Spiral 1 / Unit 5

Karnaugh Maps

Outcomes

- I know the difference between combinational and sequential logic and can name examples of each.
- I understand latency, throughput, and at least 1 technique to improve throughput
- I can identify when I need state vs. a purely combinational function
  - I can convert a simple word problem to a logic function (TT or canonical form) or state diagram
- I can use Karnaugh maps to synthesize combinational functions with several outputs
- I understand how a register with an enable functions & is built
- I can design a working state machine given a state diagram
- I can implement small logic functions with complex CMOS gates

A new way to synthesize your logic functions

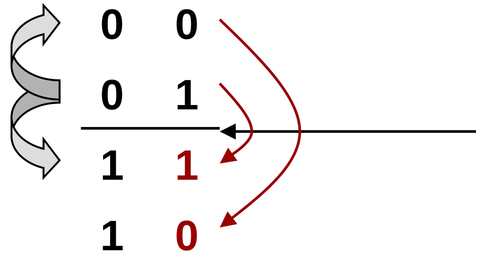
KARNAUGH MAPS

Logic Function Synthesis

- Given a function description as a T.T. or canonical form, how can we arrive at a circuit implementation or equation (i.e. perform logic synthesis)?
- First method
  - Minterms / maxterms
    - Can simplify to find minimal 2-level implementation
    - Use "off-the-shelf" decoder + 1 gate per output
- New, second method
  - Karnaugh Maps
    - Minimal 2-level implementation (though not necessarily minimal 3-, 4-, … level implementation)

Gray Code

- Different than normal binary ordering
- Reflective code
  - When you add the (n+1)th bit, reflect all the previous n-bit combinations
- Consecutive code words differ by only 1-bit



0 0

0 1

1 1

1 0

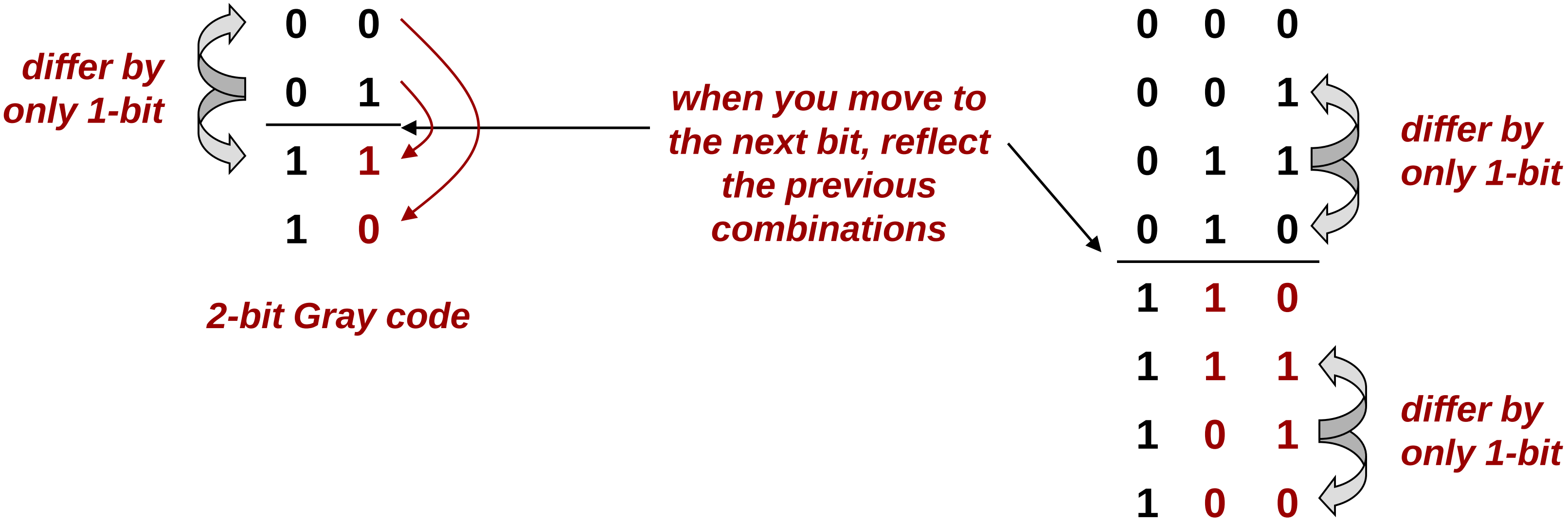
differ by only 1-bit

when you move to the next bit, reflect the previous combinations

2-bit Gray code

Gray Code

- Different than normal binary ordering
- Reflective code
  - When you add the (n+1)th bit, reflect all the previous n-bit combinations
- Consecutive code words differ by only 1-bit



0	0
0	1
1	1
1	0

2-bit Gray code

0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0

3-bit Gray code

differ by only 1-bit
when you move to the next bit, reflect the previous combinations
differ by only 1-bit
differ by only 1-bit

