflamme & Milburn, Nature 409, 46 (2001)

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Robust Quantum Control in Closed and Open Systems: Theory and Practice

C.A. Weidner, E.A. Reed, J. Monroe, B. Sheller, E. Maas, E.A. Jonckheere, F.C. Langbein, S.G. Schirmer

Abstract—Robust control of quantum systems is an increasingly relevant field of study amidst the second quantum revolution, but there remains a gap between quantum physics and robust control. To develop general theories of robust quantum control, this gap must be minimized, as general quantum systems are not amenable to analysis via classical robust control techniques, e.g., the formulation as linear, time-invariant problems. This tutorial is written for the control theorist and presents an introduction to quantum systems, issues that arise when applying classical robust control theory to quantum systems, typical methods used by quantum physicists to explore such systems, and a discussion of open problems to be addressed in the field. This tutorial's focus on general, practical applications allows the control researcher to understand and begin applying their knowledge to advance this burgeoning field.

Index Terms—Quantum Systems, Quantum Information, Quantum Control, Robust Control

I. INTRODUCTION

As quantum technologies continue to mature, their development will transition from proofs-of-principle to well-engineered systems with numerous commercial applications in computing, sensing, and networking. However, the transformation of quantum technologies into the real-world application space requires the development of robust means to control and manipulate these quantum systems. Quantum control theory has been developed to the point where a number of textbooks [1], [2] and comprehensive review papers [3]–[8] have been written on the subject. While classical robust control is extensively studied and well-understood [9], rigorous development of robust control protocols for quantum mechanical systems remains an open field of research as classical methods cannot be readily applied to quantum systems in general.

Coherent quantum control is naturally formulated in terms of bilinear control systems with time-dependent controls that do not map easily to the framework of robust linear control, and coherent quantum systems are only marginally stable. Progress in decoherence-based state preparation [10], [11] and bath engineering [12] has not strongly leveraged robust control theory. Therefore, more research is needed into the theoretical underpinnings of robust quantum control as well as practical applications and eventual implementation into real systems. The overarching questions still remain to be answered: Can a quantum system ever be inherently robust, especially in the absence of stability? What are the fundamental device limitations established by quantum robust control protocols? Will we ever be able to move past the current noisy, intermediate-scale quantum (NISQ) era and build useful, scalable, and robust devices that are promised by the second quantum revolution? This remains to be seen, but some hope can be offered by the success of related applications that rely on quantum phenomena and control such as nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI) (see, e.g., [13]–[17], among many others). If we can see a coherent signal from the many protons contained in the water that makes up (most of) the noisy, squishy, and chaotic human body, there may yet be hope for large-scale quantum computers.

As a result of the relative immaturity of robust quantum control, the barrier to entry into the field is quite high, as there are few good references for students and researchers in related fields to gain an overview of the state-of-the-art and the open questions in the area. This tutorial attempts to fill this gap. It is written for beginning graduate students and researchers from a variety of backgrounds, including quantum chemistry, optical engineering, mathematics, or quantum information, but we focus especially on the classical robust control researcher. Our goal is to introduce quantum theory and explain why classical robust control theory cannot be directly mapped onto quantum systems. We further introduce the Lie algebraic theory of geometric quantum control and discuss how quantum control is currently applied in practical settings. Where applicable, we include basic examples as well as relevant references. We do not cover measurement-based control, coherent feedback control, or Lyapunov control, as these are covered by existing tutorial papers and texts [18]–[20], with an introduction to robust control for linear quantum systems found in [21]. In particular, for a special class of quantum optical systems, the quantum system is mappable onto a linear, time-invariant (LTI) system; see [7], [22] for excellent reviews of these systems. All references listed previously are excellent resources for the interested student looking to supplement what they can learn in this tutorial. Our approach is to be as general as possible through a discussion of closed and open quantum systems, robust control challenges framed in the context of classical control, and current methods for finding optimal controls with notions of robustness in practice.

The tutorial is organized as follows: Section II introduces quantum systems, starting from fully coherent Hamiltonian systems and moving into dissipative Lindbladian systems and the Bloch representation of such systems. We also introduce the example problems that will be revisited throughout the tutorial. Section III discusses the issues that arise when applying classical robust control methods to quantum systems, particularly those stemming from the differences between classical linear systems to the bilinear systems described by quantum mechanics. We discuss the notions of controllability and observability

This work was partially supported by NSF IRIS 1829078.

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BS is with the Department of Mathematics and Computer Science, Rutgers University, Newark, NJ 07102, shellerba.math@gmail.com.

