# Contention-Aware Performance Analysis of Mobility-Assisted Routing 

Apoorva Jindal, Konstantinos Psounis<br>University of Southern California<br>E-mail: apoorvaj, kpsounis@usc.edu.


#### Abstract

A large body of work has theoretically analyzed the performance of mobility-assisted routing schemes for intermittently connected mobile networks. But a vast majority of these prior studies have ignored wireless contention. Recent papers have shown through simulations that ignoring contention leads to inaccurate and misleading results, even for sparse networks.

In this paper, we analyze the performance of routing schemes under contention. To model contention we use our recently-proposed analytical framework which is applicable to any multi-hop wireless network. Then, we take into consideration the special characteristics of intermittently connected mobile networks and compute the delays for different representative mobility-assisted routing schemes for these networks. We analyze these schemes for the random direction, random waypoint and the more realistic community-based mobility models. Finally, we use these delay expressions to answer practical questions in the context of designing more efficient mobility-assisted routing schemes.


## I. Introduction

Intermittently connected mobile networks (also referred to as delay tolerant or disruption tolerant networks) are networks where most of the time, there does not exist a complete end-to-end path from the source to the destination. Even if such a path exists, it may be highly unstable because of topology changes due to mobility. Examples of such networks include sensor networks for wildlife tracking and habitat monitoring [1], military networks [2], deep-space inter-planetary networks [3], nomadic communities networks [4], networks of mobile robots [5], vehicular ad hoc networks [6] etc.

Conventional routing schemes for mobile ad-hoc networks like DSR, AODV, etc. [7] assume that a complete path exists between a source and a destination, and they try to discover these paths before any useful data is sent. Since, no end-to-end paths exist most of the times in intermittently connected mobile networks (ICMN's), these protocols will fail to deliver any data to all but the few connected nodes. To overcome this issue, researchers have proposed to exploit node mobility to carry messages around the network as part of the routing algorithm. These routing schemes are collectively referred to as mobility-assisted or encounterbased or store-carry-and-forward routing schemes.

A number of mobility-assisted routing schemes for intermittently connected mobile networks have been proposed in the literature [8-18]. Researchers have also tried to theoretically characterize the performance of these routing schemes [17, 1924]. But, most of these analytical works ignore the effect of contention on the performance. The assumption of ignoring contention is justified by arguing that contention will not have a significant impact on performance in sparse networks. However,
recent papers $[17,25]$ have shown through simulations that this argument is not necessarily true. The assumption of no contention is valid only for very low traffic rates, irrespective of whether the network is sparse or not. For higher traffic rates, contention has a significant impact on the performance, especially of flooding based routing schemes. To demonstrate the inaccuracies which arise when contention is ignored, we use simulations to compare the delay of three different routing schemes in a sparse network, both with and without contention, in Figure 1. The plot shows that ignoring contention not only grossly underestimates the delay, but also predicts incorrect trends and leads to incorrect conclusions. For example, without contention, the spraying based scheme has the worst delay, while with contention, it has the best delay.


Fig. 1. Comparison of delay with and without contention for three different routing schemes in sparse networks. The simulations without contention ensure that there is only one packet in the network at a given time. The simulations with contention use the scheduling mechanism and interference model described in Section III-A. The routing schemes compared are: epidemic routing [8], randomized flooding [25] and spraying based routing [12].

In general, incorporating wireless contention complicates the analysis significantly because it is a very complex phenomenon manifesting itself in three ways: (i) finite bandwidth which limits the number of packets two nodes can exchange while they are within range, (ii) scheduling of transmissions between nearby nodes which is needed to avoid excessive interference, and (iii) interference from transmissions outside the scheduling area, which may be significant due to multipath fading [26]. Recently, [27] has proposed a general framework to incorporate contention in a mobile wireless multi-hop network while keeping the analysis tractable. This framework incorporates all the three
manifestations of contention, and can be used with any mobility and channel model. In this framework, loss of a transmission opportunity due to contention is modeled by a loss probability. The paper also gives a general analytical methodology to find the exact expression for this loss probability in terms of the network parameters for any given routing and scheduling scheme. (Note that previous papers like [23, 28], have also proposed modeling loss due to contention with a loss probability, but none of them discuss how to find this loss probability analytically.) We use this framework to model contention to do a contention-aware performance analysis of different representative mobility-assisted routing schemes for intermittently connected mobile networks.

In this paper, we will derive the expected delay for the following mobility-assisted routing schemes: direct transmission [16], epidemic routing [8], randomized flooding (or gossiping) [21,22,25] and spraying based routing schemes [12, 13, 22, 23]. For each of these schemes, we will find the loss probability due to contention using the framework proposed in [27] and then find the expected end-to-end delay expressions. Note that other papers have studied the performance of these routing schemes without contention in the network. For example, $[11,20]$ studied the performance of direct transmission, [19-21] studied epidemic routing, [21-23] studied randomized flooding and [12, 17] studied different spraying based schemes. $[24,29]$ are preliminary efforts of ours to analyze the performance of routing schemes under contention. Specifically, [24] studies the expected delay of epidemic routing under the random walk mobility model and [29] studies randomized flooding and a spraying based scheme under the random waypoint mobility model. Here, we generalize our prior work and provide results for more efficient routing schemes under more realistic mobility models.

We will first derive the delay expressions for different routing schemes for the two most commonly used mobility models, the random direction and the random waypoint mobility model. But, real world mobility traces show that the random direction and the random waypoint mobility models are not realistic [30,31]. Based on the intuition gained from these traces, Spyropoulos et al [20] proposed a more realistic and analytically tractable community-based mobility model. So, we also analyze these routing schemes for the more realistic community-based mobility model. The analysis for the community-based mobility model is similar to the derivations for the random direction / random waypoint mobility models. (Note that we include the analysis for the random direction / random waypoint mobility models because it is simpler, easier to understand and naturally extends to the derivations for the more complicated communitybased mobility model.) We also study a spraying-based scheme proposed to exploit the heterogeneity introduced in the network by the community-based mobility model. We then use these delay expressions to answer practical questions in the context of designing more efficient mobility-assisted routing schemes. First, we compare the performance of randomized flooding and a simple spraying scheme to conclude that spraying-based schemes outperform gossip based schemes. So, we study the spraying based schemes in more detail. We first discuss how to spray copies in the spraying phase and then study how to route each sprayed copy towards the destination so as to reduce the overall end-to-end delay.

| $N$ | Area of the 2D torus |
| :---: | :---: |
| $M$ | Number of nodes in the network |
| $K$ | The transmission range |
| $\Theta$ | The desirable SIR ratio |
| $s_{B W}$ | Bandwidth of links in units of packets per time slot |

TABLE I
NOTATION USED THROUGHOUT THE PAPER.

The outline of this paper is as follows: Section II presents our notation and assumptions, summarizes the contention framework and defines some properties of the mobility model which we will use during the course of the analysis. Then, sections IV and V find the expected delay expressions for the random direction / random waypoint mobility models and the more realistic communitybased mobility model respectively. Section VI studies the impact of the different approximations made during the analysis on its accuracy by comparing the analytical results to the simulation results. Section VII then uses the expressions derived in the previous sections to answer some pertinent questions in the context of designing more efficient routing schemes for sparse networks. Finally, Section VIII concludes the paper.

## II. Preliminaries

## A. Notation and Assumptions

We first introduce our notation and state the assumptions we will be making throughout the remainder of the paper.

1. $M$ nodes move in a two dimensional torus of area $N$.
2. Each node acts as a source sending packets to a randomly selected destination.
3. We assume a Rayleigh-Rayleigh fading model for the channel (both the desired and the interfering signals are Rayleigh distributed).
4. The signal to interference ratio should be greater than a desired threshold, which we call $\Theta$, for the transmission to be successful. For ease of analysis, we assume that two nodes will try to transmit to each other only if the link between them is in the connected region (not in the transitional or grey region). $[26,32]$ show that this is equivalent to assuming that the nodes will transmit to each other when the distance between them is less than $K$. (The value of $K$ depends on the transmit power.) Note that this does not imply that transmissions from nodes at a distance greater than $K$ are not going to interfere with the ongoing transmission or that the ongoing transmission will always be successful.

## III. Tables and Figures

## A. Contention Model

This section briefly summarizes the contention model introduced in [27].

## 1) Three Manifestations of Contention:

Finite Bandwidth: When two nodes meet, they might have more than one packet to exchange. Say two nodes can exchange $s_{B W}$ packets during a unit of time. If they move out of each other's range, they will have to wait until they meet again to transfer
more packets. The number of packets which can be exchanged in a unit of time is a function of the packet size and the bandwidth of the links.
Scheduling: We assume that a CSMA-CA like scheduling mechanism which ensures no simultaneous transmission occurs within the scheduling area of the transmitter and the receiver, is in place to avoid excessive interference. For ease of analysis, we also assume that time is slotted. At the start of the time slot, all node pairs contend for the channel and once a node pair captures the medium, it retains the medium for the entire time slot.
Interference: Even though the scheduling mechanism is ensuring that no simultaneous transmissions are taking place within each other's scheduling area, there is no restriction on simultaneous transmissions taking place outside the scheduling area. These transmissions act as noise for each other and hence can lead to packet corruption.

In the absence of contention, two nodes would exchange all the packets they want to exchange whenever they come within range of each other. Contention will result in a loss of such transmission opportunities. This loss can be caused by either of the three manifestations of contention. Next we summarize the framework proposed in [27] to find this loss probability.
2) The Framework:

Lets look at a particular packet, label it packet $A$. Suppose two nodes $i$ and $j$ are within range of each other at the start of a time slot and they want to exchange this packet. Let $p_{t x S}^{R}$ denote the probability that they will successfully exchange the packet during that time slot. (The value of $p_{t x S}^{R}$ depends on the routing mechanism $R$.) Note that $1-p_{t x S}^{R}$ denotes the loss probability due to contention.

