

Optimal Routing with Mutual Information Accumulation in Wireless Networks

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Abstract—We study the problem of minimum delay routing in multi-hop wireless networks with *rateless codes*. Rateless codes allow each node of the network to accumulate mutual information from every packet transmission. This enables a significant performance gain over conventional shortest path routing. Further, it outperforms cooperative communication techniques that are based on energy accumulation. However, determining the minimum delay routing solution requires complex and combinatorial networking decisions concerning which nodes participate in transmission, and which decode ordering to use. We identify several structural properties of the optimal solution to simplify the problem and derive an optimal greedy algorithm. Although the reduced problem still has exponential complexity, unlike prior works on such problems, our greedy algorithm is simple to use and does not require solving any linear programs. Further, using the insight obtained from the optimal solution to a line network, we propose two simple heuristics that can be implemented in polynomial time in a distributed fashion and compare them with the optimal solution. Simulations suggest that both heuristics perform very close to the optimal solution over random network topologies.

Index Terms—Mutual Information Accumulation, Rateless Codes, Minimum Delay Routing

I. INTRODUCTION

Cooperative communication promises significant gains in the performance of wireless networks over traditional techniques that treat the network as comprised of point-to-point links. Cooperative communication protocols exploit the broadcast nature of wireless transmissions and offer spatial diversity gains by making use of multiple relays for cooperative transmissions. This can increase the reliability and reduce the energy cost of data transmissions in wireless networks. See [2] for a recent comprehensive survey.

Most prior work in the area of cooperative communication has investigated physical layer techniques such as orthogonal repetition coding/signaling [3], distributed beamforming [4], distributed space-time codes [5], etc. All these techniques perform *energy accumulation* from multiple transmissions to decode a packet. In energy accumulation, a receiver can decode a packet when the total received energy from multiple transmissions of that packet exceeds a certain threshold.

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An alternate approach of recent interest is based on *mutual information accumulation* [6] [7]. In this approach, a node accumulates mutual information for a packet from multiple transmissions until it can be decoded successfully. This is shown to outperform energy accumulation based schemes, particularly in the high SNR regime, in [6] [7]. Such a scheme can be implemented in practice using rateless codes of which Fountain and Raptor codes [8]–[10] are two examples. Rateless codes encode information bits into potentially infinite-length codewords. Subsequently, additional parity bits are sent by the transmitter until the receiver is able to decode. The decoding procedure at the receiver is carried out using belief propagation algorithms. For an overview of the working of Fountain, LT, and Raptor codes, we refer to the excellent survey in [11].

In addition to allowing mutual information accumulation, rateless codes provide further advantages over traditional fixed rate schemes in the context of fading relay networks as discussed in [12] [13]. Unlike fixed rate code schemes in which knowledge of the current channel state information (CSI) is required at the transmitters, rateless codes adapt to the channel conditions without requiring CSI. This advantage becomes even more important in large networks where the cost of CSI acquisition grows exponentially with the network size. However, this introduces a deep memory in the system because mutual information accumulated from potentially multiple transmissions in the past can be used to decode a packet.

In this paper, we study the problem of optimal routing and scheduling over a multi-hop wireless network using mutual information accumulation. Specifically, we consider a network with a single source-destination pair and n relay nodes. When a node transmits, the other nodes accumulate mutual information at a rate that depends on their incoming link capacity. All nodes operate under bandwidth and transmit power constraints as described in detail in Section II. The objective is to transmit a packet from the source to the destination in minimum delay.

Related problems of optimal routing in wireless networks with multi-receiver diversity have been studied in [14]–[17] while problems of optimal cooperative diversity routing and broadcasting are treated in [18]–[21] and references therein. Although these formulations incorporate the broadcast nature of wireless transmissions, they assume that the outcome of each transmission is a binary success/failure. Further, any packet that cannot be successfully decoded in one transmission is discarded. This is significantly different from the scenario considered in this paper where nodes can accumulate partial information about a packet from different transmissions over time. This can be thought of as networking with “soft” information.

Prior work on accumulating partial information from multi-packet transmissions includes the work in [7], [22]–[27]. Specifically, [22] considers the problem of minimum energy unicast routing in wireless networks with energy accumulation and shows that it is an NP-complete problem. Similar results are obtained for the problem of minimum energy *accumulative broadcast* in [23]–[25]. A related problem of *accumulative multicast* is studied in [26]. Minimum energy unicast routing with energy accumulation only at the destination is considered in [27]. The work closest to ours is [7] which treats the minimum delay routing problem with mutual information accumulation. Both [24] [7] develop an LP based formulation for their respective problems that involves solving a linear program for *every possible ordering* of relay nodes over all subsets of relay nodes to derive the optimal solution. Thus, for a network with n relay nodes, this exhaustive approach requires solving $\sum_{m=1}^n \binom{n}{m} m! > n!$ linear programs.

The primary challenge associated with solving the problem addressed in this paper is its inherent combinatorial nature. Unlike traditional shortest path routing, the cost of routing with mutual information accumulation depends not only on the set of nodes in the routing path, but also their *relative ordering* in the transmission sequence, making standard shortest path algorithms inapplicable. Therefore, we approach the problem differently. To derive the optimal transmission strategy, we first formulate an optimization problem in Section III-B that optimizes over all possible transmission orderings over all subsets of relay nodes (similar to [24] [7]). This approach clearly has a very high complexity of $O(n!)$. Then in Section III-C, we prove a key structural property of the optimal solution that allows us to simplify the problem and derive a simple greedy algorithm that only needs to optimize over all subsets of nodes. Further, it does not require solving any linear programs. Thus, it has a complexity of $O(2^n)$. We note that this complexity, while still exponential, is a significant improvement over solving $O(n!)$ linear programs. For example, with $n = 10$, this requires $2^{10} = 1024$ runs of a simple greedy algorithm as compared to $10! = 3628800$ runs of an LP solver. Note that for small networks, (say, $n \leq 10$), it is reasonable to use our algorithm to exactly compute the optimal solution. Further, for larger n it provides a feasible way to compute the optimal solution as a benchmark when comparing against simpler heuristics. In general, we expect this problem to be NP-complete based on the results in [22]–[25]. For the special case of a line topology, we derive the exact optimal solution in Section III-E. Finally, in Section IV, we propose two simple heuristics that can be implemented in polynomial time and in a distributed fashion and compare them with the optimal solution. Simulations suggest that both heuristics perform quite close to the optimal solution over random network topologies.