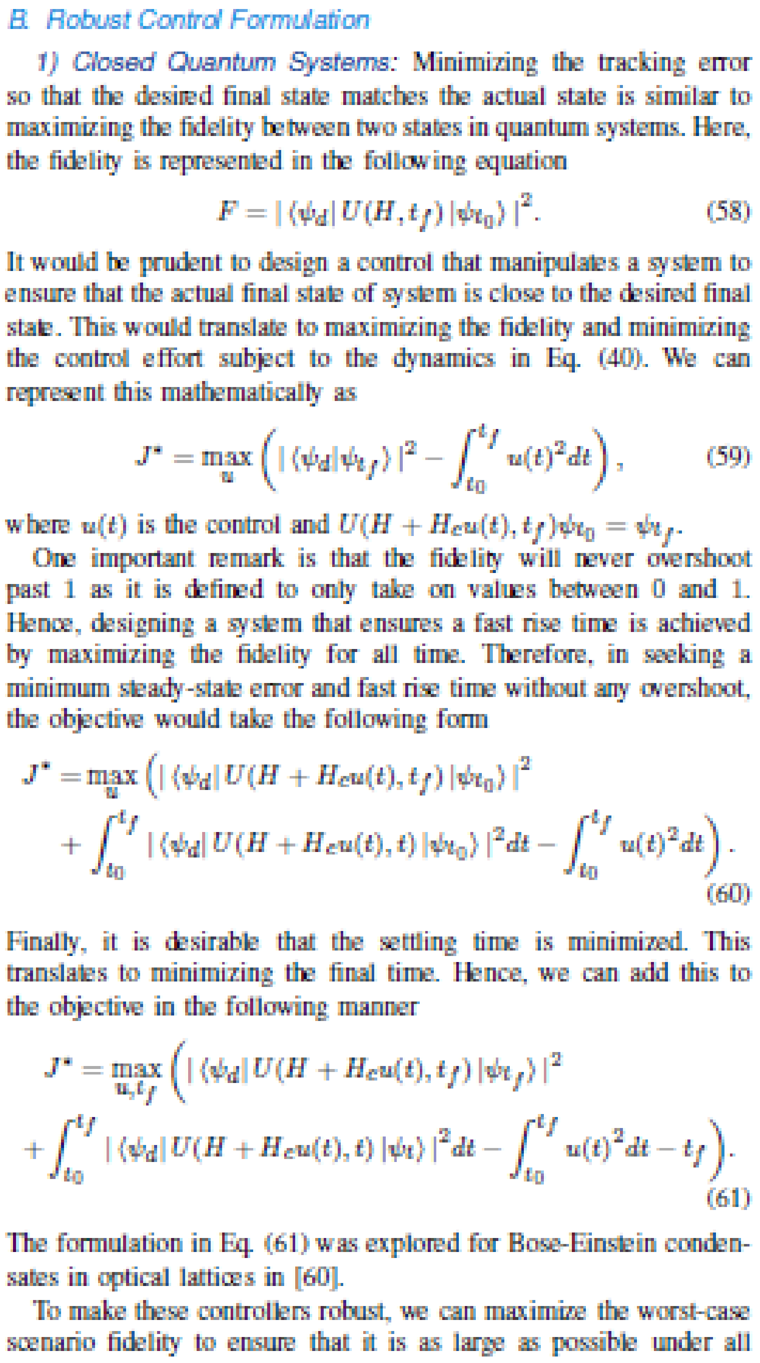
FCL is with the School of Computer Science and Informatics, Cardiff University, UK, frank@langbein.org.

SGS is with the Faculty of Science & Engineering, Swansea University, Singleton Park, Swansea, SA2 8PP, UK, lw1660@gmail.com.

Putting the students to work:

Asking them to write a survey paper on quantum control with emphasis on robustness of quantum controllers under structured uncertainties [under review for Automatica]

Robustness of quantum controllers—and its relation to the global phase—has been one of the primary foci of the Advanced Study Institute.



B. Robust Control Formulation

1) Closed Quantum Systems: Minimizing the tracking error so that the desired final state matches the actual state is similar to maximizing the fidelity between two states in quantum systems. Here, the fidelity is represented in the following equation

$$F = |\langle\psi_d| U(H, t_f) |\psi_{t_0}\rangle|^2. \qquad (58)$$

It would be prudent to design a control that manipulates a system to ensure that the actual final state of system is close to the desired final state. This would translate to maximizing the fidelity and minimizing the control effort subject to the dynamics in Eq. (40). We can represent this mathematically as

$$J^* = \max_u \left(|\langle\psi_d|\psi_{t_f}\rangle|^2 - \int_{t_0}^{t_f} u(t)^2 dt \right), \qquad (59)$$

where $u(t)$ is the control and $U(H + H_c u(t), t_f)\psi_{t_0} = \psi_{t_f}$.