Karnaugh Maps

- If used correctly, will always yield a minimal, 2-level implementation
  - There may be a more minimal 3-level, 4-level, 5-level… implementation but K-maps produce the minimal two-level (SOP or POS) implementation
- Represent the truth table graphically as a series of adjacent squares that allows a human to see where variables will cancel

Karnaugh Map Construction

- Every square represents 1 input combination
- Must label axes in Gray code order
- Fill in squares with given function values

$F=\Sigma_{XYZ}(1,4,5,6)$

Z \ XY	00	01	11	10
0	0 0	2 0	6 1	4 1
1	1 1	3 0	7 0	5 1

3 Variable Karnaugh Map

$G=\Sigma_{WXYZ}(1,2,3,5,6,7,9,10,11,14,15)$

YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

4 Variable Karnaugh Map

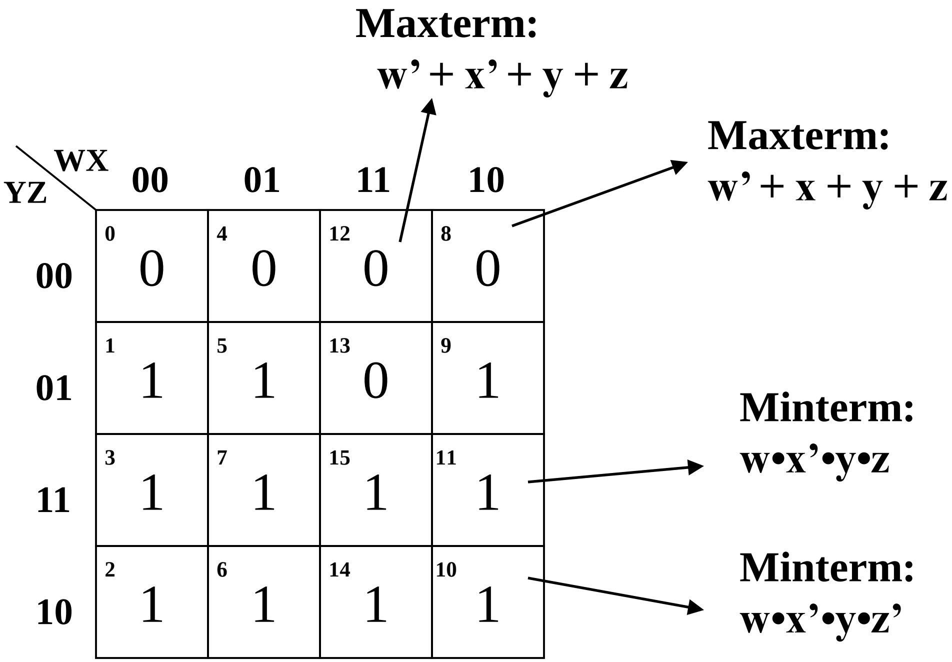
Karnaugh Maps

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

Karnaugh Maps

- Squares with a '1' represent minterms
- Squares with a '0' represent maxterms



YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

Maxterm:
w’ + x’ + y + z

Maxterm:
w’ + x + y + z

Minterm:
w•x’•y•z

Minterm:
w•x’•y•z’

Karnaugh Maps

- Groups of adjacent 1’s will always simplify to smaller product term than just individual minterms

$F=\Sigma_{XYZ}(0,2,4,5,6)$

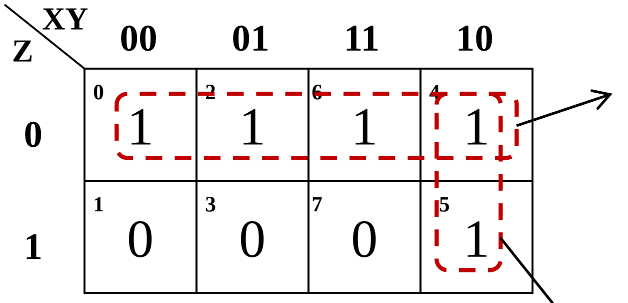
Z \ XY	00	01	11	10
0	0 1	2 1	6 1	4 1
1	1 0	3 0	7 0	5 1

3 Variable Karnaugh Map

Karnaugh Maps

- Groups of adjacent 1’s will always simplify to smaller product term than just individual minterms

$F=\Sigma_{XYZ}(0,2,4,5,6)$



Z \ XY	00	01	11	10
0	1	1	1	1
1	0	0	0	1

3 Variable Karnaugh Map

$= m0 + m2 + m6 + m4$
$= x'y'z' + x'yz' + xyz' + xy'z'$
$= z'(x'y' + x'y + xy + xy')$
$= z'(x'(y'+y) + x(y+y'))$
$= z'(x'+x)$
$= z'$