Let $E_{b w}$ denote the event that finite link bandwidth allows nodes $i$ and $j$ to exchange packet $A$, let $E_{\text {sch }}$ denote the event that the scheduling mechanism allows nodes $i$ and $j$ to exchange packets. and, let $E_{\text {inter }}$ denote the event that the transmission of packet $A$ is not corrupted due to interference given that nodes $i$ and $j$ exchanged this packet. Packet $A$ will be successfully exchanged by nodes $i$ and $j$ only if the following three events occur: (i) the scheduling mechanism allows these nodes to exchange packets, (ii) nodes $i$ and $j$ decide to exchange packet $A$ from amongst the other packets they want to exchange, and (iii) this transmission does not get corrupted due to interference from transmissions outside the scheduling area. Thus,

$$
\begin{equation*}
p_{t x S}^{R}=P\left(E_{b w}\right) \times P\left(E_{\text {sch }}\right) \times P\left(E_{\text {inter }}\right) \tag{1}
\end{equation*}
$$

[27] shows how to compute the three unknown probabilities in Equation (1).

The derivation of these unknown probabilities in [27] shows that $p_{t x S}^{R}$ depends on the routing mechanism $R$ only through the probability $p_{e x}^{R}$, which is the probability that two nodes $i$ and $j$ want to exchange a particular packet. In other words, to apply this framework to a given routing mechanism, the only variable whose value needs to be determined is $p_{e x}^{R}$. The value of all the three probabilities depend on $p_{e x}^{R}$. For example, to find $P\left(E_{s c h}\right)$, one has to figure out the number of transmitter-receiver pairs within the scheduling area of the $i-j$ pair which have a packet to exchange (transmitter-receiver pairs which do not have any packet to exchange will not contend for the channel). The probability that
two nodes have at least one packet to exchange is a function of $p_{e x}^{R}$.

We will find the value of $p_{e x}^{R}$ for each routing mechanism we analyze. Given the value of $p_{e x}^{R}$, the value of $p_{t x S}^{R}$ can be derived using the analytical methodology of [27]. We skip the derivation of $p_{t x S}^{R}$ given $p_{e x}^{R}$ in this paper. (The interested reader is referred to [27] for details.)

## B. Mobility Properties

In this section, we define three properties of a mobility model. We will use the statistics of these three properties in the analysis.
(i) Meeting Time: Let nodes $i$ and $j$ move according to a mobility model ' mm ' and start from their stationary distribution at time 0 . Let $X_{i}(t)$ and $X_{j}(t)$ denote the positions of nodes $i$ and $j$ at time $t$. The meeting time ( $M_{m m}$ ) between the two nodes is defined as the time it takes them to first come within range of each other, that is $M_{m m}=\min _{t}\left\{t:\left\|X_{i}(t)-X_{j}(t)\right\| \leq K\right\}$.
(ii) Inter-Meeting Time: Let nodes $i$ and $j$ start from within range of each other at time 0 and then move out of range of each other at time $t_{1}$, that is $t_{1}=\min _{t}\left\{t:\left\|X_{i}(t)-X_{j}(t)\right\|>K\right\}$. The inter meeting time $\left(M_{m m}^{+}\right)$of the two nodes is defined as the time it takes them to first come within range of each other again, that is $M_{m m}^{+}=\min _{t}\left\{t-t_{1}:\left\|X_{i}(t)-X_{j}(t)\right\| \leq K\right\}$.
(iii) Contact Time: Assume that nodes $i$ and $j$ come within range of each other at time 0 . The contact time $\tau_{m m}$ is defined as the time they remain in contact with each other before moving out of the range of each other, that is $\tau_{m m}=\min _{t}\{t-1$ : $\left.\left\|X_{i}(t)-X_{j}(t)\right\|>K\right\}$.

## IV. Delay Analysis for Popular Mobility Models

In this section, we find the expected end-to-end delay of five different mobility-assisted routing schemes for intermittently connected mobile networks. For each routing scheme, we first define the routing algorithm and then derive the end-to-end delay. In this section, we assume that nodes are moving around according to the mobility model ' mm ' where ' mm ' represents either the random direction or the random waypoint mobility model. The statistics of the properties defined in Section III-B for the random direction and the random waypoint mobility models were studied by [20] and [33]. The two important properties satisfied by both the mobility models, which we use during the course of the analysis are as follows: (i) The expected intermeeting time is approximately equal to the expected meeting time and (ii) The tail of the distribution of the meeting and the intermeeting times is exponential ${ }^{1}$.

## A. Direct Transmission

Direct transmission is one of the simplest possible routing schemes. Node $A$ forwards a message to another node $B$ it encounters, only if $B$ is the message's destination. We now analyze its performance with contention.

[^0]First, we find the value of $p_{e x}^{d t}$ (the probability that two nodes $i$ and $j$ want to exchange a particular packet) for direct transmission ${ }^{2}$. and then find the expected end-to-end delay.

Lemma 4.1: $p_{e x}^{d t}=\frac{2}{M(M-1)}$.
Proof: In direct transmission, each packet undergoes only one transmission, from the source to the destination. A packet has node $i$ as its source with probability $\frac{1}{M}$. The probability that $j$ is the destination given $i$ is the source is $\frac{1}{M-1}$ (the destination is chosen uniformly at random from amongst the other $M-1$ nodes). Thus, the probability that $i$ and $j$ want to exchange a particular packet is equal to $\frac{2}{M(M-1)}(i$ is the source and $j$ is the destination or vice versa).

Theorem 4.1: Let $E\left[D_{d t}^{m m}\right]$ denote the expected delay of direct transmission. Then, $E\left[D_{d t}^{m m}\right]=\frac{E\left[M_{m m}\right]}{p_{s u c c e s s}^{d t}}$, where $E\left[M_{m m}\right]$ is the expected meeting time of the mobility model ' mm ', $p_{\text {success }}^{d t}$ is the probability that when two nodes come within range of each other, they successfully exchange the packet before going out of each other's range (within the contact time $\tau_{m m}$ ) and is equal to $1-\left(1-p_{t x S}^{d t}\right)^{E\left[\tau_{m m}\right]}$.

Proof: The expected time it takes for the source to meet the destination for the first time is $E\left[M_{m m}\right]$ (the expected meeting time). $1-p_{t x S}^{d t}$ is the probability of loss of a transmission opportunity due to contention. Thus, with probability $1-p_{t x S}^{d t}$, the source and the destination are unable to exchange the packet in one time slot. They are within range of each other for $E\left[\tau_{m m}\right]$ number of time slots. (We are making an approximation here by replacing $\tau_{m m}$ by its expected value.) Then $\left(1-p_{t x S}^{d t}\right)^{E\left[\tau_{m m}\right]}$ is the probability that the source fails to deliver the packet to the destination when they came within range of each other. Thus, $p_{\text {success }}^{d t}=1-\left(1-p_{t x S}^{d t}\right)^{E\left[\tau_{m m}\right]}$.

If the two nodes fail to exchange the packet when they were within range, then they will have to wait for one inter-meeting time to come within range of each other again. If they fail yet again, they will have to wait another inter-meeting time to come within range. Thus, $E\left[D_{d t}^{m m}\right]=E\left[M_{m m}\right]+p_{\text {success }}^{d t}$ $\left(\left(1-p_{\text {success }}^{d t}\right) E\left[M_{m m}^{+}\right]+2\left(1-p_{\text {success }}^{d t}\right)^{2} \quad E\left[M_{m m}^{+}\right]+\ldots\right)=$ $E\left[M_{m m}\right]+\frac{\left(1-p_{s u c e s s}^{d t}\right) E\left[M_{m m}^{+}\right]}{p_{s u c c e s s}^{d t}}$. Since $E\left[M_{m m}^{+}\right]=E\left[M_{m m}\right]$ for both random direction and random waypoint mobility models, $E\left[D_{d t}^{m m}\right]$ evaluates to $\frac{E\left[M_{m m}\right]}{p_{s u c c e s s}^{d t}}$.

## B. Epidemic Routing

Epidemic routing [8] extends the concept of flooding to ICMN's. It is one of the first schemes proposed to enable message delivery in such networks. Each node maintains a list of all messages it carries, whose delivery is pending. Whenever it encounters another node, the two nodes exchange all messages that they don't have in common. This way, all messages are eventually spread to all nodes. The packet is delivered when the first node carrying a copy of the packet meets the destination. The packet will keep on getting copied from one node to the other node till its Time-To-Live (TTL) expires. For ease of analysis, we assume that as soon as the packet is delivered to the destination, no further copies of the packet are spread.

[^1]To find the expected end-to-end delay for epidemic routing, we first find $E\left[D_{\text {epidemic }}^{m m}(m)\right]$ which is the expected time it takes for the number of nodes having a copy of the packet to increase from $m$ to $m+1$.

Lemma 4.2: $E\left[D_{\text {epidemic }}^{m m}(m)\right]=\frac{E\left[M_{m m}\right]}{m(M-m) p_{s u c u c c e s}{ }^{\text {pidemic }}}$, where $p_{\text {success }}^{\text {epidemic }}=1-\left(1-p_{t x S}^{\text {epidemic }}\right)^{E\left[\tau_{m m}\right]^{m(M}}$.

Proof: $E\left[D_{\text {epidemic }}^{m m}(m)\right]$ is the expected time it takes for the copies of a packet to increase from $m$ to $m+1$. When there are $m$ copies of a packet in the network, if one of the $m$ nodes having a copy meets one of the other $M-m$ nodes not having a copy, there is a transmission opportunity to increase the number of copies by one. Since intermittently connected mobile networks are sparse networks, we look at the tail of the distribution of the meeting time which is exponential for both the random direction and the random waypoint mobility models. The time it takes for one of the $m$ nodes to meet one of the other $M-m$ nodes is equal to the minimum of $m(M-m)$ exponentials, which is again an exponential random variable with mean $\frac{E\left[M_{m m}\right]}{m(M-m)}$. Now when they meet, the probability that the two nodes are able to successfully exchange the packet is $p_{\text {success }}^{\text {epidemic }}$. If they fail to exchange the packet, they will have to wait one inter-meeting time to meet again. But, since $E\left[M_{m m}\right]=E\left[M_{m m}^{+}\right]$for both the random direction and the random waypoint mobility model, and both meeting and inter-meeting times have memoryless tails, the expected time it takes for one of the $m$ nodes to meet one of the other $M-m$ nodes again is still equal to $\frac{E\left[M_{m m}\right]}{m(M-m)}$. Hence, $E\left[D_{\text {epidemic }}^{\text {mm }}(m)\right]=p_{\text {success }}^{\text {epidemic }} \frac{E\left[M_{m m}\right]}{m(M-m)}+2 p_{\text {success }}^{\text {epidemic }}(1-$ $\left.p_{\text {success }}^{\text {epidemic }}\right) \frac{E\left[M_{m m}\right]}{m(M-m)}+\ldots=\frac{E\left[M_{m m}\right]}{m(M-m) p_{s u c u c e s s}^{e p i c s}}$. The value of $p_{\text {success }}^{\text {epidemic }}$ can be derived in a manner similar to the derivation of $p_{\text {success }}^{d t}$ in Theorem 4.1.