One important remark is that the fidelity will never overshoot past 1 as it is defined to only take on values between 0 and 1. Hence, designing a system that ensures a fast rise time is achieved by maximizing the fidelity for all time. Therefore, in seeking a minimum steady-state error and fast rise time without any overshoot, the objective would take the following form

$$J^* = \max_u \Big(|\langle\psi_d| U(H + H_c u(t), t_f) |\psi_{t_0}\rangle|^2 + \int_{t_0}^{t_f} |\langle\psi_d| U(H + H_c u(t), t) |\psi_{t_0}\rangle|^2 dt - \int_{t_0}^{t_f} u(t)^2 dt \Big). \qquad (60)$$

Finally, it is desirable that the settling time is minimized. This translates to minimizing the final time. Hence, we can add this to the objective in the following manner

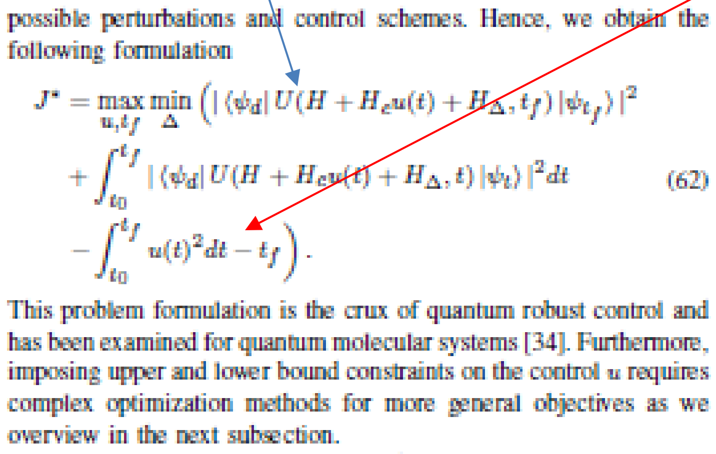
$$J^* = \max_{u,t_f} \Big(|\langle\psi_d| U(H + H_c u(t), t_f) |\psi_{t_f}\rangle|^2 + \int_{t_0}^{t_f} |\langle\psi_d| U(H + H_c u(t), t) |\psi_t\rangle|^2 dt - \int_{t_0}^{t_f} u(t)^2 dt - t_f \Big). \qquad (61)$$

The formulation in Eq. (61) was explored for Bose-Einstein condensates in optical lattices in [60].

To make these controllers robust, we can maximize the worst-case scenario fidelity to ensure that it is as large as possible under all