Before proceeding, we note that in [1], we have used the techniques developed in this paper to get similar structural results for related problems of (i) minimum energy routing subject to delay constraint, and (ii) minimum delay broadcast, both involving mutual information accumulation. We also note that these techniques can be applied to similar problems that use energy accumulation instead of mutual information

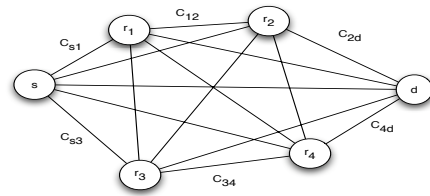


Fig. 1. Example network with source, destination and 4 relay nodes. When a node transmits, every other node that has not yet decoded the packet accumulates mutual information at a rate given by the capacity of the link between the transmitter and that node.

accumulation.

II. NETWORK MODEL

The network model consists of a source s , destination d and n relays r_1, r_2, \dots, r_n as shown in Fig. 1. There are no time variations in the channel states. This models the scenario where the coherence time of the channels is larger than any considered transmission time of the encoded bits. In the first two problems, the source has a packet to be delivered to the destination. In the third problem, the source packet must be delivered to all nodes in the network.

Each node i transmits at a fixed power spectral density (PSD) P_i (in units of joules/sec/Hz) that is uniform across its transmission band. However, the transmission duration for a node is variable and is a design parameter. The total available bandwidth is W Hz. A node can transmit the packet only if it has fully decoded the packet. For this, it must accumulate *at least* I_{max} bits of total mutual information.

All transmissions happen on orthogonal channels in time or frequency and at most one node can transmit over a frequency channel at any given time. The channel gain between nodes i and j is given by h_{ij} which is independent of the frequency channel. However, this is not necessarily known at the transmitting nodes.

Under this assumption, the minimum transmission time under the two orthogonal schemes (where nodes transmit in orthogonal time vs. frequency channels) is the same. In the following, we will focus on the case where transmissions are orthogonal in time. When a node i transmits, every other node j that does not have the full packet yet, receives mutual information at a rate that depends on the transmission capacity C_{ij} (in units of bits/sec/Hz) of link $i - j$. This transmission capacity itself depends on the transmit power P_i and channel strength h_{ij} . For example, for an AWGN channel, using Shannon's formula, this is given by $C_{ij} = \log_2 \left[1 + \frac{h_{ij} P_i}{N_0} \right]$ where $N_0/2$ is the PSD of the noise process. Specifically, if node i transmits for duration Δ over bandwidth W , then node j accumulates $\Delta W C_{ij}$ bits of information. In the following, we assume $W = 1$ for simplicity. We assume that nodes use independently generated *ideal* rateless codes so that the mutual information collected by a node from different transmissions add up. We can incorporate the non-idealities of practical rateless codes by multiplying C_{ij} with a factor $1/(1+\epsilon)$ where $\epsilon \geq 0$ is the overhead. A similar model has been considered in [7].

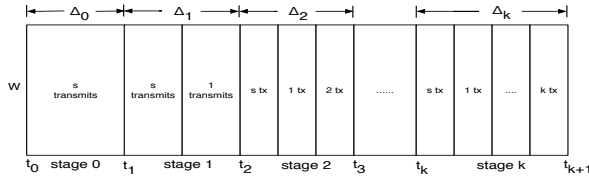


Fig. 2. Example timeslot and transmission structure. In each stage, nodes that have already decoded the full packet transmit on orthogonal channels in time.

III. MINIMUM DELAY ROUTING

Under the modeling assumptions discussed in Section II, the problem of routing a packet from the source to the destination with minimum delay consists of the following sub-problems:

- First, which subset of relay nodes should take part in forwarding the packet?
- Second, in what order should these nodes transmit?
- And third, what should be the transmission durations for these nodes?

We next discuss the transmission structure of a general policy under this model.

A. Timeslot and Transmission Structure

Consider any transmission strategy \mathcal{G} for routing the packet to the destination in the model described above. This includes the choice of the relay set, the transmission order for this set, and the transmission durations for each node in this set. Let \mathcal{R} denote the subset of relay nodes that take part in the routing process under strategy \mathcal{G} . By this, we mean that each node in \mathcal{R} is able to decode the packet before the destination and then transmits for a non-zero duration. There could be other nodes that are able to decode the packet before the destination, but these do not take part in the forwarding process and are therefore not included in the set \mathcal{R} .