$= m4 + m5$
$= xy'z' + xy'z = xy'(z'+z)$
$= xy'$

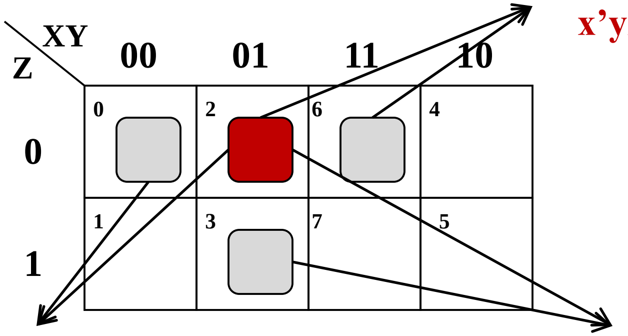
Karnaugh Maps

- Adjacent squares differ by 1-variable
  - This will allow us to use T10 = AB + AB'= A or T10' = (A+B')(A+B) = A

3 Variable Karnaugh Map

Difference in X: 010 & 110

x'yz' + xyz' = yz'



Difference in Y: 010 & 000

x'yz' + x'y'z' = x'z'

Difference in Z: 010 & 011

x'yz' + x'yz = x'y

$0 = 0\underline{0}0$

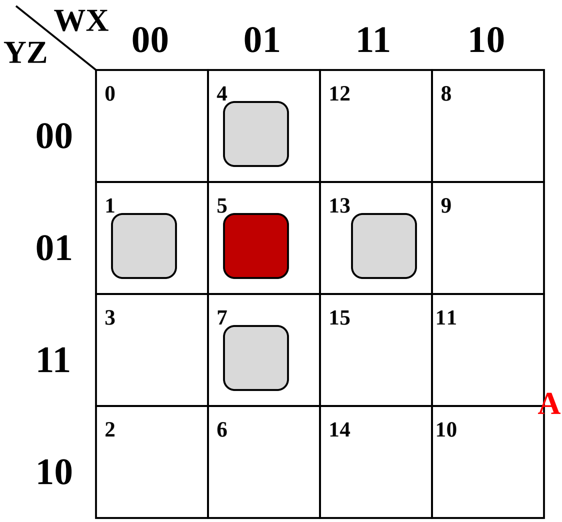
$2 = \underline{010}$

$3 = 01\underline{1}$

$6 = \underline{1}10$

Adjacent squares differ by 1-bit

4 Variable Karnaugh Map



$1 = 0\underline{0}01$

$4 = 010\underline{0}$

$5 = \underline{0101}$

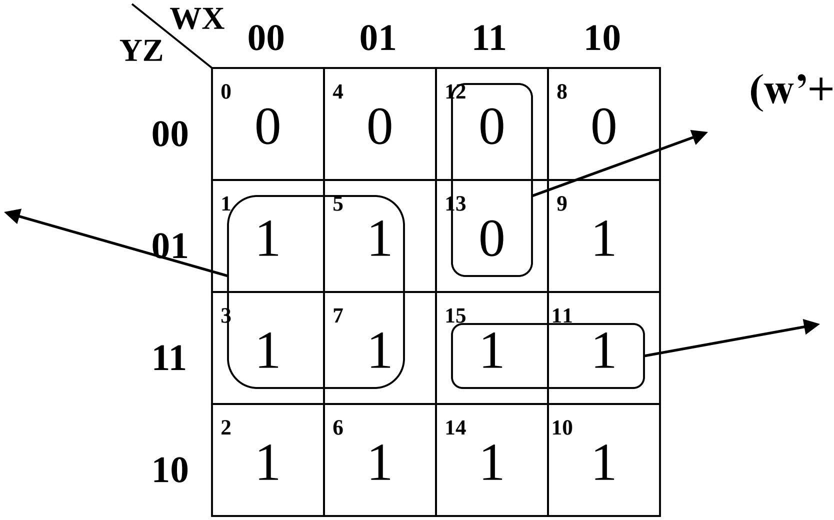
$7 = 01\underline{1}1$

$13 = \underline{1}101$

Adjacent squares differ by 1-bit

Karnaugh Maps

- 2 adjacent 1's (or 0's) differ by only one variable
- 4 adjacent 1's (or 0's) differ by two variables
- 8, 16, … adjacent 1's (or 0's) differ by 3, 4, … variables
- By grouping adjacent squares with 1's (or 0's) in them, we can come up with a simplified expression using T10 (or T10' for 0's)



YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

$w'\bullet x'\bullet y'\bullet z + w'\bullet x'\bullet y\bullet z + w'\bullet x\bullet y'\bullet z + w'\bullet x\bullet y\bullet z$

$= \mathbf{w'\bullet z}$

w'z are constant while all combos of x and y are present (x'y', x'y, xy', xy)

$\mathbf{(w'+x'+y+z)\bullet(w'+x'+y+z') = (w'+x'+y)}$

$\mathbf{w\bullet x\bullet y\bullet z + w\bullet x'\bullet y\bullet z = w\bullet y\bullet z}$

K-Map Grouping Rules

- Cover the 1's [=on-set] or 0's [=off-set] with as few groups as possible, but make those groups as large as possible
  - Make them as large as possible even if it means "covering" a 1 (or 0) that's already a member of another group
- Make groups of 1, 2, 4, 8, ... and they must be rectangular or square in shape.
- Wraparounds are legal