Now, we find the value of $p_{e x}^{e p i d e m i c}$ for epidemic routing and then find the expected end-to-end delay.

Lemma 4.3: $p_{e x}^{e p i d e m i c}=\sum_{m=1}^{M-1} \frac{2 m(M-m)}{M(M-1)} \sum_{i=m}^{M-1} \frac{1}{M-1}$ $\frac{\frac{1}{m(M-m)}}{\sum_{j=1}^{i} \bar{j}(M-j)}$.

Proof: Given that only $m$ nodes have a copy of the packet, the probability that one of the nodes has it and the other one doesn't, follows from elementary combinatorics. To complete the proof, we have to find the probability that only $m$ nodes have a copy of the packet. Then, applying the law of total probability over the random variable $m$ will yield the result. Please see the Appendix for proof details.

Theorem 4.2: Let $E\left[D_{\text {epidemic }}^{m m}\right]$ denote the expected delay of epidemic routing. Then, $E\left[D_{\text {epidemic }}^{\text {mm }}\right]=$ $\sum_{i=1}^{M-1} \frac{1}{M-1} \sum_{m=1}^{i} \frac{E\left[M_{m m}\right]}{m(M-m) p_{s u c z e c e s i c}^{\text {pide }}}$.

Proof: The probability that the destination is the $i^{\text {th }}$ node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2 \leq i \leq M$. The amount of time it takes for the $i^{t h}$ copy to be delivered is equal to $\sum_{m=1}^{i} E\left[D_{\text {epidemic }}^{m m}(m)\right]$. Applying the law of total probability over the random variable $i$ and substituting the value of $E\left[D_{\text {epidemic }}^{m m}(m)\right]$ from Lemma 4.2 yields $E\left[D_{\text {epidemic }}^{m m}\right]$.

## C. Randomized Flooding

Randomized flooding [21,25] has been proposed to reduce the overhead and improve the performance of epidemic routing.

Under this scheme, a message is forwarded to another node with some probability $p$ smaller than one (that is data is gossiped instead of flooded). When $p=0$, the scheme reduces to direct transmission, while when $p=1$, it reduces to standard epidemic routing.

We first find the value of $E\left[D_{r f}^{m m}(m)\right]$, then we find the value of $p_{e x}^{r f}$ and finally, we derive the expected end-to-end delay for randomized flooding.

Lemma 4.4: $E\left[D_{r f}^{m m}(m)\right]=\frac{E\left[M_{m m}\right]}{\left(m(M-m-1) p_{s u c c e s s}^{1}\right)+\left(m p_{\text {success }}^{2}\right)}$, where $p_{\text {success }}^{1}=1-\left(1-p p_{t x S}^{r f}\right)^{E\left[\tau_{m m}\right]}$ and $p_{\text {success }}^{2}=1-$ $\left(1-p_{t x S}^{r f}\right)^{E\left[\tau_{m m}\right]}$.

Proof: The proof runs along similar lines as the proof of Lemma 4.2. The only difference is that whenever one of the $m$ nodes having a copy of the packet meet one of the $(M-m-1)$ non-destination nodes which don't have a copy of the packet, they will try to exchange the packet with probability $p$ only.
Lemma 4.5: $p_{e x}^{r f}=p \sum_{m=1}^{M-1} \frac{2 m(M-m)}{M(M-1)} \sum_{i=m}^{M-1} \frac{1}{M-1}$ $\frac{\frac{1}{m(M-m)}}{\sum_{j=1}^{i} \frac{1}{\bar{j}(M-j)}}$.

Proof: See Appendix.
Theorem 4.3: Let $E\left[D_{r f}^{m m}\right]$ denote the expected delay of randomized flooding. Then, $E\left[D_{r f}^{m m}\right] \quad=\quad \sum_{i=1}^{M-1} p_{\text {dest }}^{r f}(i) \sum_{m=1}^{i} E\left[D_{r f}^{m m}(m)\right]$
where $\quad p_{\text {dest }}^{r f}(i)=\frac{i p_{\text {success }}^{2}}{\left(i(M-i-1) p_{\text {success }}^{\text {s. }}\right)+\left(i p_{\text {success }}^{2}\right)}$ $\prod_{m=1}^{i-1} \frac{m(M-m-1) p_{s u c c e s s}^{1}}{\left(m(M-m-1) p_{s u c c e s s}^{1}\right)+\left(m p_{s \text { sccess }}^{2}\right)}$ is the probability that the destination is the $(i+1)^{t h}$ node to receive a copy of the packet.

Proof: The proof runs along similar lines as the proof of Theorem 4.2.

## D. Spraying a small fixed number of copies

Another approach to route packets in sparse networks is that of controlled replication or spraying [12, 13, 22, 23]. A small, fixed number of copies are distributed to a number of distinct relays. Then, each relay routes its copy independently towards the destination. By having multiple relays routing a copy independently and in parallel towards the destination, these protocols create enough diversity to explore the sparse network more efficiently while keeping the resource usage per message low.

Different spraying schemes may differ in how they distribute the copies and, or how they route each copy. We study two different spraying based routing schemes here. These two differ in the way they distribute their copies.

1) Source Spray and Wait: Source spray and wait is one of the simplest spraying schemes proposed in the literature [12]. For this scheme, the source node forwards all the copies (lets label the number of copies being sprayed as $L$ ) to the first $L$ distinct nodes it encounters. (In other words, no other node except the source node can forward a copy of the packet.) And, once these copies get distributed, each copy performs direct transmission.

First, we find the value $E\left[D_{s s w}^{m m}(m)\right]$, then we find $p_{e x}^{s s w}$ and finally, we derive the expected end-to-end delay for source spray and wait.
 where $p_{s u c c e s s}^{s s w}=1-\left(1-p_{t x S}^{s s w}\right)^{E\left[\tau_{m m}\right]}$.

Proof: See Appendix.
Lemma 4.7: $p_{e x}^{s s w}=\left(\frac{2 L p_{d e s t}^{s s w}(L)}{M(M-1)} \frac{E\left[D_{s s w}^{m m}(L)\right]}{\sum_{k=1}^{L} E\left[D_{s s w}^{m m}(k)\right]}\right)+$ $\left(\frac{2}{M-1} \sum_{m=1}^{L-1} \sum_{i=m}^{L} p_{\text {dest }}^{s s w}(i) \frac{E\left[D_{s s w}^{m m}(m)\right]}{\sum_{k=1}^{i} E\left[D_{s s w}^{m m}(k)\right]}\right)$, where
$p_{\text {dest }}^{s s w}(i)=\left\{\begin{array}{cc}\left(\prod_{j=1}^{i-1} \frac{M-j-1}{M-1}\right) \frac{i}{M-1} & 1 \leq i<L \\ \left(\prod_{j=1}^{i-1} \frac{M-j-1}{M-1}\right) & i=L\end{array} \quad\right.$ is the
probability that the destination is the $(i+1)^{t h}$ node to receive a copy of the packet.

Proof: The proof runs along the same lines as the proof of Lemma 4.3.

Theorem 4.4: Let $E\left[D_{s s w}^{m m}\right]$ denote the expected delay of source spray and wait. Then, $E\left[D_{s s w}^{m m}\right]=$ $\sum_{i=1}^{L} p_{\text {dest }}^{s s w}(i) \sum_{m=1}^{i} E\left[D_{s s w}^{m m}(m)\right]$.

Proof: The proof runs along similar lines as the proof of Theorem 4.2.
2) Fast Spray and Wait: In Fast Spray and Wait, every relay node can forward a copy of the packet to a non-destination node which it encounters in the spray phase. (Recall that in source spray and wait, only the source node can forward copies to non-destination nodes.) There is a centralized mechanism which ensures that after $L$ copies of the packet have been spread, no more copies get transmitted to non-destination nodes. And, once these copies get distributed, each copy performs direct transmission. Since fast spray and wait spreads copies whenever there is any opportunity to do so, it has the minimum spraying time when there is no contention in the network. We now derive the expected delay of fast spray and wait with contention in the network.

First, we find the value $E\left[D_{f s w}^{m m}(m)\right]$, then we find $p_{e x}^{f s w}$ and finally, we derive the expected end-to-end delay for fast spray and wait. All the derivations are very similar to the corresponding derivations for epidemic routing. The only difference is that when $m=L$ nodes have a copy of the packet, a transmission opportunity will arise only when one of these $m=L$ nodes meet the destination.
Lemma 4.8: $E\left[D_{f s w}^{m m}(m)\right]$
$\left\{\begin{array}{cc}\frac{E\left[M_{m m}\right]}{m(M-m) p_{s u c e s s}^{f s w}} & 1 \leq m<L \\ \frac{E\left[M_{m m}\right]}{L p_{s u c c e s s}^{f s w}} & m=L\end{array} \quad\right.$ where $\quad p_{\text {success }}^{f s w} \quad=$
$1-\left(1-p_{t x S}^{f s w}\right)^{E\left[\tau_{m m}\right]}$.
Proof: The proof runs along the same lines as the proof of Lemma 4.6.
Lemma 4.9: $p_{\text {ex }}^{f s w}=\left(\frac{2 L p_{\text {dest }}^{f s w}(L)}{M(M-1)} \frac{E\left[D_{f s w}^{m m}(L)\right]}{\sum_{k=1}^{L} E\left[D_{f s w}^{m m}(k)\right]}\right)+$ $\left(\sum_{m=1}^{L-1} \frac{2 m(M-m)}{M(M-1)} \sum_{i=m}^{L} p_{\text {dest }}^{f s w}(i) \frac{E\left[D_{f s w}^{m m}(m)\right]}{\sum_{k=1}^{i} E\left[D_{f s w}^{m m}(k)\right]}\right)$,
where $p_{\text {dest }}^{f s w}(i)=\left\{\begin{array}{cc}\frac{1}{M-1} & 1 \leq i<L \\ \frac{M-L}{M-1} & i=L\end{array} \quad\right.$ is the probability that the destination is the $(i+1)^{t h}$ node to receive a copy of the packet.

Proof: The proof runs along the same lines as the proof of Lemma 4.3.