Let $k = |\mathcal{R}|$ be the size of this set. Also, let \mathcal{O} be the ordering of nodes in \mathcal{R} that describes the sequence in which nodes in \mathcal{R} successfully decode the packet under strategy \mathcal{G} . Without loss of generality, let the relay nodes in the ordering \mathcal{O} be indexed as $1, 2, 3, \dots, k$. Also, let the source s be indexed as 0 and the destination d be indexed as $k + 1$. Initially, only the source has the packet. Let t_0 be the time when it starts its transmission and let t_1, t_2, \dots, t_k denote the times when relays $1, 2, \dots, k$ in the ordering \mathcal{O} accumulate enough mutual information to decode the packet. Also, let t_{k+1} be the time when the destination decodes the packet. By definition, $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_k \leq t_{k+1}$. We say that the transmission occurs over $k + 1$ stages, where stage $j, j \in \{0, 1, 2, \dots, k\}$ represents the interval $[t_j, t_{j+1}]$. The state of the network at any time is given by the set of nodes that have the full packet and the mutual information accumulated so far at all the other nodes. Note that in any stage j , the first j nodes in the ordering \mathcal{O} and the source have the fully decoded packet. Thus, any subset of these nodes (including potentially all of them) may transmit during this stage. Then the time-slot structure for the transmissions can be depicted as in Fig. 2. Note that in each stage, the set of relays that have successfully decoded the packet increases by one (we ignore those relays that are not part of the set \mathcal{R}).

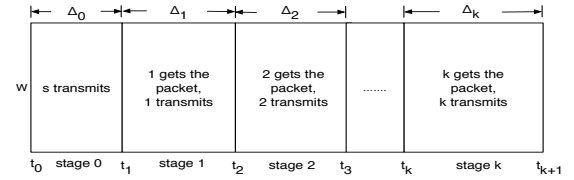


Fig. 3. Optimal timeslot and transmission structure. In each stage, only the node that decodes the packet at the beginning of that stage transmits.

We are now ready to formulate the problem of minimum delay routing with mutual information accumulation.

B. Problem Formulation

For each j , define the duration of stage j as $\Delta_j = t_{j+1} - t_j$. Also, let A_{ij} denote the transmission duration for node i in stage j under strategy \mathcal{G} . Note that $A_{ij} = 0$ if $i > j$, else $A_{ij} \geq 0$. This is because node i does not have the full packet until the end of stage $i - 1$. The total time to deliver the packet to the destination T_{tot} is given by $T_{tot} = t_{k+1} - t_0 = \sum_{j=0}^k \Delta_j$. For any transmission strategy \mathcal{G} that uses the subset of relay nodes \mathcal{R} with an ordering \mathcal{O} , the minimum delay is given by the solution to the following optimization problem:

$$\begin{aligned}
 &\text{Minimize: } T_{tot} = \sum_{j=0}^k \Delta_j \\
 &\text{Subject to: } \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} A_{ij} C_{im} \geq I_{max} \quad \forall m \in \{1, 2, \dots, k+1\} \\
 &\quad \sum_{i=0}^j A_{ij} \leq \Delta_j \quad \forall j \in \{0, 1, 2, \dots, k\} \\
 &\quad A_{ij} \geq 0 \quad \forall i \in \{0, 1, 2, \dots, k\}, j \in \{0, 1, 2, \dots, k\} \\
 &\quad A_{ij} = 0 \quad \forall i > j \\
 &\quad \Delta_j \geq 0 \quad \forall j \in \{0, 1, 2, \dots, k\}
 \end{aligned} \tag{1}$$

Here, the first constraint captures the requirement that node m in the ordering must accumulate at least I_{max} amount of mutual information by the end of stage $m - 1$ using all transmissions in all stages up to stage $m - 1$. The second constraint means that in every stage j , the total transmission time for all nodes that have the fully decoded packet in that stage cannot exceed the length of that stage.

It can be seen that the above problem is a linear program and thus can be solved efficiently for a given relay set \mathcal{R} and its ordering \mathcal{O} . Indeed, this is the approach taken in [7] that proposes solving such a linear program for *every possible ordering of relays* for each subset of the set of relay nodes. While such an approach is guaranteed to find the optimal solution, it has a huge computational complexity of $O(n!)$ linear programs. In the next section, we show that the above computation can be simplified significantly by making use of a structural property of the optimal solution.

C. Characterizing the Optimal Solution of (1)

Let \mathcal{R}_{opt} denote the subset of relay nodes that take part in the routing process in the optimal solution. Let $k = |\mathcal{R}_{opt}|$ be the size of this set. Also, let \mathcal{O}_{opt} be the optimal ordering.

Note that, by definition, each node in \mathcal{R}_{opt} transmits for a non-zero duration (else, we can remove it from the set without affecting the minimum total transmission time). Then, we have the following:

Theorem 1: Under the optimal solution to the minimum delay routing problem (1), in each stage j , it is optimal for only one node to transmit, and that node is node j .

Fig. 3 shows the timeslot structure under the optimal solution. The above theorem shows that only one node transmits in each stage, and that the optimal transmission ordering is the same as the ordering that nodes in the set \mathcal{R}_{opt} decode the packet. Comparing this with the general timeslot structure in Fig. 2, it can be seen that Theorem 1 simplifies problem (1) significantly. Specifically, Theorem 1 implies that, *given the optimal relay set \mathcal{R}_{opt}* , the optimal transmission structure (i.e., the decoding order and the transmission durations) can be computed in a *greedy* fashion as follows. First, the source starts to transmit and continues to do so until any relay node in this set gets the packet. Once this relay node gets the packet, we know from Theorem 1 that the source does not transmit in any of the remaining stages. This node then starts to transmit until another node in the set gets the packet. This process continues until the destination is able to decode the packet. The optimal solution to (1) can then be obtained by applying this greedy transmission strategy to all subsets of relay nodes and picking one that yields the minimum delay.¹ Note that applying this greedy transmission strategy does not require solving an LP. While searching over all subsets still has an exponential complexity of $O(2^n)$, it can be used to compute the optimal solution as a benchmark.

Theorem 1 also implies that multiple copies of the packet need not be maintained across the network. For example, note that the source need not transmit after the first relay has decoded the packet and therefore can drop the packet from its queue.