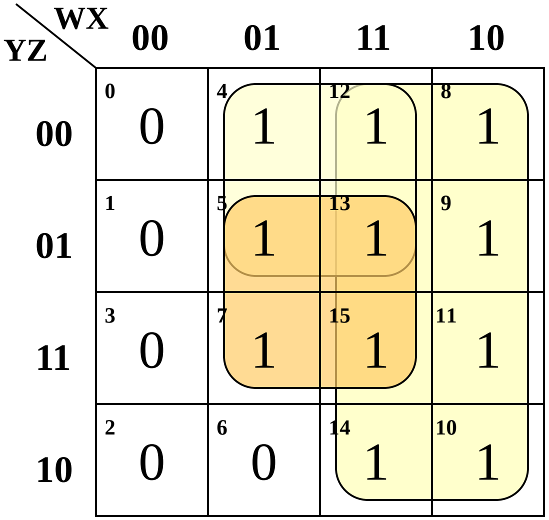
K-Map Grouping Rules

XY Z	00	01	11	10
0	0 0	2 1	6 0	4 0
1	1 1	3 0	7 0	5 0

XY Z	00	01	11	10
0	0 1	2 1	6 0	4 0
1	1 1	3 0	7 0	5 0

WX YZ	00	01	11	10
00	0 0	4 0	12 1	8 1
01	1 1	5 1	13 1	9 0
11	3 1	7 1	15 1	11 0
10	2 0	6 0	14 0	10 1

Karnaugh Maps

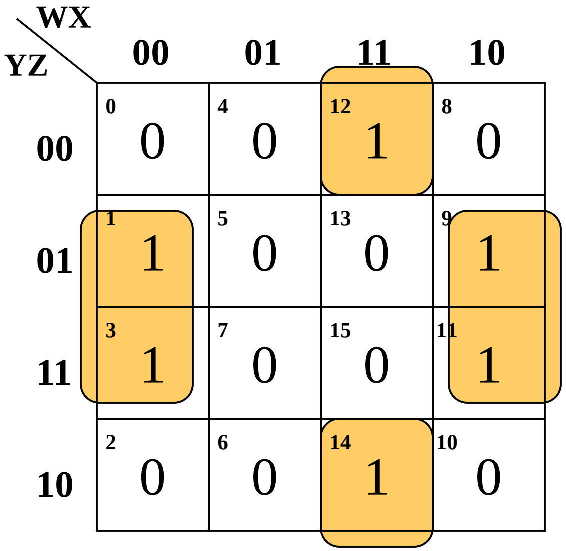


YZ \ WX	00	01	11	10
00	0 0	4 1	12 1	8 1
01	1 0	5 1	13 1	9 1
11	3 0	7 1	15 1	11 1
10	2 0	6 0	14 1	10 1

- Cover the remaining ‘1’ with the largest group possible even if it “reuses” already covered 1’s

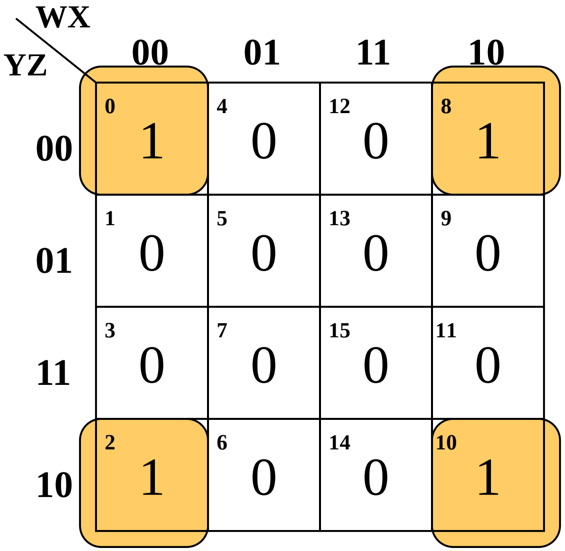
Karnaugh Maps

- Groups can wrap around from:
    - Right to left
    - Top to bottom
    - Corners



WX YZ	00	01	11	10
00	0 0	4 0	12 1	8 0
01	1 1	5 0	13 0	9 1
11	3 1	7 0	15 0	11 1
10	2 0	6 0	14 1	10 0

F = X’Z + WXZ’



WX YZ	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 0	5 0	13 0	9 0
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 0	10 1

F = X’Z’

Group This

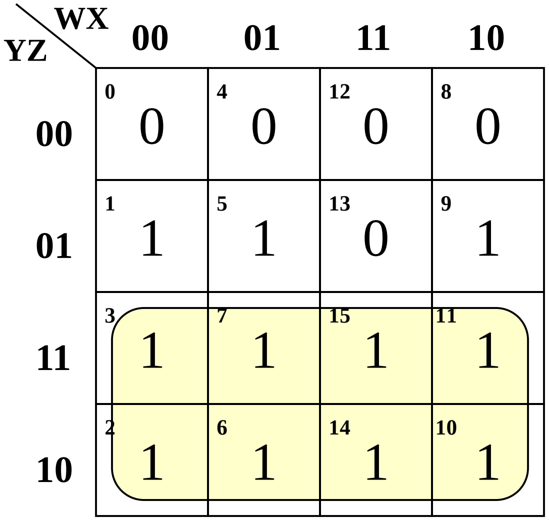
YZ \ WX	00	01	11	10
00	^{0} 0	^{4} 0	^{12} 0	^{8} 0
01	^{1} 1	^{5} 1	^{13} 0	^{9} 1
11	^{3} 1	^{7} 1	^{15} 1	^{11} 1
10	^{2} 1	^{6} 1	^{14} 1	^{10} 1