Theorem 4.5: Let $E\left[D_{f s w}^{m m}\right]$ denote the expected
delay of fast spray and wait. Then, $E\left[D_{f s w}^{m m}\right]=$ $\sum_{i=1}^{L} p_{\text {dest }}^{f s w}(i) \sum_{m=1}^{i} E\left[D_{f s w}^{m m}(m)\right]$.

Proof: The proof runs along similar lines as the proof of Theorem 4.2.

## V. Delay Analysis for Realistic Mobility Models

Real world mobility traces show that mobility models which assume that all nodes are homogeneous and move randomly all around the network, like the random direction and the random waypoint mobility models, are not realistic [30,31]. Nodes usually have some locations where they spend a large amount of time. Additionally, node movements are not identically distributed. Different nodes visit different locations more often, and some nodes are more mobile than others. Based on this intuition, Spyropoulos et al [20] proposed a more realistic and analytically tractable community-based mobility model. Later, Hsu et al [34] showed that the statistics of real traces match with a time varying version of this community-based mobility model further proving that this model captures real world mobility properties.

In this section, we analyze different routing schemes for the community-based mobility model. We have two different objectives in doing so: (i) To show how the analysis presented in the previous section can be modified to analyze the same routing schemes for more realistic mobility models. (ii) Study how heterogeneity in the network can be used to improve the performance of routing schemes.

We first define the family of Community-based mobility models: The model consists of two states, namely the 'local' state and the 'roaming' state. The model alternates between these two states. Each node inside the network moves as follows: (i) Each node $i$ has a local community $C_{i}$ of size $\left\|C_{i}\right\|=c^{2} N, c \in(0,1]$. A node's movement consists of local and roaming epochs. (ii) A local epoch is a random direction movement restricted inside area $C_{i}$. (iii) A roaming epoch is a random direction movement inside the entire network. (iv) (Local state $L$ ) If the previous epoch of the node was a local one, the next epoch is a local one with probability $p_{l}$, or a roaming epoch with probability $1-p_{l}$. (v) (Roaming state $R$ ) If the previous epoch of the node was a roaming one, the next epoch is a roaming one with probability $p_{r}$, or a local one with probability $1-p_{r}$. (Note that nodes are more likely to be found within the community than outside the community.)

The Community-based mobility model can be used to model a large number of scenarios by tuning its parameters. We choose a specific scenario closely resembling reality where there is a finite number of communities (denoted by $r$ ) in the network. Further, we assume that these communities are small such that all nodes within a community are within each other's range. We also assume that the nodes spend most of their time within their respective communities. This scenario corresponds to several office buildings on a campus or several conference rooms in a hotel. This model is more realistic than one where all nodes choose their community area uniformly at random from the entire network because communities in general are already fixed, like office buildings and conference rooms, and many nodes share these rather than each node choosing its own community area uniformly at random. Finally, for ease of exposition, we assume
that the number of nodes sharing a community is equal across all $r$ communities, that is the number of nodes sharing a community is equal to $\frac{M}{r}{ }^{3}$.
The success probability of a transmission inspite of contention for the community-based mobility model is derived in a manner similar to the derivation described in Section III-A. A difference here is that the success probability depends on whether the two nodes met within a community or outside. Let the probabilities that two nodes outside the communities and two nodes within a community are able to successfully exchange a particular packet inspite of contention be denoted by $p_{t x S 1}^{R}$ and $p_{t x S 2}^{R}$ respectively. The value of these two probabilities is derived in [27].
The statistics of the meeting time, inter-meeting time and contact time for the community-based mobility model is studied in [33]. Nodes which share the same community have different statistics than nodes which belong to different communities. (Its easy to see thats nodes which share the same community meet faster and stay in contact for a longer duration.) Let $E\left[M_{\text {comm,same }}\right]\left(E\left[M_{\text {comm,diff }}\right]\right), E\left[M_{\text {comm,same }}^{+}\right]$ $\left(E\left[M_{\text {comm,diff }}^{+}\right]\right)$and $E\left[\tau_{\text {comm,same }}\right]\left(E\left[\tau_{\text {comm,diff }}\right]\right)$ denote the expected meeting time, inter-meeting and contact time for nodes which belong to the same community (belong to different communities) respectively. Please refer to [33] for their exact values. The two important properties which we use during the course of the analysis are as follows: (i) $E\left[M_{\text {comm,diff }}\right]=E\left[M_{\text {comm,diff }}^{+}\right]$. Though, note that $E\left[M_{\text {comm,same }}\right] \neq E\left[M_{\text {comm,same }}^{+}\right]$. (ii) Even though the overall statistics of the meeting and inter-meeting times for a communitybased mobility model is not exponential, however, after conditioning on whether the two nodes under consideration share the community or not, these statistics become exponential [35].

Now, we derive the expected delay values for four different routing schemes. We first analyze direct transmission and epidemic routing as these two form the basic building block for all routing schemes. Then, we analyze two different spraying based schemes: fast spray and wait and fast spray and focus. Fast spray and focus [15] is a spraying based scheme designed to improve the performance of nodes with heterogeneous mobility. Note that the value of $p_{e x}^{R}$ for each routing scheme remains the same as derived in Section IV. The derivation of the expected delay for the community-based mobility model uses arguments similar to the ones used in the derivation of the expected delay for the random direction / random waypoint mobility model. The proofs which are very similar are not discussed to keep the exposition interesting. (Due to lack of space and to avoid repetition, we do not present the analysis for gossip based schemes or source spray and wait here.)

## A. Direct Transmission

Theorem 5.1: Let $E\left[D_{d t}^{c o m m}\right]$ denote the expected delay of direct transmission for community-based mobility model. Then, $E\left[D_{d t}^{c o m m}\right]=\frac{(r-1) m}{r(m-1)} \frac{E\left[M_{\text {com } m, d i f f}\right]}{p_{s u \text { ccess } 1}^{d t}}+\frac{m-r}{r(m-1)}\left(E\left[M_{\text {comm }, \text { same }}\right]+\right.$ $\left.\frac{\left(1-p_{\text {success } 2}^{d t}\right) E\left[M_{\text {comm }, \text { same }}^{+}\right]}{p_{\text {success } 2}^{d t}}\right)$, where $p_{\text {success } 1}^{d t}=1-(1-$ $\left.p_{t x S 1}^{d t}\right)^{E\left[\tau_{c o m m, d i f f}\right]}$ is the probability that when two nodes which

[^2]do not share a community come within each other's range, they successfully exchange the packet before going out of each other's range and $p_{s u c c e s s 2}^{d t}=1-\left(1-p_{t x S 2}^{d t}\right)^{E\left[\tau_{c o m m, s a m e}\right]}$ is the probability that when two nodes belonging to the same community come within each other's range, they successfully exchange the packet before going out of each other's range.

Proof: The probability that the destination belongs to a different community than the source is equal to $\frac{(r-1) m}{r(m-1)}$. The derivation of the expected delay after conditioning on whether the source and the destination belong to the same community or not is similar to the derivation of $E\left[D_{d t}^{m m}\right]$ in Theorem 4.1. Finally, using the law of total probability to remove the conditioning yields $E\left[D_{d t}^{c o m m}\right]$.

## B. Epidemic Routing

This section derives the expected delay of epidemic routing for the community-based mobility model. Since each node spends most of its time within its community (which implies $E\left[M_{c o m m, d i f f}\right] \gg E\left[M_{c o m m, s a m e}\right]$ ), we make an approximation to simplify the exposition by assuming that with high probability, a node starting from its stationary location distribution will first meet a node within its own community than a node belonging to a different community. This implies that once a node gets a copy of a packet, all members of its community will get the copy before any node outside its community, with high probability. A simple outcome of this is that the first $\frac{M}{r}-1$ nodes to get a copy of the packet belong to the source's community.

We first study how much time it takes for all nodes within the source's community to get a copy of the packet. This derivation is different from all the derivations in Section IV because $E\left[M_{\text {comm,same }}\right] \neq E\left[M_{\text {comm,same }}^{+}\right]$. Thus, we need to keep track of which pair of nodes have met in the past but were unable to successfully exchange the packet. We model the system using the following state space: $\left(m, m_{p}\right)$ where $1 \leq m \leq \frac{M}{r}$ is the number of nodes which have a copy of the packet and $0 \leq m_{p} \leq m\left(\frac{M}{r}-m\right)$ is the number of node pairs such that only one of them has a copy of the packet, they have met at least once after the node (which has the copy) received its copy, and they were unable to successfully exchange this packet in their past meetings.

Lemma 5.1: Let $E\left[D_{\text {in }}(m)\right]$ denote the expected time it takes for the number of nodes to increase from $m$ to $m+1$ given $m<\frac{M}{r}$ (which implies that all nodes within the source's community have not yet received a copy of the packet). Then, $E\left[D_{i n}(m)\right]=\sum_{m_{p}=0}^{m\left(\frac{M}{r}-m\right)} p_{m, m_{p}} \frac{E\left[T_{m, m_{p}}\right]}{1-p_{m, m_{p}}^{\text {silf }}}$, where $E\left[T_{m, m_{p}}\right]$ is the expected time elapsed till one of the nodes not having a copy meets a node having a copy of the packet given that the system is in state $\left(m, m_{p}\right), p_{m, m_{p}}^{s e l f}$ is the probability that the system remains in the state ( $m, m_{p}$ ) after these nodes (which met after $E\left[T_{m, m_{p}}\right]$ ) are unable to successfully exchange the packet, $p_{s u c c e s s 2}^{\text {epidemic }}=1-\left(1-p_{t x S 2}^{\text {epidemic }}\right)^{E\left[\tau_{\text {com } m, s a m e}\right]}$, and $p_{m, m_{p}}$ is the probability that the system visits state $\left(m, m_{p}\right)$.