“nature-made”
fidelity criterion
independent of global phase

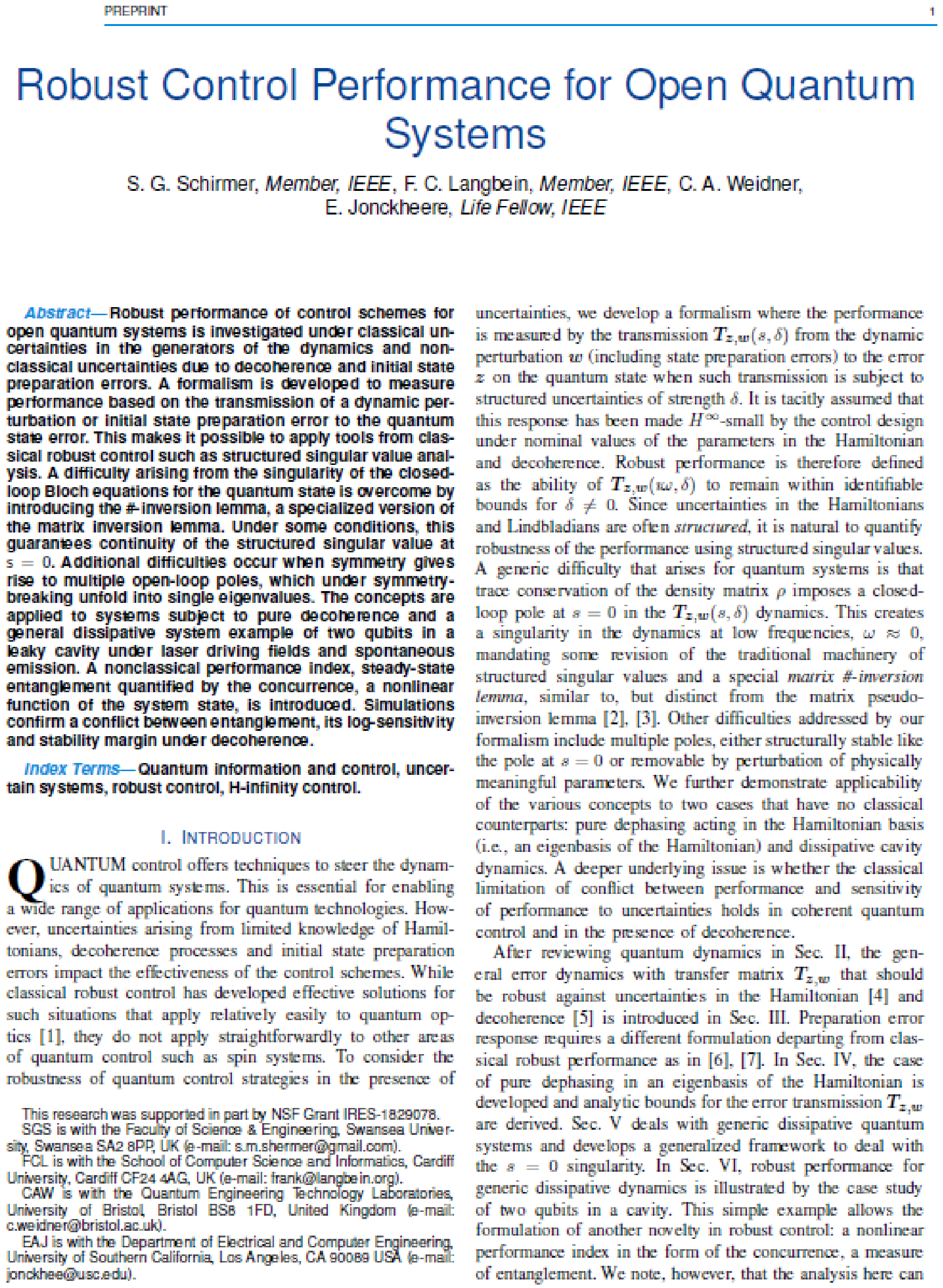
“man-made”
energy and time criteria
depending on global phase



possible perturbations and control schemes. Hence, we obtain the following formulation

$$J^* = \max_{u,t_f} \min_{\Delta} \Big(|\langle\psi_d| U(H + H_c u(t) + H_\Delta, t_f) |\psi_{t_f}\rangle|^2 + \int_{t_0}^{t_f} |\langle\psi_d| U(H + H_c u(t) + H_\Delta, t) |\psi_t\rangle|^2 dt - \int_{t_0}^{t_f} u(t)^2 dt - t_f \Big). \qquad (62)$$

This problem formulation is the crux of quantum robust control and has been examined for quantum molecular systems [34]. Furthermore, imposing upper and lower bound constraints on the control u requires complex optimization methods for more general objectives as we overview in the next subsection.



PREPRINT 1

Robust Control Performance for Open Quantum Systems

S. G. Schirmer, *Member, IEEE*, F. C. Langbein, *Member, IEEE*, C. A. Weidner,
E. Jonckheere, *Life Fellow, IEEE*

***Abstract*—Robust performance of control schemes for open quantum systems is investigated under classical uncertainties in the generators of the dynamics and nonclassical uncertainties due to decoherence and initial state preparation errors. A formalism is developed to measure performance based on the transmission of a dynamic perturbation or initial state preparation error to the quantum state error. This makes it possible to apply tools from classical robust control such as structured singular value analysis. A difficulty arising from the singularity of the closed-loop Bloch equations for the quantum state is overcome by introducing the #-inversion lemma, a specialized version of the matrix inversion lemma. Under some conditions, this guarantees continuity of the structured singular value at $s = 0$. Additional difficulties occur when symmetry gives rise to multiple open-loop poles, which under symmetry-breaking unfold into single eigenvalues. The concepts are applied to systems subject to pure decoherence and a general dissipative system example of two qubits in a leaky cavity under laser driving fields and spontaneous emission. A nonclassical performance index, steady-state entanglement quantified by the concurrence, a nonlinear function of the system state, is introduced. Simulations confirm a conflict between entanglement, its log-sensitivity and stability margin under decoherence.**