K-Map Translation Rules

- When translating a group of 1's, find the variable values that are constant for each square in the group and translate only those variables values to a product term
- Grouping 1's yields SOP
- When translating a group of 0's, again find the variable values that are constant for each square in the group and translate only those variable values to a sum term
- Grouping 0's yields POS

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

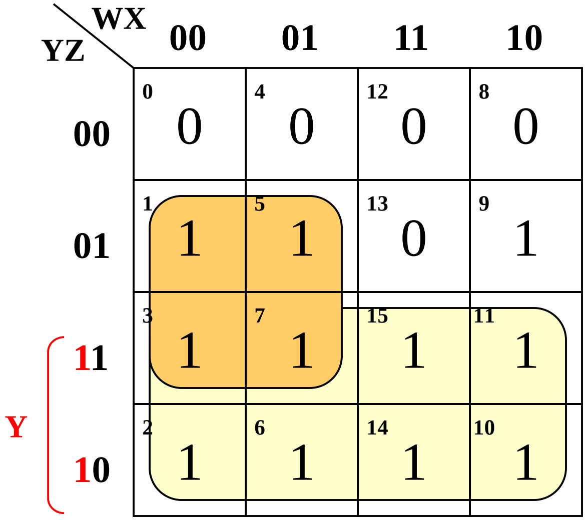


YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

F =

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

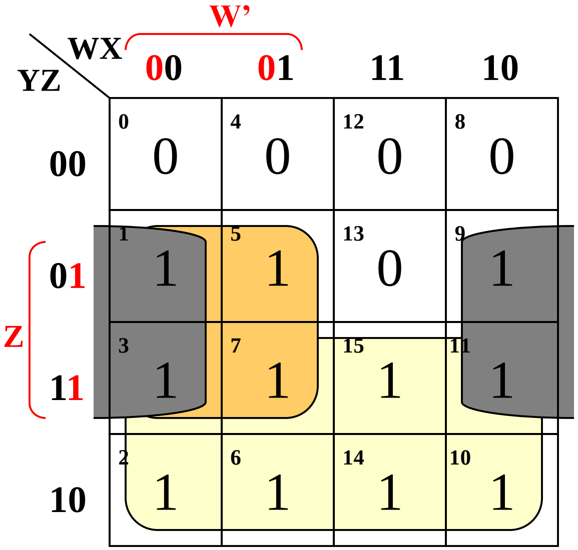


YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

F = Y

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

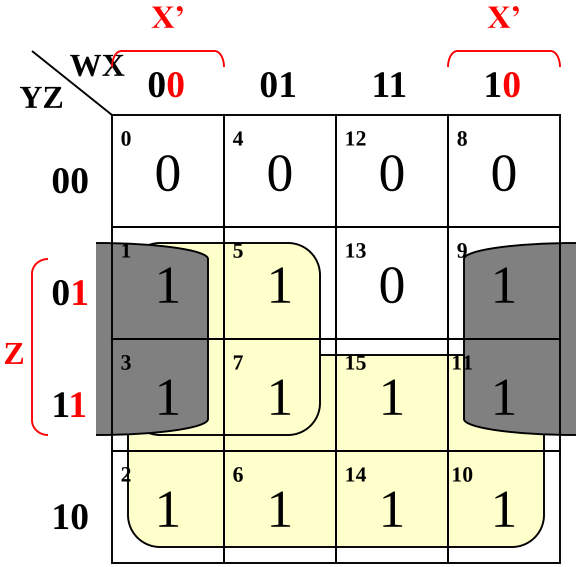


YZ \ WX	00	01	11	10
00	0	0	0	0
01	1	1	0	1
11	1	1	1	1
10	1	1	1	1

F = Y + W’Z + …

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



YZ \ WX	00	01	11	10
00	0: 0	4: 0	12: 0	8: 0
01	1: 1	5: 1	13: 0	9: 1
11	3: 1	7: 1	15: 1	11: 1
10	2: 1	6: 1	14: 1	10: 1

F = Y + W'Z + X'Z

Karnaugh Maps (POS)

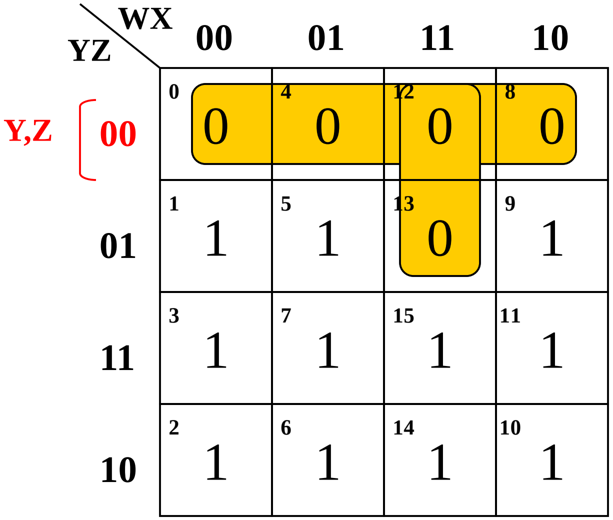
W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