We emphasize that the optimal transmission structure suggested by Theorem 1 is not obvious. For example, at the beginning of any stage, the newest addition to the set of relay nodes with the full packet may not have the best links (in terms of transmission capacity) to all the remaining nodes, including the destination. This would suggest that under the optimal solution, in general in each stage, nodes with the full packet should take turns transmitting the packet. However, Theorem 1 states that such time-sharing is not required.

Before proceeding, we present a preliminary lemma that is used in the proof of Theorem 1. Consider any linear program:

$$\begin{aligned} \text{Minimize:} & \quad c^T x \\ \text{Subject to:} & \quad Ax = b \\ & \quad x \geq 0 \end{aligned} \quad (2)$$

where $x \in \mathbf{R}^n$. Then we have the following:

Lemma 1: Let x^* be an optimal solution to the problem (2) such that $x^* > 0$ (where the inequality is taken entry wise).

¹We note that the transmission structure characterized by Theorem 1 is similar to the *wavepath property* shown in [22] for the problem of minimum energy unicast routing with energy accumulation in wireless networks. However, our proof technique is significantly different.

Then x^* is still an optimal solution when the constraint $x \geq 0$ is removed.

Proof: See [1]. ■

Lemma 1 implies that removing an *inactive* constraint does not affect the optimal solution of the linear program.

D. Proof of Theorem 1

Note that Theorem 1 trivially holds in stage 0 (since only the source has the full packet in this stage). Next, it is easy to see that in the last stage (i.e., stage k), only the node with the *best* link (in terms of transmission capacity) to the destination in the set \mathcal{R}_{opt} should transmit in order to minimize the total delay. This is because this node will take the smallest time to transmit the remaining amount of mutual information needed by d to decode the packet. Further, we claim that this node must be the node k in the ordering \mathcal{O}_{opt} . This can be argued as follows. Assume that the node with the best link to the destination in the set \mathcal{R}_{opt} has the full packet at some stage $(k - j)$ (where $0 < j < k$) *before* the start of stage k . Then a smaller delay can be achieved by having only this node transmit after it has decoded the full packet from that stage onwards. Thus, the other nodes labeled $k - j, \dots, k - 1$ in the transmission order do not transmit, a contradiction. This shows that under the optimal solution, in the last stage k , only node k in the ordering \mathcal{O}_{opt} transmits. Using induction, we now show that in every prior stage $(k - j)$ where $1 \leq j \leq k - 1$, only one node needs to transmit and that this node must be node $k - j$ in the ordering \mathcal{O}_{opt} .

Consider the $(k - 1)^{th}$ stage. At time t_{k-1} , all nodes except k and d have decoded the packet. Let the mutual information state at nodes k and d at time t_{k-1} be $I_k(t_{k-1})$ and $I_d(t_{k-1})$ respectively. Also, suppose in the $(k - 1)^{th}$ stage, relay nodes $1, 2, \dots, k - 1$ and the source transmit a fraction $\alpha_1^{k-1}, \alpha_2^{k-1}, \dots, \alpha_{k-1}^{k-1}$ and α_0^{k-1} of the total duration of stage $(k - 1)$, i.e., Δ_{k-1} respectively. Note that these fractions must add to 1 since it is suboptimal to have any idle time (where no one is transmitting). Then, the optimal solution must solve the following optimization problem:

$$\begin{aligned} \text{Minimize:} & \quad \Delta_{k-1} + \Delta_k \\ \text{Subject to:} & \quad I_k(t_{k-1}) + \Delta_{k-1} \sum_{i=0}^{k-1} \alpha_i^{k-1} C_{ik} \geq I_{max} \\ & \quad I_d(t_{k-1}) + \Delta_{k-1} \sum_{i=0}^{k-1} \alpha_i^{k-1} C_{id} + \Delta_k C_{kd} \geq I_{max} \\ & \quad 0 \leq \alpha_0^{k-1}, \alpha_1^{k-1}, \dots, \alpha_{k-1}^{k-1} \leq 1 \\ & \quad \sum_{i=0}^{k-1} \alpha_i^{k-1} = 1 \\ & \quad \Delta_{k-1} \geq 0, \Delta_k \geq 0 \end{aligned} \quad (3)$$

Here, the first constraint states that relay k must accumulate at least I_{max} bits of mutual information by the end of stage $(k - 1)$. The second constraint states that the destination must accumulate at least I_{max} bits of mutual information by the end of stage k . Note that in the last term of the left hand side

of the second constraint, we have used the fact that only node k transmits during stage k .

It is easy to see that under the optimal solution, the first and second constraints must be met with equality. This simply follows from the definition of the beginning of any stage j as the time when node j has just decoded the packet. Next, let $\beta_i = \Delta_{k-1} \alpha_i^{k-1}$ for all $i \in \{0, 1, 2, \dots, k-1\}$. Since $\sum_{i=0}^{k-1} \alpha_i^{k-1} = 1$, we have that $\sum_{i=0}^{k-1} \beta_i = \Delta_{k-1}$ and (3) is equivalent to:

$$\begin{aligned} \text{Minimize: } & \sum_{i=0}^{k-1} \beta_i + \Delta_k \\ \text{Subject to: } & I_k(t_{k-1}) + \sum_{i=0}^{k-1} \beta_i C_{ik} = I_{max} \\ & I_d(t_{k-1}) + \sum_{i=0}^{k-1} \beta_i C_{id} + \Delta_k C_{kd} = I_{max} \\ & \Delta_k \geq 0, \beta_i \geq 0 \quad \forall i \in \{0, 1, 2, \dots, k-1\} \quad (4) \end{aligned}$$

Note that problems (3) and (4) are equivalent because we can transform (4) to the original problem by using the relations $\Delta_{k-1} = \sum_{i=0}^{k-1} \beta_i$ and $\alpha_i^{k-1} = \frac{\beta_i}{\Delta_{k-1}}$. The degenerate case where $\Delta_{k-1} = 0$ does not arise because if $\Delta_{k-1} = 0$, then no node transmits in stage $(k-1)$ and we transition to stage k in which only node k transmits. This means node $k-1$ never transmits, contradicting the fact that it is part of the optimal transmission schedule.