Proof: Let the system be in state $\left(m, m_{p}\right)$. We first derive the expected time duration after which the system moves to another state. A transmission opportunity will arise only when one of the $m$ nodes carrying a copy of the packet meet one
of the $\frac{M}{r}-m$ not having a copy of the packet. There are a total of ${ }_{m}^{r}\left(\frac{M}{r}-m\right)$ such node pairs of which $m_{p}$ have already met before. Since, both the meeting and inter-meeting times have exponential tails, the expected time elapsed till one of these $m\left(\frac{M}{r}-m\right)$ node pairs come within range is $E\left[T_{m, m_{p}}\right]=$ $\frac{1}{m\left(\frac{M}{r}-m\right)-m_{p}}$. If the two nodes which met are $\overline{\frac{m\left(\frac{M}{r}-m\right)-m_{p}}{E\left[M_{\text {comm }, \text { same }}\right]}+\frac{m_{p}}{E\left[M_{\text {comm }, \text { same }}^{+}\right]}}$
not able to successfully exchange the packet, then the system will remain in the same state if these two nodes were one of the $m_{p}$ node pairs which have already met at least once in the past, otherwise the system will move to $\left(m, m_{p}+1\right)$. Thus, the probability that the system remains in the same state is $p_{m, m_{p}}^{\text {self }}=\left(1-p_{\text {success } 2}^{\text {epidemic }}\right) \frac{m_{p} E\left[T_{\left.m, m_{p}\right]}^{E\left[M_{\text {comm }, \text { same }}^{+}\right]} \text {, where }\right.}{\text { success } 2}=$ $1-\left(1-p_{t x S 2}^{\text {epidemic }}\right)^{E\left[\tau_{\text {comm }, \text { same }]}\right]}$. If the system remains in the same state, then it will take yet another time duration equal to $E\left[T_{m, m_{p}}\right]$ for a transmission possibility. Again, with $p_{m, m_{p}}^{s e l f}$ the system will remain in the same state. Thus, the expected amount of time the system remains in state $\left(m, m_{p}\right)$ is equal to $\frac{E\left[T_{m, m_{p}}\right]}{1-p_{m, m_{p}}^{s e l f}}$.

In a manner similar to the derivation of $p_{m, m_{p}}^{s e l f}$, the probability that the system moves to $\left(m, m_{p}+1\right)$ is derived to be ( $1-$ $\left.p_{\text {success } 2}^{\text {epidemic }}\right)\left(1-\frac{m_{p} E\left[T_{\left.m, m_{p}\right]}\right]}{E\left[M_{\text {comm,same }]}^{+}\right]}\right)$. The transmission is successful with probability $p_{\text {success } 2 \text {, }}^{\text {epidemic }}$, in which case the system moves to the state $\left(m+1, m_{p}-\frac{m_{p}}{\frac{M}{r}-m}\right)$. Since each node not having a copy of the packet has met on an average $\left(\frac{m_{p}}{\frac{M}{r}-m}\right)$ nodes which have a copy of the packet, when a new node receives the packet, this number has to be subtracted from $m_{p}$.

Now, we find the probability that the system will visit the state $\left(m, m_{p}\right)$ (denoted by $\left.p_{m, m_{p}}\right)$. The system can move to state $\left(m, m_{p}\right)$ from states $\left(m-1, m_{p}+\frac{m_{p}}{\frac{M}{r}-m-1}\right)$ (with probability $\left.p_{\text {success } 2}^{\text {epidemic }}\right)$ and $\left(m, m_{p}-1\right)$ (with probability ( $1-$ $\left.p_{\text {success } 2}^{\text {epidemic }}\right)\left(1-\frac{\left(m_{p}-1\right) E\left[T_{m, m_{p}-1}\right]}{E\left[M_{\text {comm }}^{+} \text {same }\right]}\right)$. Thus,

$$
\begin{aligned}
& \left(p_{\text {success } 2}^{\text {epidemic }} p_{m-1, m_{p}+\frac{m_{p}}{\frac{M}{r}-m-1}}\right. \\
& +\left(1-\frac{\left(m_{p}-1\right) E\left[T_{\left.m, m_{p}-1\right]}\right]}{E\left[M_{\text {comm }, \text { same } e}^{+}\right]}\right) \quad m>1
\end{aligned}
$$

Solving this set of linear equations yields $p_{m, m_{p}}$.
Now, we find $E\left[D_{\text {epidemic }}^{c o m m}(m)\right]$ which is the expected time it takes for the number of nodes having a copy of the packet to increase from $m$ to $m+1$.

Lemma 5.2: $E\left[D_{\text {epidemic }}^{\text {comm }}(m)\right]$

where
$p_{\text {success } 1}^{\text {epidemic }}=1-\left(1-p_{t x S 1}^{\text {epidemic }}\right)^{E\left[\tau_{\text {comm }, \text { diff }}\right]}$ and rem $(x, y)$ is the remainder left after dividing $x$ by $y$.

Proof: As previously discussed, the first $\frac{M}{r}-1$ nodes to receive a copy of the packet are the nodes belonging to the source's community. Then, a node belonging to another community (lets label it community $Y$ ) will receive a copy from one of the nodes belonging to the source's community. After
that, the next $\frac{M}{r}-1$ nodes to get a copy of the packet are the ones which belong to community $Y$. Even though there are other nodes which have a copy of the packet (belonging to the source's community), with high probability, the nodes in community $Y$ will receive a copy of the packet from a node belonging to its own community. Thus, the expected time for the copies to spread within community $Y$ is equal to the expected time for the copies to spread within the source's community. Similarly, the expected time for the copies to spread within any community after a node belonging to that community obtains a copy, is equal to the expected time for the copies to spread within the source's community (irrespective of how many nodes outside the community have copies of the packet). However, when $\operatorname{rem}\left(m, \frac{M}{r}\right)=0$, either all or no nodes in a community have a copy of the packet and the expected time for the copies to increase for this scenario can be found in a manner similar to the derivation of $E\left[D_{\text {epidemic }}^{m m}(m)\right]$ in Lemma 4.2.

Finally, we derive the expected delay of epidemic routing for the community based mobility model in terms of $E\left[D_{\text {epidemic }}^{\text {comm }}(m)\right]$ using the same argument used to derive $E\left[D_{\text {epidemic }}^{m m}\right]$ in Theorem 4.2.

Theorem 5.2: Let $E\left[D_{\text {epidemic }}^{\text {comm }}\right]$ denote the expected delay of epidemic routing for the community-based mobility model. Then, $E\left[D_{\text {epidemic }}^{\text {comm }}\right]=\sum_{i=1}^{M-1} \frac{1}{M-1} \sum_{m=1}^{i} E\left[D_{\text {epidemic }}^{\text {comm }}(m)\right]$.

## C. Fast Spray and Wait

This section derives the expected delay of fast spray and wait routing scheme for the community-based mobility model. As before, first we derive the value of $E\left[D_{f s w}^{c o m m}(m)\right]$. For $m<L$ (in the spray phase), the value of $E\left[D_{f s w}^{c o m m}(m)\right]$ is derived in a manner similar to the derivation of $E\left[D_{\text {epidemic }}^{\text {comm }}(m)\right]$ as flooding is used to spread the $L$ copies in the spray phase.

Now, we derive the value of $E\left[D_{f s w}^{c o m m}(L)\right]$ which is the expected time to find the destination in the wait phase.

Lemma 5.3: $E\left[D_{f s w}^{c o m m}(L)\right]$
$\frac{\frac{M}{r}-\hat{l}}{M-L}\left(\sum_{m_{p}=0}^{\hat{l}\left(\frac{M}{r}-\hat{l}\right)} p_{\hat{l}, m_{p}} E\left[T_{m_{p}}^{\frac{M}{r}-\hat{l}}\right]\right)+\left(1-\frac{M}{\frac{M}{r}-\hat{l}} \underset{M-L}{ }\right) \frac{E\left[M_{\text {comm }, \text { diff }]}\right.}{L p_{\text {success } 1}^{f s w}}$, where $\hat{l}=\operatorname{rem}\left(L, \frac{M}{r}\right), E\left[T_{s}\right]$ is the expected time till the destination receives a copy of the packet given there are $s$ nodes belonging to the destination's community which were unable to successfully exchange the packet with the destination in the past, and $p_{s u c c e s s 1}^{f s w}=1-\left(1-p_{t x S 1}^{f s w}\right)^{E\left[\tau_{c o m m, d i f f}\right]}$.

Proof: After the spray phase (after $L$ copies have been spread), there is a community which has only $\hat{l}=\operatorname{rem}\left(L, \frac{M}{r}\right)$ nodes carrying a copy of the packet. The probability that the destination is one of the remaining $\frac{M}{r}-\hat{l}$ nodes belonging to this community is equal to $\frac{\frac{M}{r}-\hat{l}}{M-L}$. First we will derive the expected delay in the wait phase when the destination belongs to this community. Then, we derive the expected delay when the destination does not belong to this community. And then combining everything together by using the law of total probability yields the result. Please see the Appendix for proof details.

Finally, we derive the expected delay of fast spray and wait for the community based mobility model in terms of $E\left[D_{f s w}^{c o m m}(m)\right]$ using the same argument used to derive $E\left[D_{f s w}^{m m}\right]$.

Theorem 5.3: Let $E\left[D_{f s w}^{c o m m}\right]$ denote the expected delay of fast spray and wait for the community-based mobility model.

Then, $E\left[D_{f s w}^{c o m m}\right]_{1}=\sum_{i=1}^{L} p_{\text {dest }}^{f s w}(i) \sum_{m=1}^{i} E\left[D_{f s w}^{c o m m}(m)\right]$, where $p_{\text {dest }}^{f s w}(i)=\left\{\begin{array}{cl}\frac{1}{M-1} & i<L \\ \frac{M-L}{M-1} & i=L\end{array}\right.$.

## D. Fast Spray and Focus

Spray and Focus schemes [15] differ from spray and wait schemes in how each relay routes the copy towards the destination. Instead of doing direct transmission, each relay does a utility-based forwarding towards the destination, that is, whenever a relay carrying a copy of the packet meets another node (label it node $B$ ) which has a higher utility, the relay gives its copy to node $B$. Node $B$ now does a utility based forwarding towards the destination and the relay drops the packet from its queue. [15] showed that spray and focus has huge performance gains over spray and wait for heterogeneous networks (networks where each node is not the same). Community-based mobility model introduces an inherent heterogeneity in the network as nodes differ depending on which community do they belong to. So, we study a spray and focus scheme for the community-based mobility model, and later we compare it to the corresponding spray and wait scheme.

Fast spray and focus performs fast spraying in the spray phase. To be able to do utility-based forwarding in the focus phase, [15] maintained last encounter timers to build the utility function. For community-based mobility models, [18] proposed the use of a simpler function as a utility function for their 'Label' scheme: If a relay meets a node which belongs to the same community as the destination, the relay hands over its copy to the new node. We use this simple utility function to route copies of the packet in the focus phase.