WX YZ	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

F =

Karnaugh Maps (POS)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

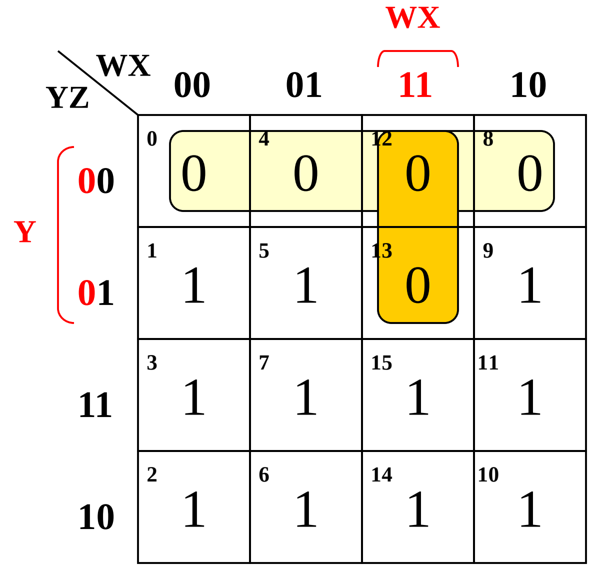


YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

F = (Y+Z)

Karnaugh Maps (POS)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

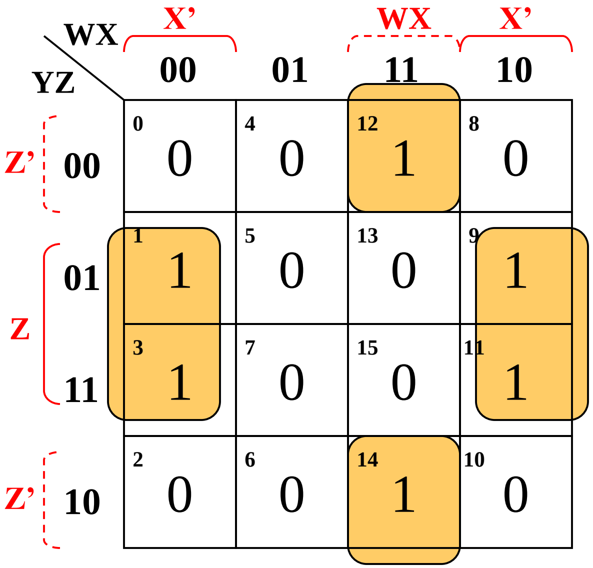


YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

F = (Y+Z)(W’+X’+Y)

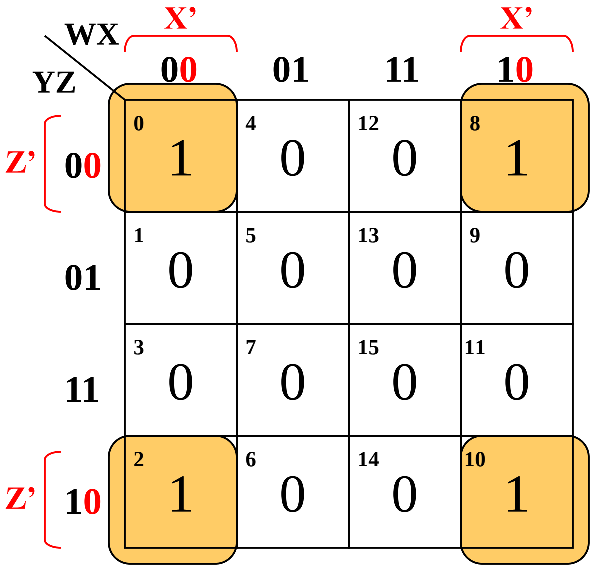
Karnaugh Maps

- Groups can wrap around from:
  - Right to left
  - Top to bottom
  - Corners



YZ \ WX	00	01	11	10
00	0 0	4 0	12 1	8 0
01	1 1	5 0	13 0	9 1
11	3 1	7 0	15 0	11 1
10	2 0	6 0	14 1	10 0

F = X'Z + WXZ'



YZ \ WX	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 0	5 0	13 0	9 0
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 0	10 1

F = X'Z'

Exercises

YZ \ WX	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 1	5 0	13 0	9 1
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 1	10 1