Since we know that under the optimal solution, $\Delta_k > 0$, we can remove the constraint $\Delta_k \geq 0$ from (4) without affecting the optimal solution (using Lemma 1). Next we multiply the minimization objective in (4) by C_{kd} without changing the problem. Then, using the second equality constraint to eliminate Δ_k from the objective and ignoring the constant terms, (4) can be expressed as:

$$\begin{aligned} \text{Minimize: } & \sum_{i=0}^{k-1} \beta_i (C_{kd} - C_{id}) \\ \text{Subject to: } & I_k(t_{k-1}) + \sum_{i=0}^{k-1} \beta_i C_{ik} = I_{max} \\ & \beta_i \geq 0 \quad \forall i \in \{0, 1, 2, \dots, k-1\} \quad (5) \end{aligned}$$

This optimization problem is linear in β_i with a single linear equality constraint and thus the solution is of the form where all except one β_i are zero. Since $\alpha_i^{k-1} = \frac{\beta_i}{\Delta_{k-1}}$, we have that in the optimal solution, exactly one of the fractions $\alpha_0^{k-1}, \alpha_1^{k-1}, \dots, \alpha_{k-1}^{k-1}$ is equal to 1 and rest must be 0. This implies that only one node transmits in this stage. Further, this node must be the relay node $k-1$ that decoded the packet at the beginning of this stage. Else, node $k-1$ never transmits. This is because by definition of stage $(k-1)$, node $k-1$ does not have the packet before the beginning of stage $(k-1)$ and hence cannot transmit before stage $(k-1)$. Since only node k transmits when stage $(k-1)$ ends, if node $k-1$ is not the node chosen for stage $(k-1)$, it never transmits, contradicting the fact that it is part of the optimal set.

Now consider the $(k-j)^{th}$ stage and suppose Theorem 1

holds for all stages after stage $(k-j)$ where $2 \leq j \leq k-1$. This means that in every stage after stage $(k-j)$, only the node that has just decoded the packet transmits. At time t_{k-j} , all nodes except $k-j+1, k-j+2, \dots, k$ and d have decoded the packet. Let the mutual information state at these nodes at time t_{k-j} be $I_{k-j+1}(t_{k-j}), I_{k-j+2}(t_{k-j}), \dots, I_k(t_{k-j})$ and $I_d(t_{k-j})$, respectively. Also, suppose in the $(k-j)^{th}$ stage, the source and the relay nodes $1, 2, \dots, k-j$ transmit a fraction $\alpha_0^{k-j}, \alpha_1^{k-j}, \alpha_2^{k-j}, \dots, \alpha_{k-j}^{k-j}$ of the total duration of stage $(k-j)$, i.e., Δ_{k-j} respectively. Then, the optimal solution must solve the following optimization problem:

$$\begin{aligned} \text{Minimize: } & \sum_{m=0}^j \Delta_{k-j+m} \\ \text{Subject to: } & I_{k-j+1}(t_{k-j}) + \Delta_{k-j} \left[\sum_{i=0}^{k-j} \alpha_i^{k-j} C_{i,k-j+1} \right] = I_{max} \\ & I_{k-j+n}(t_{k-j}) + \Delta_{k-j} \left[\sum_{i=0}^{k-j} \alpha_i^{k-j} C_{i,k-j+n} \right] \\ & + \sum_{i=1}^{n-1} \Delta_{k-j+i} C_{k-j+i,k-j+n} = I_{max} \quad \forall n \in \{2, \dots, j+1\} \\ & 0 \leq \alpha_0^{k-j}, \alpha_1^{k-j}, \dots, \alpha_{k-j}^{k-j} \leq 1, \sum_{i=0}^{k-j} \alpha_i^{k-j} = 1 \\ & \Delta_{k-j} \geq 0, \Delta_{k-j+1} \geq 0, \dots, \Delta_k \geq 0 \quad (6) \end{aligned}$$

where the first constraint states that relay $k-j+1$ must accumulate I_{max} bits of mutual information by the end of stage $(k-j)$. The second set of constraints state that every subsequent node $k-j+n$ (where $2 \leq n \leq j+1$) including the destination in the ordering \mathcal{O}_{opt} must accumulate I_{max} bits of mutual information by the end of stage $(k-j+n)$. In the last term of the left hand side of each such constraint, we have used the induction hypothesis that in every stage after stage $(k-j)$, only the node that just decoded the packet transmits. Using the transform $\beta_i = \Delta_{k-j} \alpha_i^{k-j}$ for all $i \in \{0, 1, 2, \dots, k-j\}$, and $\sum_{i=0}^{k-j} \alpha_i^{k-j} = 1$, we have the equivalent problem:

$$\begin{aligned} \text{Minimize: } & \sum_{i=0}^{k-j} \beta_i + \Delta_{k-j+1} + \dots + \Delta_{k-1} + \Delta_k \\ \text{Subject to: } & I_{k-j+1}(t_{k-j}) + \sum_{i=0}^{k-j} \beta_i C_{i,k-j+1} = I_{max} \\ & I_{k-j+n}(t_{k-j}) + \sum_{i=0}^{k-j} \beta_i C_{i,k-j+n} \\ & + \sum_{i=1}^{n-1} \Delta_{k-j+i} C_{k-j+i,k-j+n} = I_{max} \quad \forall n \in \{2, \dots, j+1\} \\ & \beta_i \geq 0 \quad \forall i \in \{0, 1, 2, \dots, k-j\}, \Delta_{k-j+1} \geq 0, \dots, \Delta_k \geq 0 \quad (7) \end{aligned}$$