This section derives the expected delay of fast spray and focus for the community-based mobility model. $p_{e x}^{f s f}$ can be derived in a manner similar to the derivation of $p_{e x}^{f s w}$. To avoid repetition, we skip the derivation of $p_{e x}^{f s f}$ here.

As before, first we derive $E\left[D_{f s f}^{c o m m}(m)\right]$. Since, flooding is used to spread the copies in the spray phase, $E\left[D_{f s f}^{\text {comm }}(m)\right]$ for $m<L$ can be derived in a manner similar to the derivation of $E\left[D_{\text {epidemic }}^{\text {comm }}(m)\right]$. The next lemma derives the value of $E\left[D_{f s f}^{c o m m}(L)\right]$ which is the expected time it takes for the packet to get delivered to the destination in the focus phase. This derivation uses arguments similar to the derivation of $E\left[D_{f s w}^{c o m m}(L)\right]$ in Lemma 5.3.
Lemma 5.4: $E\left[D_{f s f}^{\text {comm }}(L)\right]$

$$
\frac{\frac{M}{r}-\hat{l}}{M-L}\left(\sum_{m_{p}=0}^{\hat{l}\left(\frac{M}{r}-\hat{l}\right)} p_{\hat{l}, m_{p}} E\left[T_{\frac{m_{p}}{\frac{M}{r}-\hat{l}}}\right]\right)
$$

$$
=
$$

$$
\left(1-\frac{\frac{M}{r}-\hat{l}}{M-L}\right)\left(\frac{E\left[M_{\text {comm }, \text { diff }}\right]^{\frac{M}{r}-l}}{L \frac{M}{r} p_{\text {succeess } 1}^{\text {fsf }}}+\frac{\frac{M}{r}-1}{\frac{M}{r}}\right.
$$

$\left.\left(E\left[M_{\text {comm }, \text { same }}\right]+\frac{\left(1-p_{s u c c e s s s 2}^{f s f}\right) E\left[M_{\text {comm }, \text { same }}^{+}\right]}{p_{\text {successs } 2}^{f s f}}\right)\right), \quad$ where
$\hat{l}=\operatorname{rem}\left(L, \frac{M}{r}\right), p_{\text {success } 1}^{f s f}=1-\left(1-p_{t x S 1}^{f s w}\right)^{E\left[\tau_{c o m m, d i f f}\right]}$ and $p_{\text {success } 2}^{f s f}=1-\left(1-p_{t x S 2}^{f s w}\right)^{E\left[\tau_{\text {comm }, \text { same }}\right]}$.

Proof: See Appendix.
Now we derive the expected delay of fast spray and focus for the community based mobility model in terms of $E\left[D_{f s w}^{c o m m}(m)\right]$.

Theorem 5.4: Let $E\left[D_{f s f}^{\text {comm }}\right]$ denote the expected delay of fast spray and focus for the community-based mobility model.

Then, $E\left[D_{f s f}^{c o m m}\right]=\sum_{i=1}^{L} p_{\text {dest }}^{f s f}(i) \sum_{m=1}^{i} E\left[D_{f s f}^{\text {comm }}(m)\right]$, where $p_{\text {dest }}^{f s f}(i)=\left\{\begin{array}{ll}\frac{1}{M-1} & i<L \\ \frac{M-L}{M-1} & i=L\end{array}\right.$.

Proof: The proof runs along similar lines as the proof of Theorem 4.2.

## VI. Accuracy of Analysis

We made the following four approximations during the delay analysis: (i) we replace the contact time by its expected value in the expression of $p_{\text {success }}^{R}$ in the delay analysis of all routing schemes, (ii) while analyzing epidemic routing, randomized flooding and source spray and wait, we assume the entire meeting and inter-meeting time distribution to be exponential, and (iii) while analyzing the delay of routing schemes for the communitybased mobility model, we make an additional approximation by assuming that $E\left[M_{\text {comm,diff }}\right] \gg E\left[M_{\text {comm,same }}\right]$. (iv) We use an approximate value of $p_{e x}^{r f}$ in the derivation of $E\left[D_{r f}^{m m}\right]$. We use simulations to verify that these approximations do not have a significant impact on the accuracy of the analysis. We use a custom simulator written in $\mathrm{C}++$ for simulations. The simulator avoids excessive interference by implementing a scheduling scheme which prohibits any simultaneous transmission within one hop from both the transmitter and the receiver. It incorporates interference by adding the received signal from other simultaneous transmissions (outside the scheduling area) and comparing the signal to interference ratio to the desired threshold. The simulator allows the user to choose from different physical layer, mobility and traffic models. We choose the Rayleigh-Rayleigh fading model for the channel, random waypoint model / communitybased mobility model for node mobility and Poisson arrivals in our simulations.
The robustness of all the approximations can be studied by varying $K$ and $M$. Figures 2(a)-2(h) and 3(a)-3(f) compare the expected end-to-end delay for different routing schemes obtained through analysis and simulations for different values of $K$ and $M$ for the random waypoint mobility model and the community based mobility model. We have compared the analytical and simulation results for a large number of scenarios, but due to limitations of space, we present some representative results for each routing scheme. Since both the simulation and the analytical curves are close to each other in all the scenarios, we conclude that the analysis is fairly accurate.

## VII. Application to Protocol Design

This section uses the expressions derived in the previous sections to answer some pertinent questions in the context of designing more efficient routing schemes for sparse networks.

## A. Gossip Based vs Spraying Based Routing

Both gossip based and spraying based routing techniques were proposed to achieve good delay performance with a lower resource usage than epidemic routing. So, first we compare which of the two performs better. [15, 17] have compared these two approaches with simulations, but having an analytical expression allows us to first find the optimal parameters for both the schemes and then comparing them, which ensures a fair comparison. [22] compared these two approaches analytically, but their analysis
ignored contention. The performance of gossip based schemes degrades significantly due to contention, hence ignoring contention when comparing a gossip based scheme to another scheme will lead to an unfair comparison.
We choose randomized flooding and source spray and wait as representative gossip based and spraying based routing schemes for comparison. Both are simple routing schemes and neither of them uses utiities to aid routing. To compare the two schemes, we study how much resource does each scheme use to achieve a given target expected delay. We measure resource consumption in terms of the average number of transmissions required to reach the destination. Larger the number of transmissions, higher is the buffer usage and energy consumption.

We will choose the least value of $p$ and $L$ which will achieve a specific target expected delay because lower the value of $p$ and $L$, lower will be the resource consumption. We numerically solve the expressions for expected delay derived in Theorems 4.3 and 4.4 to find the minimum value of $p$ and $L$ which achieve the target delay. The average number of transmissions required to deliver the packet to the destination is equal to $\sum_{i=1}^{M-1} i_{\text {dest }}^{R}(i)$, where $p_{\text {dest }}^{R}(i)$ is the probability that the destination is the $(i+1)^{t h}$ node to receive a copy of the packet for the routing scheme $R$. We derived the value of $p_{\text {dest }}^{R}(i)$ in Theorem 4.3 and Lemma 4.7 for randomized flooding and source spray and wait respectively. Figure 4 plots the average number of transmissions required to reach the destination versus the required target delay for both the schemes for two different network densities. We make the following two observations from this figure: (i) source spray and wait is able to achieve lower values of target expected delay than randomized flooding, and (ii) for a target expected delay which can be achieved by both the schemes, source spray and wait uses less resources to achieve it than randomized flooding. The superiority of source spray and wait becomes more prominent as the network density increases.


Fig. 4. Comparison of source spray and wait and randomized flooding: Average number of transmissions required to deliver the packet to the destination vs target expected delay. Network parameters: $N=100 \times 100$ square units, $K=6, \Theta=5, s_{B W}=1$ packet/time slot. Delay is expressed in time slots. (a) $M=50$. Note that target delays below 130 time units are not achievable by either routing scheme. (b) $M=100$. Note that target delays below 170 time units for randomized flooding and 120 time units for source spray and wait are not achievable.


Fig. 2. Simulation and analytical results for the expected delay. Network parameters: $N=150 \times 150$ square units, $\Theta=5, s_{B W}=1$ packet/time slot. The transmission range, $K$, is expressed in the same distance units as $\sqrt{N}$. The delay is expressed in time slots. (a) Direct transmission. (b) Epidemic routing. (c) Randomized flooding with $p=0.3$. (d) Randomized flooding with $p=0.7$. (e) Source spray and wait with $L=5$. (f) Source spray and wait with $L=15$. (g) Fast spray and wait with $L=5$. (h) Fast spray and wait with $L=15$. (Note that the number of instances for which each Monte Carlo simulation is run is chosen so as to ensure that the $90 \%$ confidence interval is within $5 \%$ of the simulation value.)

## B. Spraying Based Routing: How to Spray Multiple Copies

Since spraying based techniques outperform gossip based schemes, we now study the spraying schemes in more detail. We first discuss how to spray copies in the spraying phase so as to reduce the overall end-to-end delay. Intuitively, spraying copies as fast as possible should minimize the delay as once the copies are spread, the expected amount of time it takes to deliver the packet will be the same for all schemes. So, is trying to spray the copies as fast as possible the optimal way. To answer this question, we compare the two different spraying schemes introduced in Section IV-D, source spray and wait and fast spray and wait. Since fast spray and wait spreads copies whenever there is any opportunity to do so, it has the minimum spraying time when there is no
contention in the network. On the other hand, since source spray and wait does not use relays to forward copies, it is one of the slower spraying mechanisms when there is no contention in the network.

Now we study how fast do the two schemes spread copies of a packet when there is contention in the network. Figure 5 plots the number of copies spread as a function of the time elapsed since the packet was generated. Somewhat surprisingly, depending on the density of the network, source spray and wait can spray copies faster than fast spray and wait. This occurs because fast spray and wait generates more contention as it tries to transmit at every possible transmission opportunity. In general, unless the network is very sparse, strategies which spray


Fig. 3. Simulation and analytical results for the expected delay for the community-based mobility model. Network parameters: $N=500 \times 500$ square units, $\Theta=5, p_{l}=0.8, p_{r}=0.15, r=4, s_{B W}=1$ packet/time slot. The transmission range, $K$, is expressed in the same distance units as $\sqrt{N}$. The delay is expressed in time slots. (a) Direct transmission. (b) Epidemic routing. (c) Fast spray and wait with $L=5$. (d) Fast spray and wait with $L=8$. (e) Fast spray and focus with $L=5$. (f) Fast spray and focus with $L=8$. (Note that the number of instances for which each Monte Carlo simulation is run is chosen so as to ensure that the $90 \%$ confidence interval is within $5 \%$ of the simulation value.)
copies slower yield better performance than more aggressive schemes thanks to reducing contention. In ongoing work, we are trying to find the optimal spraying algorithm and design practical and implementable heuristics which achieve performance very close to the optimal. [36] is a first step in this direction. It derives the optimal spraying scheme and a simple heuristic which performs very close to the optimal, but it assumes that there is no contention in the network. Currently, we are merging this work with the contention framework proposed in [27] to find the optimal spraying scheme with contention in the network.