***Index Terms*—Quantum information and control, uncertain systems, robust control, H-infinity control.**

I. INTRODUCTION

QUANTUM control offers techniques to steer the dynamics of quantum systems. This is essential for enabling a wide range of applications for quantum technologies. However, uncertainties arising from limited knowledge of Hamiltonians, decoherence processes and initial state preparation errors impact the effectiveness of the control schemes. While classical robust control has developed effective solutions for such situations that apply relatively easily to quantum optics [1], they do not apply straightforwardly to other areas of quantum control such as spin systems. To consider the robustness of quantum control strategies in the presence of uncertainties, we develop a formalism where the performance is measured by the transmission $T_{z,w}(s,\delta)$ from the dynamic perturbation w (including state preparation errors) to the error z on the quantum state when such transmission is subject to structured uncertainties of strength δ. It is tacitly assumed that this response has been made H^∞-small by the control design under nominal values of the parameters in the Hamiltonian and decoherence. Robust performance is therefore defined as the ability of $T_{z,w}(\imath\omega,\delta)$ to remain within identifiable bounds for $\delta \neq 0$. Since uncertainties in the Hamiltonians and Lindbladians are often *structured*, it is natural to quantify robustness of the performance using structured singular values. A generic difficulty that arises for quantum systems is that trace conservation of the density matrix ρ imposes a closed-loop pole at $s = 0$ in the $T_{z,w}(s,\delta)$ dynamics. This creates a singularity in the dynamics at low frequencies, $\omega \approx 0$, mandating some revision of the traditional machinery of structured singular values and a special *matrix #-inversion lemma*, similar to, but distinct from the matrix pseudo-inversion lemma [2], [3]. Other difficulties addressed by our formalism include multiple poles, either structurally stable like the pole at $s = 0$ or removable by perturbation of physically meaningful parameters. We further demonstrate applicability of the various concepts to two cases that have no classical counterparts: pure dephasing acting in the Hamiltonian basis (i.e., an eigenbasis of the Hamiltonian) and dissipative cavity dynamics. A deeper underlying issue is whether the classical limitation of conflict between performance and sensitivity of performance to uncertainties holds in coherent quantum control and in the presence of decoherence.

After reviewing quantum dynamics in Sec. II, the general error dynamics with transfer matrix $T_{z,w}$ that should be robust against uncertainties in the Hamiltonian [4] and decoherence [5] is introduced in Sec. III. Preparation error response requires a different formulation departing from classical robust performance as in [6], [7]. In Sec. IV, the case of pure dephasing in an eigenbasis of the Hamiltonian is developed and analytic bounds for the error transmission $T_{z,w}$ are derived. Sec. V deals with generic dissipative quantum systems and develops a generalized framework to deal with the $s = 0$ singularity. In Sec. VI, robust performance for generic dissipative dynamics is illustrated by the case study of two qubits in a cavity. This simple example allows the formulation of another novelty in robust control: a nonlinear performance index in the form of the concurrence, a measure of entanglement. We note, however, that the analysis here can

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For such nonlinear measure as the concurrence (or entanglement) error, some form of the classical *S+T=I* limitation reappears!

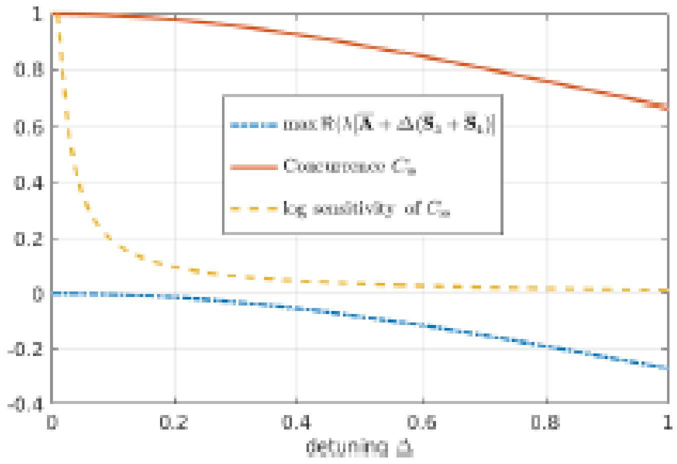

Fig. 4: Maximum of the real part of the eigenvalues λ of $\overline{A} + \Delta(\overline{S}_3 + \overline{S}_4)$, concurrence of steady-state and log-sensitivity of steady-state concurrence as function of detuning Δ for $\alpha = \gamma = 1$. All three figures of merit are concordant, i.e. they decrease with increasing detuning.