The problems (6) and (7) are equivalent because we can transform (7) to the original problem by using the relations $\Delta_{k-j} = \sum_{i=0}^{k-j} \beta_i$ and $\alpha_i^{k-j} = \frac{\beta_i}{\Delta_{k-j}}$. The degenerate case where $\Delta_{k-j} = 0$ does not arise because if $\Delta_{k-j} = 0$, then no node transmits in stage $(k-j)$. We know from the induction

$$\begin{bmatrix} \sum_{i=0}^{k-j} \beta_i C_{i,k-j+2} \\ \sum_{i=0}^{k-j} \beta_i C_{i,k-j+3} \\ \vdots \\ \sum_{i=0}^{k-j} \beta_i C_{i,d} \end{bmatrix} + \begin{bmatrix} C_{k-j+1,k-j+2} & \cdots & 0 \\ C_{k-j+1,k-j+3} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_{k-j+1,d} & \cdots & C_{k,d} \end{bmatrix} \begin{bmatrix} \Delta_{k-j+1} \\ \Delta_{k-j+2} \\ \vdots \\ \Delta_k \end{bmatrix} = \begin{bmatrix} I_{max} - I_{k-j+2}(t_{k-j}) \\ I_{max} - I_{k-j+3}(t_{k-j}) \\ \vdots \\ I_{max} - I_d(t_{k-j}) \end{bmatrix}$$

TABLE I
SECOND SET OF CONSTRAINTS IN (7) IN MATRIX FORM.

hypothesis that only the nodes after node $k-j$ in the ordering \mathcal{O}_{opt} transmit after stage $(k-j)$. This means that node $k-j$ never transmits, a contradiction.

The second set of constraints in problem (7) can be written in matrix form as $\mathbf{B} + \mathbf{C}\Delta = \mathbf{I}$ as shown in Table I where \mathbf{C} is a lower triangular matrix with all diagonal entries being non-zero. Note that if we are using the optimal relay set such that each node transmits for a non-zero duration, then none of the capacity values in the diagonal of \mathbf{C} can be zero. This is because each such term denotes the capacity value between successive relay nodes in the optimal transmission schedule. If any of these is zero, it would imply that a successor relay node is able to decode the packet without requiring its predecessor node to transmit. Then, one can simply remove the predecessor node from the optimal relay set, leading to a contradiction.

Thus, we have: $\Delta = \mathbf{C}^{-1}(\mathbf{I} - \mathbf{B})$. Therefore each of the terms $\Delta_{k-j+1}, \Delta_{k-j+2}, \dots, \Delta_{k-1}, \Delta_k$ is linear in the variables $\beta_0, \beta_1, \dots, \beta_{k-j}$. Using this, the objective in (7) can be expressed as a linear function of these variables. Let this be denoted by $f(\beta_0, \beta_1, \dots, \beta_{k-j})$. Also we know that under the optimal solution, $\Delta_{k-j+1} > 0, \dots, \Delta_k > 0$. Thus, we can remove the last set of constraints from (7) without affecting the optimal solution (using Lemma 1). Thus, (7) becomes:

$$\begin{aligned} \text{Minimize:} & \quad f(\beta_0, \beta_1, \dots, \beta_{k-j}) \\ \text{Subject to:} & \quad I_{k-j+1}(t_{k-j}) + \sum_{i=0}^{k-j} \beta_i C_{i,k-j+1} = I_{max} \\ & \quad \beta_i \geq 0 \quad \forall i \in \{0, 1, 2, \dots, k-j\} \end{aligned} \quad (8)$$

Similar to the stage $(k-1)$ case, this optimization problem is linear in β_i with a single linear equality constraint and thus the solution is of the form where all except one β_i are zero. Since $\alpha_i^{k-j} = \frac{\beta_i}{\Delta_{k-j}}$, we have that in the optimal solution, exactly one of the fractions $\alpha_0^{k-j}, \alpha_1^{k-j}, \dots, \alpha_{k-j}^{k-j}$ is equal to 1 and rest must be 0. This implies that only one node transmits in this stage. Further, this node must be the relay node $k-j$ that decoded the packet at the beginning of this stage. Else, node $k-j$ never transmits. This is because by definition of stage $(k-j)$, node $k-j$ does not have the packet before the beginning of stage $(k-j)$ and hence cannot transmit before stage $(k-j)$. By induction hypothesis, only nodes $k-j+1, k-j+2, \dots, k$ transmit when stage $(k-j)$ ends. Thus, if node $k-j$ is not the node chosen for stage $(k-j)$, it never transmits, contradicting the fact that it is part of the optimal set. This proves the Theorem.

E. Exact Solution for a Line Topology

In this section, we present the optimal solution for a special case of line topologies. Specifically, all nodes are located on a line as shown in Fig. 4 and no two nodes are co-located. We assume that each node transmits at the same PSD P . Further, the transmission capacity C_{ij} between any two nodes i and j depends only on the distance d_{ij} between the two nodes and is a monotonically decreasing function of d_{ij} . For example, we may have that $C_{ij} = \log_2(1 + \frac{h_{ij}P}{N_0})$ where P is the PSD and $h_{ij} = \frac{1}{d_{ij}^\alpha}$ where $\alpha \geq 2$ is the path loss coefficient. Under these assumptions, we can determine the optimal cooperating set for the problem of routing with mutual information accumulation as follows.

Lemma 2: The optimal cooperating set for the line topology as described above is given by the set of all relay nodes located between the source and the destination.

Proof: (*Lemma 2*): Consider the line network as shown in Fig. 4. We first show that the optimal cooperating set cannot contain any relay node that lies to the left of the source. Suppose the optimal set contains one or more such nodes. Then, we can replace all transmissions by these nodes with a source transmission and get a smaller delay. This is because the source has a strictly higher transmission capacity to *all* nodes to its right than each of these nodes.