## C. Spraying Based Routing: How to Route Individual Copies

In this section, we study how each copy should be routed towards the destination for heterogeneous networks. Specifically, we compare the performance of two different schemes, fast spray and wait which does not exploit the heterogeneity in the network and fast spray and focus which makes more transmissions than fast spray and wait but is specifically designed to reduce delay in a heterogeneous network. We study how much performance gains are achieved by spray and focus and how does the value of gain depend on the network parameters. [15] compared the two schemes using simulations. Having an analytical expression allows us to calculate the optimal value of $L$ to ensure a fair comparison between the two schemes.

To compare the two schemes, we plot the minimum value of the average number of transmissions it takes to achieve a given target


Fig. 5. Comparison of fast spray and wait and source spray and wait: Expected number of copies spread vs time elapsed since the packet was generated. Network parameters: $N=100 \times 100$ square units, $K=5, \Theta=5, s_{B W}=1$ packet/time slot, $L=20$. Time is expressed in time slots.
expected delay for both the schemes. Recall that the average number of transmissions is equal to $\sum_{i=1}^{L} i p_{\text {dest }}^{R}(i)$. (We derived the value of $p_{\text {dest }}^{R}(i)$ for both the schemes in Theorems 5.3 and 5.4). Figure VII-C plots the average number of transmissions required to reach the destination against the target expected delay
for both the schemes for two different values of $r$. As expected, fast spray and focus performs better than fast spray and wait with gains upto three times (the gains are this big because $E\left[M_{\text {comm,diff }}\right] \gg E\left[M_{\text {comm,same }}\right]$ ) and this gain reduces as the number of nodes per community decreases (in other words, increase $r$ while keeping $M$ the same). The gain reduces because as the number of nodes per community decreases, the number of good nodes towards the destination decreases which increases the delay of the focus phase.

## VIII. CONCLUSIONS

This paper finds the expected delay for representative routing schemes for intermittently connected mobile networks: direct transmission, epidemic routing, randomized flooding and spraying based schemes, with contention in the network for the random direction, random waypoint and more realistic community-based mobility models. This paper uses a recently proposed framework to model wireless contention. We use these expressions to conclude that spraying based schemes outperform randomized flooding. So, we study spraying based schemes in more detail. After analyzing two different ways to spread copies in the spraying phase, we conclude that strategies which spray copies as fast as possible generate more contention and are not the best way to spread copies. Then, we study the gains achieved by modifying how individual copies are routed towards the destination after the spray phase to exploit the heterogeneity in the network introduced due to the community-based mobility model.

## References

[1] P. Juang, H. Oki, Y. Wang, M. Martonosi, L. S. Peh, and D. Rubenstein, "Energy-efficient computing for wildlife tracking: design tradeoffs and early experiences with zebranet," in Proceedings of ACM ASPLOS, 2002.
[2] "Disruption tolerant networking," http://www.darpa.mil/ato/solicit/DTN/.
[3] S. Burleigh, A. Hooke, L. Torgerson, K. Fall, V. Cerf, B. Durst, and K. Scott, "Delay-tolerant networking: an approach to interplanetary internet," IEEE Communications Magazine, vol. 41, 2003.
[4] A. Doria, M. Udn, and D. P. Pandey, "Providing connectivity to the Saami nomadic community", in Proc. 2nd Int. Conf. on Open Collaborative Design for Sustainable Innovation, Dec. 2002.
[5] A. F. Winfield, "Distributed sensing and data collection via broken ad hoc wireless connected networks of mobile robots," Distributed Autonomous Robotic Systems, pp. 273-282, 2000.
[6] H. Wu, R. Fujimoto, R. Guensler, and M. Hunter, "Mddv: Mobility-centric data dissemination algorithm for vehicular networks," in Proceedings of ACM SIGCOMM workshop on Vehicular Ad Hoc Networks (VANET), 2004.
[7] C. Perkins, Ad Hoc Networking, 1st ed. Addison-Wesley, 2001.
[8] A. Vahdat and D. Becker, "Epidemic routing for partially connected ad hoc networks," Duke University, Tech. Rep. CS-200006, Apr. 2000.
[9] S. Jain, K. Fall, and R. Patra, "Routing in a delay tolerant network," in Proceedings of ACM SIGCOMM, Aug. 2004.
[10] W. Zhao, M. Ammar, and E. Zegura, "A message ferrying approach for data delivery in sparse mobile ad hoc networks," in Proceedings of ACM MOBIHOC, 2004.
[11] A. Lindgren, A. Doria, and O. Schelen, "Probabilistic routing in intermittently connected networks," SIGMOBILE Mobile Computing and Communication Review, vol. 7, no. 3, 2003.
[12] T. Spyropoulos, K. Psounis, and C. S. Raghavendra, "Spray and wait: Efficient routing in intermittently connected mobile networks," in Proceedings of ACM SIGCOMM workshop on Delay Tolerant Networking (WDTN), 2005.
[13] Y. Wang, S. Jain, M. Martonosi, and K. Fall, "Erasure coding based routing for opportunistic networks," in Proceedings of ACM SIGCOMM workshop on Delay Tolerant Networking (WDTN), 2005.
[14] E. Jones, L. Li, and P. Ward, "Practical routing in delay-tolerant networks," in Proceedings of ACM SIGCOMM workshop on Delay Tolerant Networking (WDTN), 2005.
[15] T. Spyropoulos, K. Psounis, and C. S. Raghavendra, "Spray and focus: Efficient mobility-assisted routing for heterogeneous and correlated mobility," in Proceedings of IEEE PerCom Workshop on Intermittently Connected Mobile Ad Hoc Networks, 2007.
[16] _-, "Efficient routing in intermittently connected mobile networks: The single-copy case," to appear in IEEE Transactions on Networking, 2007.
[17] ——, "Efficient routing in intermittently connected mobile networks: The multi-copy case," to appear in IEEE Transactions on Networking, 2007.
[18] P. Hui and J. Crowcroft, "How small labels create big improvements," in Proceedings of IEEE PerCom Workshop on Intermittently Connected Mobile Ad Hoc Networks, 2007.
[19] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," in Performance, 2005.
[20] T. Spyropoulos, K. Psounis, and C. S. Raghavendra, "Performance analysis of mobility-assisted routing," in Proceedings of ACM MOBIHOC, 2006.
[21] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "Performance modeling of epidemic routing," in Networking, 2005.
[22] G. Neglia and X. Zhang, "Optimal delay-power tradeoff in sparse delay tolerant networks: a preliminary study," in Proceedings of ACM SIGCOMM workshop on Challenged Networks (CHANTS'06), 2006.
[23] T. Small and Z. Haas, "Resource and performance tradeoffs in delay-tolerant wireless networks," in Proceedings of ACM SIGCOMM workshop on Delay Tolerant Networking (WDTN), 2005.
[24] A. Jindal and K. Psounis, "Performance analysis of epidemic routing under contention," in Proceedings of Workshop on Delay Tolerant Mobile Networking (DTMN) held in conjunction with IWCMC, 2006.
[25] Y.-C. Tseng, S.-Y. Ni, Y.-S. Chen, and J.-P. Sheu, "The broadcast storm problem in a mobile ad hoc network," Wireless Networks, vol. 8, no. 2/3, 2002.
[26] D. Aguuayo, J. Bicket, S. Biswas, G. Judd, and R. Morris, "Link-level measurements from an 802.11b mesh network," in Proceedings of ACM SIGCOMM, 2004.
[27] A. Jindal and K. Psounis, "Wireless contention in mobile multi-hop networks," USC, Tech. Rep. CENG-2007-4, 2007.
[28] J. Padhye, R. Draves, and B. Zill, "Routing in multi-radio, multi-hop wireless mesh networks," in Proceedings of ACM MOBICOM, 2004.
[29] A. Jindal and K. Psounis, "Contention-aware analysis of routing schemes for mobile opportunistic networks," in Proceedings of ACM MOBISYS Workshop on Mobile Opportunistic Networks, 2007.
[30] T. Henderson, D. Kotz, and I. Abyzov, "The changing usage of a mature campus-wide wireless network," in Proceedings of ACM MOBICOM, 2004.
[31] M. McNett and G. Voelker, "Access and mobility of wireless PDA users," ACM Mobile Computing and Communications Review, 2003.
[32] M. Zuniga and B. Krishnamachari, "Analyzing the transitional region in low power wireless links," in Proceedings of IEEE SECON, 2004.
[33] A. Jindal and K. Psounis, "Fundamental properties of mobility for realistic performance analysis of mobility-assisted networks," in Proceedings of IEEE PerCom Workshop on Intermittently Connected Mobile Ad Hoc Networks, 2007.
[34] W. Hsu, T. Spyropoulos, K. Psounis, and A. Helmy, "Modeling timevariant user mobility in wireless mobile networks," in Proceedings of IEEE INFOCOM, 2004.
[35] T. Spyropoulos, A. Jindal, and K. Psounis, "A theoretical framework for analyzing the performance of mobility-assisted routing," USC, Tech. Rep. CENG-2007-?, 2007.
[36] A. Jindal and K. Psounis, "Optimizing multi-copy routing schemes for resource constrained intermittently connected mobile networks," in Proceedings of IEEE Asilomar Conference on Signals, Systems and Computers, 2006.