F_{SOP}=

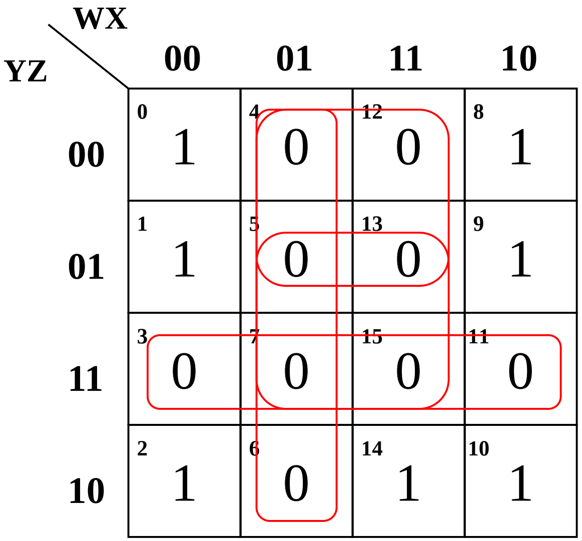
YZ \ WX	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 1	5 0	13 0	9 1
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 1	10 1

F_{POS}=

$P=\Sigma_{XYZ}(2,3,5,7)$

P=

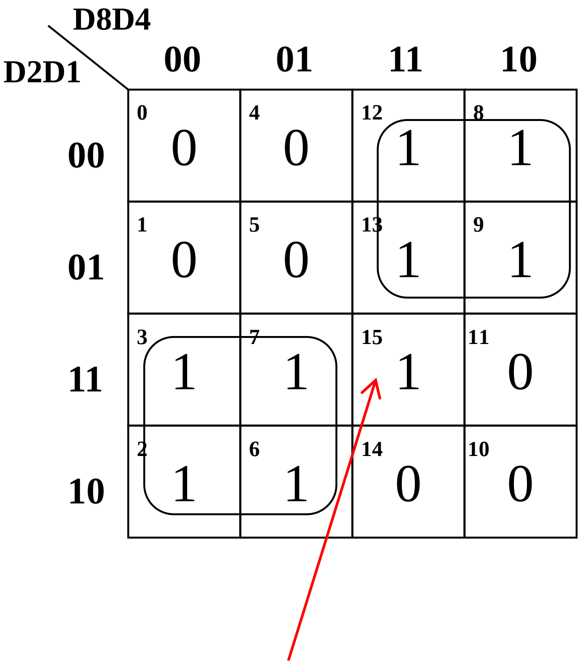
No Redundant Groups



YZ \ WX	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 1	5 0	13 0	9 1
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 1	10 1

Multiple Minimal Expressions

- For some functions, multiple minimal groupings exist which will lead to alternate minimal expressions…Pick one

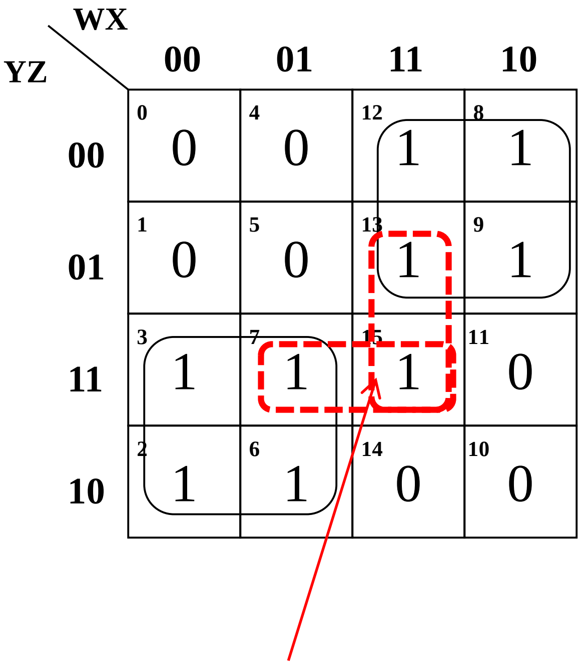


D2D1 \ D8D4	00	01	11	10
00	0 0	4 0	12 1	8 1
01	1 0	5 0	13 1	9 1
11	3 1	7 1	15 1	11 0
10	2 1	6 1	14 0	10 0

Best way to cover this ‘1’??

Multiple Minimal Expressions

- For some functions, multiple minimal expressions (multiple minimal groups) exist…Pick one



YZ \ WX	00	01	11	10
00	0 0	4 0	12 1	8 1
01	1 0	5 0	13 1	9 1
11	3 1	7 1	15 1	11 0
10	2 1	6 1	14 0	10 0

Pick either one

Terminology

- Implicant: A product term (grouping of 1's) that covers a subset of cases where F=1
  - the product term is said to "imply" F because if the product term evaluates to '1' then F='1'
- Prime Implicant: The largest grouping of 1's (smallest product term) that can be made
- Essential Prime Implicant: A prime implicant (product term) that is needed to cover all the 1's of F