Next, we show that the optimal cooperating set must contain all the nodes that are located between s and d . We know that s is the first node to transmit. The first relay node that decodes the packet is node 1, since link $s-1$ has the smallest distance and therefore the highest transmission capacity among all links from s to nodes to the right of s . From Theorem 1, we know that once node 1 has decoded the packet, it should start transmitting if it is part of the optimal set. Else, it never transmits and the source continues to transmit until another node can decode the packet. Suppose that the optimal set does not contain node 1. Then, we can get a smaller delay by having node 1 transmit instead of s once it has decoded the packet. This is because node 1 has a strictly higher transmission capacity to *all* nodes to its right than s . Thus, we have that the optimal set must contain node 1.

The above argument can now be applied to each of the nodes $2, 3, \dots, n$ as in Fig. 4. This proves the Lemma. \blacksquare

To get an idea of the reduction in delay achieved by using mutual information accumulation over traditional routing, consider the line topology above with n nodes placed between s and d at equal distance such that $d_{i,i+1} = 1$ for all i . Also, suppose the transmission capacity on link $i-j$ is given by $C_{ij} = \frac{\gamma P}{d_{ij}^2}$ where $\gamma > 0$ is a constant. Then the capacity of link $s-1$ is γP , the capacity of link $s-2$ is $\frac{\gamma P}{4}$, the capacity of link

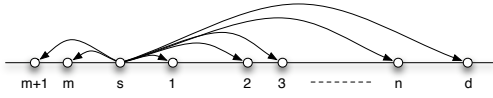


Fig. 4. A line topology.

link $s-3$ is $\frac{\gamma P}{9}$, and so on. Define $\theta \triangleq \gamma P$. Then, the minimum delay for routing with mutual information accumulation is given by $\sum_{i=0}^n \Delta_i$ where:

$$\begin{aligned} \Delta_0 &= \frac{I_{max}}{C_{s1}} = \frac{I_{max}}{\theta}, \Delta_1 = \frac{I_{max} - \Delta_0 C_{s2}}{C_{12}} = \frac{I_{max} - \Delta_0 \frac{\theta}{4}}{\theta} \\ &\vdots \\ \Delta_n &= \frac{I_{max} - \sum_{i=0}^{n-1} \Delta_i C_{i,n+1}}{C_{n,n+1}} = \frac{I_{max} - \sum_{i=0}^{n-1} \Delta_i \frac{\theta}{(n+1-i)^2}}{\theta} \end{aligned}$$

For simplicity, let us ignore the contribution of nodes that are more than 3 units away from a receiver. Then, we have:

$$\begin{aligned} \sum_{i=0}^n \Delta_i &= \frac{(n+1)I_{max} - \frac{\theta}{4} \sum_{i=0}^{n-1} \Delta_i - \frac{\theta}{9} \sum_{i=0}^{n-2} \Delta_i}{\theta} \\ \Rightarrow \sum_{i=0}^n \Delta_i &= \frac{(n+1)I_{max} + \frac{\theta}{4} \Delta_n + \frac{\theta}{9} (\Delta_n + \Delta_{n-1})}{\theta(1 + \frac{1}{4} + \frac{1}{9})} \\ &< \frac{(n+1)I_{max} + \frac{\theta}{4} \Delta_0 + \frac{\theta}{9} 2\Delta_0}{\theta(1 + \frac{1}{4} + \frac{1}{9})} = \frac{I_{max}}{\theta} \left(\frac{n+1 + \frac{1}{4} + \frac{2}{9}}{1 + \frac{1}{4} + \frac{1}{9}} \right) \end{aligned}$$

where we used the fact that $\Delta_n, \Delta_{n-1} < \Delta_0$. The minimum delay for traditional routing is simply $(n+1)\Delta_0 = (n+1)\frac{I_{max}}{\theta}$. Thus, for this network, the delay under mutual information accumulation is smaller than that under traditional routing at least by a factor $\frac{n+1 + \frac{1}{4} + \frac{2}{9}}{(n+1)(1 + \frac{1}{4} + \frac{1}{9})}$ that approaches $\frac{36}{49} = 73\%$ for large n .

IV. DISTRIBUTED HEURISTICS AND SIMULATIONS

The greedy algorithm presented in Section III-C to compute the optimal solution to problem (1) has an exponential computational complexity and is centralized. In this section, we present two simple heuristics that can be implemented in polynomial time and in a distributed fashion. We compare the performance of these heuristics with the optimal solution on general network topologies. We also show the performance of the traditional minimum delay route that does not use mutual information accumulation.

Heuristic 1: Here, first the traditional minimum delay route is computed using, say, Dijkstra's shortest path algorithm on the weighted graph (where the weight w_{ij} of link $i-j$ is defined as the time required to deliver a packet from i to j , i.e., $w_{ij} = \frac{I_{max}}{C_{ij}}$). Let \mathcal{M} denote the set of relay nodes that form this minimum delay shortest path. Then the greedy algorithm as described in Section III-C is applied on the set of nodes in \mathcal{M} . Note that we are not searching over all subsets of \mathcal{M} .² Thus, the complexity of this heuristic is same as that

²It may be possible to get further gains by searching over all subsets of \mathcal{M} , but the worst case complexity of doing so would again be exponential. Our goal here is to develop polynomial time algorithms.