Proof: (Lemma 4.3) Let there be $m$ copies of a particular packet in the network. Then the probability that node $i$ has a copy is equal to $\frac{m}{M}$ and the probability that node $j$ does not have a copy given that node $i$ has one is equal to $\frac{(M-m)}{M-1}$. Thus, the probability that nodes $i$ and $j$ want to exchange the packet given that there are $m$ copies of the packet in the network is equal to $\frac{2 m(M-m)}{M(M-1)}$. Now, we find the probability that there are $m$ copies of the packet in the network. The copies of a packet keep on increasing till the packet is delivered to the destination. The probability that the destination is the $k^{t h}$ node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2 \leq k \leq M$. A packet will have $m$ copies in the network only if the destination wasn't amongst the first $m-1$ nodes to receive a copy. The amount of time a


Fig. 6. Comparison of fast spray and wait and fast spray and focus. Average number of transmissions required to deliver the packet to the destination vs target expected delay. Network parameters: $N=500 \times 500$ square units, $M=40, K=20, \Theta=5, s_{B W}=1$ packet/time slot, $p_{l}=0.8, p_{r}=0.15$. Time is expressed in time slots. (a) $r=4$ (b) $r=10$.
packet has $m$ copies in the network is equal to $E\left[D_{\text {epidemic }}^{m m}(m)\right]$. Hence, the probability that there are $m$ copies of a packet in the network equals $\sum_{i=m}^{M-1} \frac{1}{M-1} \frac{E\left[D_{\text {epidemic }}^{m m}(m)\right]}{\sum_{j=1}^{i} E\left[D_{\text {epidemic }}^{m}(j)\right]}$. Applying the law of total probability over the random variable $m$ and substituting the value of $E\left[D_{\text {epidemic }}^{m m}(m)\right]$ from Lemma 4.2 gives $p_{e x}^{e p i d e m i c}$.

Proof: (Lemma 4.5) The proof runs along similar lines as the proof of Lemma 4.3. Given that there are $m$ copies of the packet in the network, the probability that nodes $i$ and $j$ want to exchange a particular packet is equal to $p \frac{2 m(M-m)}{M(M-1)}$. The probability that there are $m$ copies of a packet in the network equals $\sum_{i=m}^{M-1} \frac{1}{M-1} \frac{E\left[D_{r f}^{m m}(m)\right]}{\sum_{j=1}^{i} E\left[D_{r f}^{m m}(j)\right]}$. To simplify the exposition, we make an approximation here by replacing $\left(m(M-m-1) p_{\text {success }}^{1}\right)+\left(m p_{\text {success }}^{2}\right)$ in the denominator of the expression for $E\left[D_{r f}^{m m}(m)\right.$ by $m(M-m) p_{\text {success }}^{1}$. We use simulations to verify that this approximation does not have a significant effect on the accuracy of the analysis. Note that this approximation is made only during the derivation of $p_{e x}^{r f}$.

Proof: (Lemma 4.6) The proof runs along the same lines as the proof of Lemma 4.2. When there are $1 \leq m<L$ copies of a packet in the network, there are $m$ nodes which can deliver a copy to the destination only, and there is one source node which can deliver a copy to any of the $M-m-1$ other nodes which do not have a copy of the packet. Hence, there are a total of $m+M-m-1=M-1$ node pairs, which when meet, have an opportunity to increase the number of copies from $m$ to $m+1$. The expected time it takes for one of these $M-1$ node pairs to meet is $\frac{E\left[M_{m m}\right]}{M-1}$. Using the same argument as in the proof of Lemma 4.2, $E\left[D_{s p r a y}^{m m}(m)\right]$ can be derived to be $\frac{E\left[M_{m m}\right]}{(M-1) p_{\text {sucuceses }}^{\text {splat }}}$.

When there are $L$ copies of a packet in the network, there are $L$ nodes which can deliver a copy to the destination but even if the source meets some other node which does not have a copy, it cannot attempt to transmit a copy to the other node. The expression for $E\left[D_{s p r a y}^{m m}(L)\right]$ is derived in a manner similar to the derivation of Lemma 4.2 to be $\frac{E\left[M_{m m}\right]}{L p_{s u c a}^{s p a}=}$.

Proof: (Lemma 5.3) After the spray phase (after $L$ copies have been spread), there is a community which has only $\hat{l}=$ rem $\left(L, \frac{M}{r}\right)$ nodes carrying a copy of the packet. The probability that the destination is one of the remaining $\frac{M}{r}-\hat{l}$ nodes belonging
to this community is equal to $\frac{\frac{M}{r}-\hat{l}}{M-L}$. First we will derive the expected delay in the wait phase when the destination belongs to this community. The probability that the system state is $\left(\hat{l}, m_{p}\right)$ (where $m_{p}$ denotes the number of node pairs in the community which want to exchange this packet, and had an opportunity in the past to exchange this packet but were unable to do so due to contention) is equal to $p_{\hat{l}, m_{p}}$. (The value of $p_{\hat{l}, m_{p}}$ was derived in Lemma 5.1.) Given the system state in which the spray phase ended is $\left(\hat{l}, m_{p}\right)$, the number of nodes which had an opportunity to deliver the packet to the destination but were unable to do so is equal to $\frac{m_{p}}{\frac{M}{r}-\hat{l}}$. (As discussed in the proof of Lemma 5.1, each node not having a copy of the packet has met on an average $\frac{m_{p}}{\frac{M}{r}-\hat{l}}$ nodes which have a copy of the packet.) To derive the delay associated with the wait phase, we define a new system state: $(s)$ where $s$ is the number of nodes in the destination's community which had an opportunity to deliver the packet to the destination but were unable to do so due to contention. Let $T_{s}$ denote the additional time it will take to deliver the packet to the destination given the current system state is $(s)$. Then, given that nodes in the destination's community have a copy of the packet, $E\left[D_{f s w}^{c o m m}(L)\right]$ is equal to $\left(\sum_{m_{p}=0}^{\hat{l}\left(\frac{M}{r}-\hat{l}\right)} p_{\hat{l}, m_{p}} E\left[T_{\frac{m_{p}}{\frac{M}{r}-\hat{l}}}\right]\right)$.

To complete the previous proof, we now describe how to derive the value of $E\left[T_{s}\right]$. One of the nodes carrying the packet meets the destination after an expected time duration of $\frac{1}{\frac{\hat{c}}{E\left[M_{\text {comm same }]}\right]}+\frac{s}{E\left[M_{\text {comm }, \text { same }}^{+}\right]}}$. With probability $p_{s u c c e s s 2}^{f s w}$, this node is able to deliver the packet to the destination (where $p_{s u c c e s s 2}^{f s w}=1-\left(1-p_{t x S 2}^{f s w}\right)^{E\left[\tau_{\text {com } m, s a m e}\right]}$. With probability $p_{s}=\frac{\frac{\hat{i}-s}{E\left[M_{\text {comm }, \text { same }]}\right.}+\frac{s}{E\left[M_{\text {comm }, \text { same }]}^{+}\right.}}{\frac{E\left[M_{\text {comm }, \text { same }}^{+}\right.}{s}}$, the node which meets the destination is one of the $s$ nodes which have missed an opportunity to deliver the packet to the destination in the past. Hence, with probability $p_{s}\left(1-p_{\text {success } 2}^{f s w}\right)$ the packet does not get delivered to the destination and the system remains in state $s$ and will take an additional $E\left[T_{s}\right]$ time to deliver the packet to the destination. On the other hand, with probability $\left(1-p_{s}\right)\left(1-p_{\text {success } 2}^{f s w}\right)$, the packet does not get delivered to the destination and the system moves to state $s+1$ (as one
more node belonging to the destination's community has missed an opportunity to deliver the packet to the destination) and will take an additional $E\left[T_{s+1}\right]$ time to deliver the packet to the destination. Thus, $E\left[T_{s}\right]=\frac{i-s}{\frac{1}{E\left[M_{\text {comm }, \text { same }]}+\frac{s}{E\left[M_{\text {comm, same }}^{+}\right]}\right.}+}$
$p_{s}\left(1-p_{\text {success } 2}^{f s w}\right) E\left[T_{s}\right]+\left(1-p_{s}\right)\left(1-p_{\text {success } 2}^{f s w}\right) E\left[T_{s+1}\right]$. This set of linear equations can be solved to find $E\left[T_{s}\right]$.
Now, with probability $1-\left(\frac{\frac{M}{r}-\hat{l}}{M-L}\right)$, none of the nodes belonging to the destination's community have a copy of the packet and the expected time it takes for the $L$ nodes to deliver the packet to destination can be derived in manner similar to the derivation of Lemma 4.2 to be equal to $\frac{E\left[M_{c o m m, d i f f}\right]}{L p_{s u c c e s s}^{f s w}}$.

Finally combining everything together by using the law of total probability to remove the condition on whether a node belonging to the destination's community had a copy of the packet after the spray phase or not, yields the result.

Proof: (Lemma 5.4) After the spray phase (after $L$ copies have been spread), there is a community which has only $\hat{l}=$ rem $\left(L, \frac{M}{r}\right)$ nodes carrying a copy of the packet. The probability that the destination is one of the remaining $\frac{M}{r}-\hat{l}$ nodes belonging to this community is equal to $\frac{\frac{M}{r}-\hat{l}}{M-L}$. The expected delivery delay to the destination for this scenario is derived in a manner similar to the derivation of $E\left[D_{f s w}^{c o m m}(L)\right]$ in Lemma 5.3.

Now we derive the delivery delay for the scenario when the nodes in the destination's community do not have a copy of the packet. The expected time it takes for the $L$ nodes carrying a copy to deliver a copy to one of the $\frac{M}{r}$ in the destination's community is equal to $\frac{E\left[M_{\text {comm,diff }}\right]}{L \frac{M}{r} p_{s u c}^{f s f} \text { fess } 1}$. (This is derived in a manner similar to the derivation of Lemma 4.2). With probability $\frac{\frac{M}{r}-1}{\frac{M}{r}}$, the packet copy is received by a node which itself is not the destination but belongs to the destination's community. This node does a direct transmission to the destination which takes an additional time whose expected value is equal to $E\left[M_{\text {comm,same }}\right]+\frac{\left(1-p_{s u c c e s s s 2}^{f s f}\right) E\left[M_{\text {com } m, s a m e}^{+}\right]}{p_{s u c c e s s s}^{f s} 2}$. (This is derived in a manner similar to the derivation of Lemma 5.1.)


[^0]:    ${ }^{1}$ Note that the analysis presented in this section can be easily modified for any mobility model which satisfies these two properties.

[^1]:    ${ }^{2}$ Note that the value of $p_{t x S}^{R}$ depends on the routing mechanism through $p_{e x}^{R}$ only. Given the value of $p_{e x}^{R}$, one can derive the value of $p_{t x S}^{R}$ in terms of the network parameters using the framework proposed in [27].

[^2]:    ${ }^{3}$ The final assumption is not critical to the analysis, but making this assumption simplifies the presentation.