Statistically Characterising Robustness and Fidelity of Quantum Controls

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Robustness of quantum operations or controls is important to build reliable quantum devices. The *robustness-infidelity measure* (RIM_p) is introduced to statistically quantify the robustness and fidelity of a controller as the p-order Wasserstein distance between the fidelity distribution of the controller under any uncertainty and an ideal fidelity distribution. The RIM_p is the p-th root of the p-th raw moment of the infidelity distribution. Using a metrization argument, we motivate why RIM_1, or the average infidelity, suffices as a practical robustness measure. Exploiting the Wasserstein distance, we define an algorithmic robustness-infidelity measure (ARIM) to quantify the robustness and infidelity of controller acquisition strategies. The utility of the RIM and ARIM is demonstrated by considering the problem of robust control of spin-$\frac{1}{2}$ networks using energy landscape shaping subject to Hamiltonian uncertainty. The robustness and infidelity of individual control solutions as well as the robustness of different popular quantum control algorithms with respect to the noise model are characterized. For algorithm comparisons, stochastic and non-stochastic optimization objectives are considered, with the goal of effective target RIM optimization in the latter. Although high fidelity and robustness are often conflicting objectives, some high fidelity, robust controllers can usually be found, irrespective of the choice of the quantum control algorithm. However, for noisy optimization objectives adaptive sequential decision making approaches such as reinforcement learning have a cost advantage compared to standard control algorithms and, in contrast, the infidelities obtained are more consistent with higher RIM for low noise levels.

I. INTRODUCTION

Fault-tolerance is crucial for quantum technology and presents a particular challenge for noisy Intermediate-Scale Quantum (NISQ) devices [1]. Broadly, there are three proposed ways to deal with noise and errors and achieve fault-tolerance: (1) via error correction protocols, e.g., Shor codes [2–4] and syndrome measurements [5]; (2) using error mitigation schemes, e.g., reversing noisy dynamics [6–9], active variational noise minimization [10], or parametric modelling of architecture defects in trapped qubits [11, 12]; (3) robust solutions engineering, e.g., landscape shaping of the quantum control optimization problem in search of noise-free regions [13–15], decoherence-free subspaces [16, 17], or noise spectral density based filter functions [18, 19]. Uncertainties that require fault-tolerance in quantum devices have two flavors: (a) interaction with the environment that leads to non-unitary dynamics; (b) inaccuracies in the control model representing a specific physical implementation that affect the evolution but do not cause non-unitary evolution.

Standard quantum control methods for steering quantum devices mostly focus on finding controls that have high fidelity using mathematical models [20–22]. However, if the operation of quantum devices is subject to noise, high fidelity itself is insufficient to gauge performance of a control scheme, and extra effort is required to systematically search for solutions that are both robust against noise and have high fidelity [23, 24]. This requires a notion of robustness and ideally a single measure that can capture robustness and fidelity, allowing for the identification and construction of more efficient methods to find controls that satisfy both properties.

In this paper, we introduce a general statistical diagnostic based on the Wasserstein distances of order p [25] to evaluate the robustness and fidelity of quantum control solutions and the schemes used to generate them, which is applicable to any quantum control problem where the fidelity is a random variable with a probability distribution. The Wasserstein distance between probability distributions is a measure of the minimal costs of probability mass transport between two distributions. In Sec. II, the p-th order *Robustness-Infidelity Measure* (RIM_p) is defined to quantify the robustness and fidelity of a quantum controller. It is based on the p-th order Wasserstein distance between the probability distribution for the fidelity induced by model uncertainties and the ideal distribution for a perfectly robust controller, described by a Dirac delta function at fidelity 1. We show that the RIM_p is the p-th root of the p-th raw moment of the infidelity