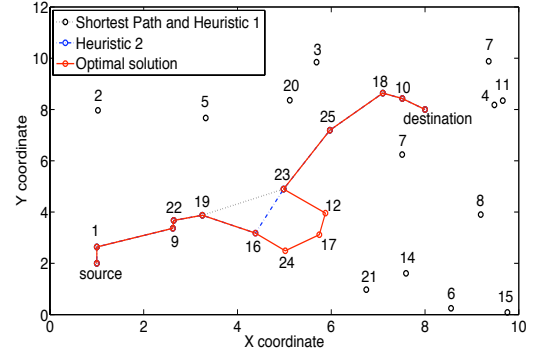


Fig. 5. A 25 node network where the routes for traditional minimum delay, Heuristics 1 and 2, and optimal mutual information accumulation are shown.

of any shortest path algorithm, i.e., $O(|\mathcal{M}|^2)$.

Heuristic 2: Here, we start with \mathcal{M} as the initial cooperative set. Then, while applying the greedy algorithm of Section III-C, if other nodes that are not in \mathcal{M} happen to decode the packet before the next node (where the next node is defined as that node in \mathcal{M} that would decode the packet if the current transmitter continued its transmission), then these nodes are added to the cooperative set if they have a better channel to the next node than the current transmitter. The intuition behind this heuristic is that while \mathcal{M} is expected to be a good cooperative set, this allows the algorithm to explore more nodes and potentially improve over Heuristic 1.

In our simulations, we consider a network of a source, destination, and n relay nodes located in a 10×10 area. The location of source (1.0, 2.0) and destination (8.0, 8.0) is fixed while the locations of the other nodes are chosen uniformly at random. The link gain between any two nodes i and j is chosen from a Rayleigh distribution with mean 1. For simplicity, all nodes have the same PSD. The total bandwidth W and packet size I_{max} are both normalized to one unit. The transmission capacity of link $i-j$ is assumed to be $C_{ij} = \log_2 \left(1 + \frac{h_{ij}}{d_{ij}^\alpha} \right)$ where d_{ij} is the distance between nodes i and j and α is the path loss exponent. We choose $\alpha = 3$ for all simulations.

In the first simulation, $n = 25$ and the network topology is fixed as shown in Fig. 5. We then compute the traditional minimum delay route and the optimal solution for routing with mutual information accumulation using the greedy algorithm of Section III-C. We also implement Heuristics 1 and 2 on this network. Fig. 5 shows the results. It is seen that the traditional minimum delay route is given by $[s, 1, 9, 22, 19, 23, 25, 18, 10, d]$ while the optimal mutual information accumulation route (according to the decoding order) is given by $[s, 1, 9, 22, 19, 16, 24, 17, 12, 23, 25, 18, 10, d]$. The decoding order of nodes under Heuristic 1 is same as that under the traditional minimum delay route while that under Heuristic 2 is given by $[s, 1, 9, 22, 19, 16, 23, 25, 18, 10, d]$. The total delay under traditional minimum delay routing, Heuristic 1, Heuristic 2, and optimal mutual information accumulation routing was found to be 29.84, 23.73, 22.99 and 22.19 seconds respectively.

This example demonstrates that the optimal route under mutual information accumulation can be quite different from

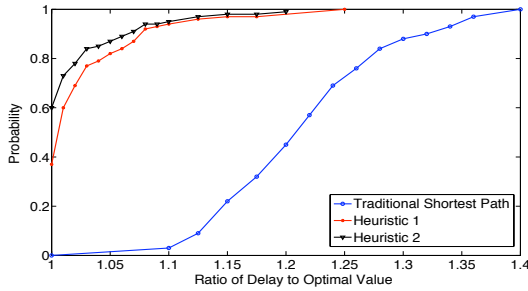


Fig. 6. The CDF of the ratio of the minimum delay under the two heuristics and the traditional shortest path to the minimum delay under the optimal mutual information accumulation solution.

the traditional minimum delay path. It is also interesting to note that the set of nodes in \mathcal{M} is a subset of the cooperative relay set in this example. However, this does not hold in general. We also note that the delay under both Heuristic 1 and 2 is close to the optimal value. Finally, while Heuristic 1 only uses the nodes in \mathcal{M} , Heuristic 2 explores more and ends up using node 16 as well.

In the second simulation, we choose $n = 20$. The source and destination locations are fixed as before but the locations of the relay nodes are varied randomly over 100 instances. For each topology instance, we compute the minimum delay obtained by these 4 algorithms. In Fig. 6, we plot the cumulative distribution function (CDF) of the ratio of the minimum delay under the two heuristics and the traditional shortest path to the minimum delay under the optimal mutual information accumulation solution. From this, it can be seen that both Heuristic 1 and 2 perform quite well over general network topologies. In fact, they are able to achieve the optimal performance 40% and 60% of the time respectively. Further, they are within 10% of the optimal at least 90% of the time and within 15% of the optimal at least 98% of the time. Also, Heuristic 2 is seen to outperform Heuristic 1 in general. Finally, the average delay in routing with mutual information accumulation was found to be 77% of the average delay of traditional shortest path routing (where the average is taken over the 100 random topologies). It is interesting to note that once the optimal relay set is computed, the only overhead of a mutual information accumulation based scheme over traditional shortest path is in the decoding and re-encoding operations. In particular, such schemes only require a 1-bit feedback from a receiver when it has successfully decoded the packet and do not require sophisticated synchronization, coordination, or extensive channel state information feedback.

V. CONCLUSIONS

In this work, we considered the problem of minimum delay routing over a multi-hop wireless network using mutual information accumulation. We formulated the general problem as combinatorial optimization problem and then made use of several structural properties to simplify the solution and derive an optimal greedy algorithm. A key feature of our algorithm is that unlike prior works on such problems, it does not require solving any linear programs to compute the optimal solution. While the greedy algorithm still has exponential complexity,

it is significantly simpler than prior schemes and allows us to compute the optimal solution as a benchmark. We also proposed two simple and practical heuristics that exhibit very good performance when compared to the optimal solution.

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