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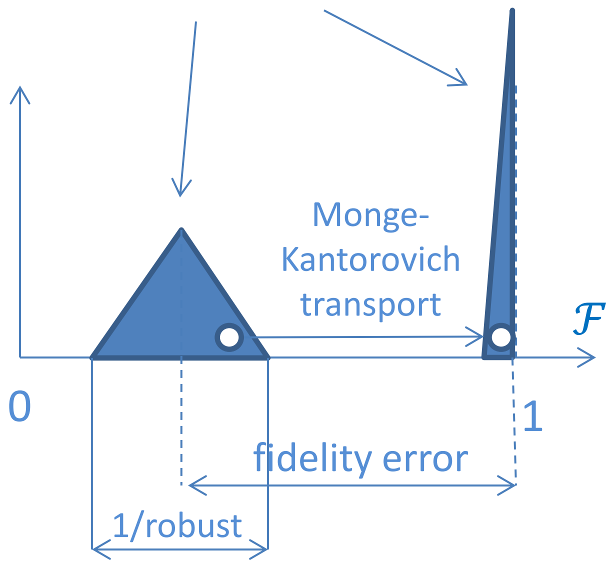
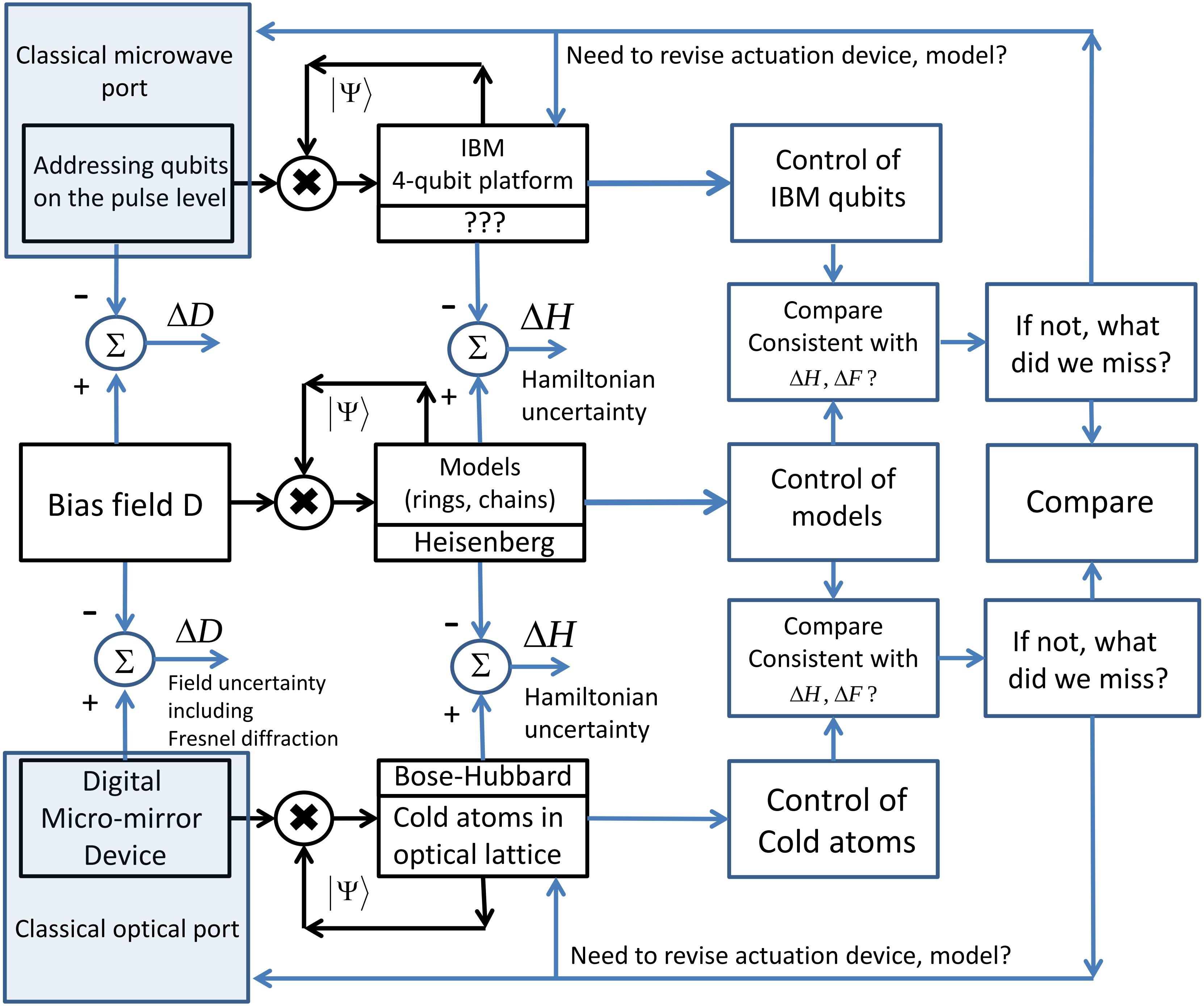
The key-point of this article is **a new—physically inspired—measure that captures both the fidelity and its robustness.** It is the p-Wasserstein distance between the ideal fidelity distribution, a Dirac δ_1 at 1, and the probability density $p_\mathcal{F}$ of the fidelity $\mathcal{F}$ subject to uncertainties:

$$W_p(p_\mathcal{F}, \delta_1)$$



Conclusion?

Confront models and models and reality


Classical microwave port
Addressing qubits on the pulse level
$|\Psi\rangle$
IBM 4-qubit platform
???
Need to revise actuation device, model?
Control of IBM qubits
-
+
Σ
ΔD
-
+
Σ
ΔH
Hamiltonian uncertainty
Compare Consistent with $\Delta H, \Delta F$?
If not, what did we miss?
Bias field D
$|\Psi\rangle$
Models (rings, chains)
Heisenberg
Control of models
Compare
-
+
Σ
ΔD
Field uncertainty including Fresnel diffraction
-
+
Σ
ΔH
Hamiltonian uncertainty
Compare Consistent with $\Delta H, \Delta F$?
If not, what did we miss?
Digital Micro-mirror Device
Classical optical port
Bose-Hubbard
Cold atoms in optical lattice
$|\Psi\rangle$
Control of Cold atoms
Need to revise actuation device, model?

US-UK international student research in robust control of quantum networks

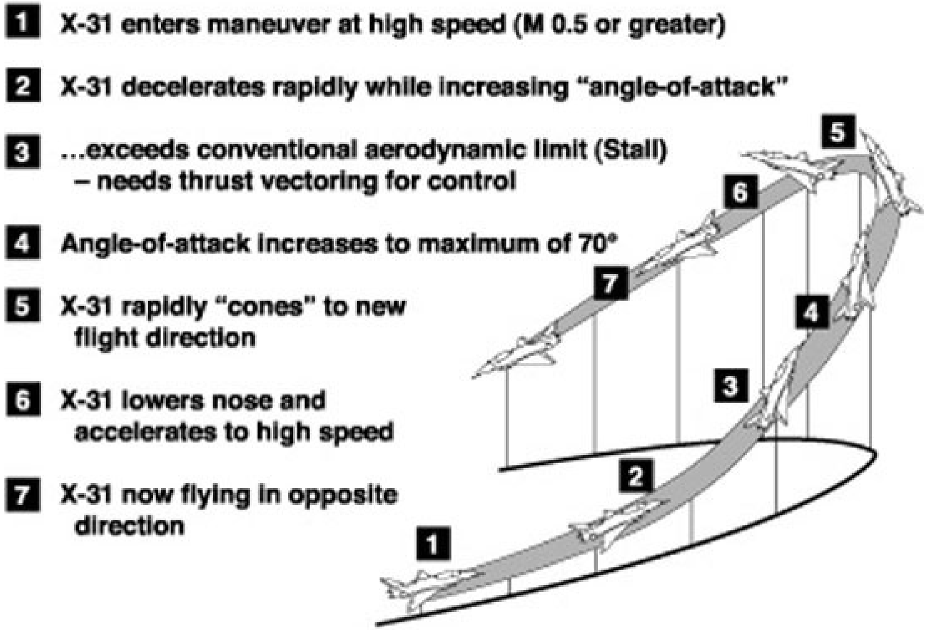
- People (students, postdoc, faculty) involved
- Venue: Wales, United Kingdom
- First week program in Cardiff: spintronics control
- Second week program in Swansea: photonics
- Products
  - Classical versus quantum fundamental limitations
  - Structurally uncertain quantum network robustness
- Social (or aerospace?) activity: Wales Air show

End Second Week Program
in Swansea
and end of Advanced Study Institute

Red Arrows pilot reveals one key secret to not crashing into other jet planes.

In control jargon, “formation control”

Demonstration of the Herbst maneuver



1. X-31 enters maneuver at high speed (M 0.5 or greater)
2. X-31 decelerates rapidly while increasing "angle-of-attack"
3. ...exceeds conventional aerodynamic limit (Stall) – needs thrust vectoring for control
4. Angle-of-attack increases to maximum of 70°
5. X-31 rapidly "cones" to new flight direction
6. X-31 lowers nose and accelerates to high speed
7. X-31 now flying in opposite direction

A Fixed H^∞ Controller for a Supermaneuverable Fighter Performing the Herbst Maneuver

R. Y. CHIANG, M. G. SAFONOV,K. HAIGES,II K. MADDEN and J. TEKAWY

A non-scheduled H^∞ robust flight controller has been designed for a supermaneuverable fighter to fly the Herbst maneuver. The complete design approach and plant uncertainties are documented with detailed linear robustness analysis and nonlinear six degree-of-freedom simulation.

The "canards" of the Eurofighter Typhoon
making it supermaneuverable,
but statistically unstable

© PA

Thank you for your attention!

Questions?

Video clip of Prof. Schirmer’s statement

https://urldefense.com/v3/__https://www.youtube.com/watch?v=Mc62lEgg-mg__;!!LIr3w8kk_Xxm!oWbJWQN4N4ZP7Ufm0DQ1b-5ktcgXlMl60_ozMwknnD9hi09ejOI3PmZqELosRELBAoxGEFI-aWv2-Q$