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

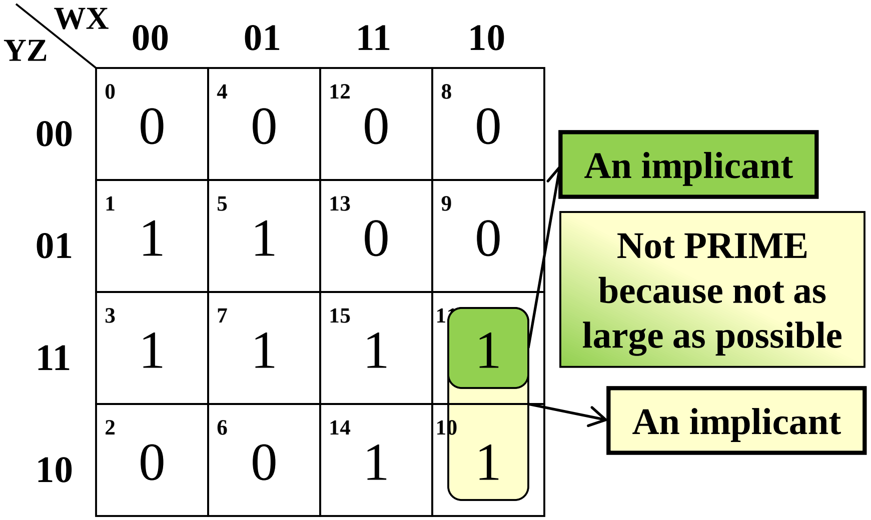
YZ \ WX	**00**	**01**	**11**	**10**
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 0
11	3 1	7 1	15 1	11 1
10	2 0	6 0	14 1	10 1

An implicant

Not PRIME because not as large as possible

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



YZ \ WX	00	01	11	10
00	0: 0	4: 0	12: 0	8: 0
01	1: 1	5: 1	13: 0	9: 0
11	3: 1	7: 1	15: 1	11: 1
10	2: 0	6: 0	14: 1	10: 1

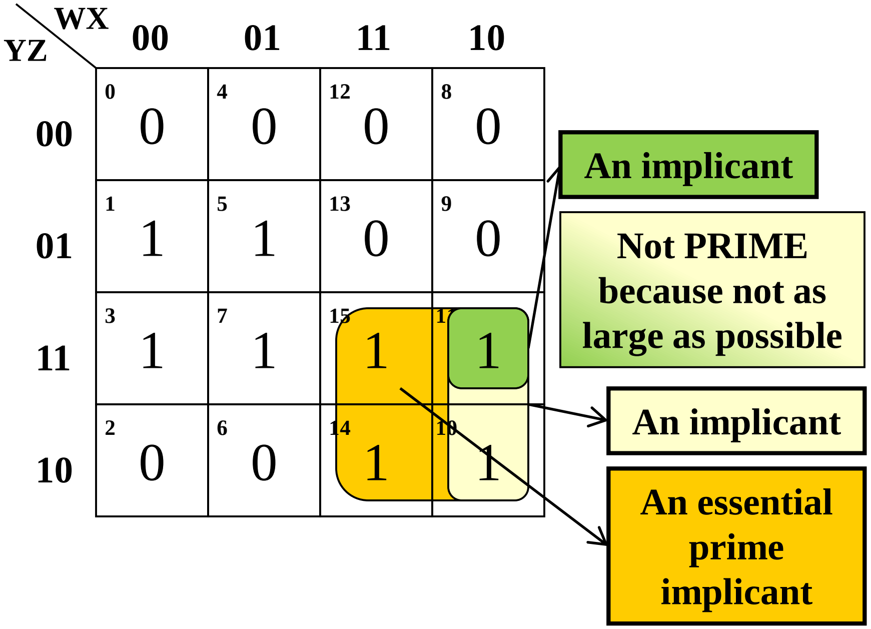
An implicant

Not PRIME because not as large as possible

An implicant

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



YZ \ WX	00	01	11	10
00	0: 0	4: 0	12: 0	8: 0
01	1: 1	5: 1	13: 0	9: 0
11	3: 1	7: 1	15: 1	11: 1
10	2: 0	6: 0	14: 1	10: 1

An implicant

Not PRIME because not as large as possible

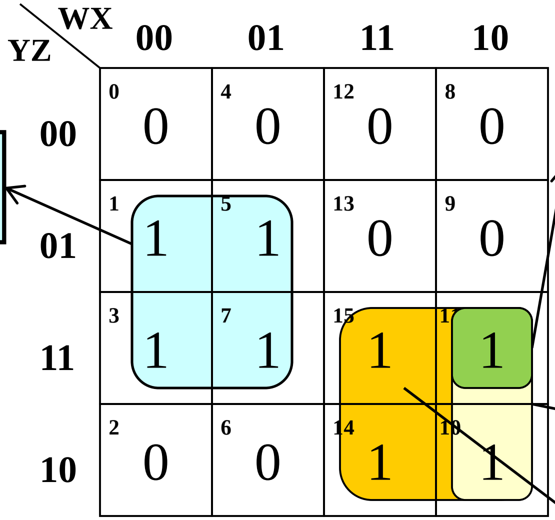
An implicant

An essential prime implicant

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 0
11	3 1	7 1	15 1	11 1
10	2 0	6 0	14 1	10 1

An essential prime implicant

An implicant

Not PRIME because not as large as possible

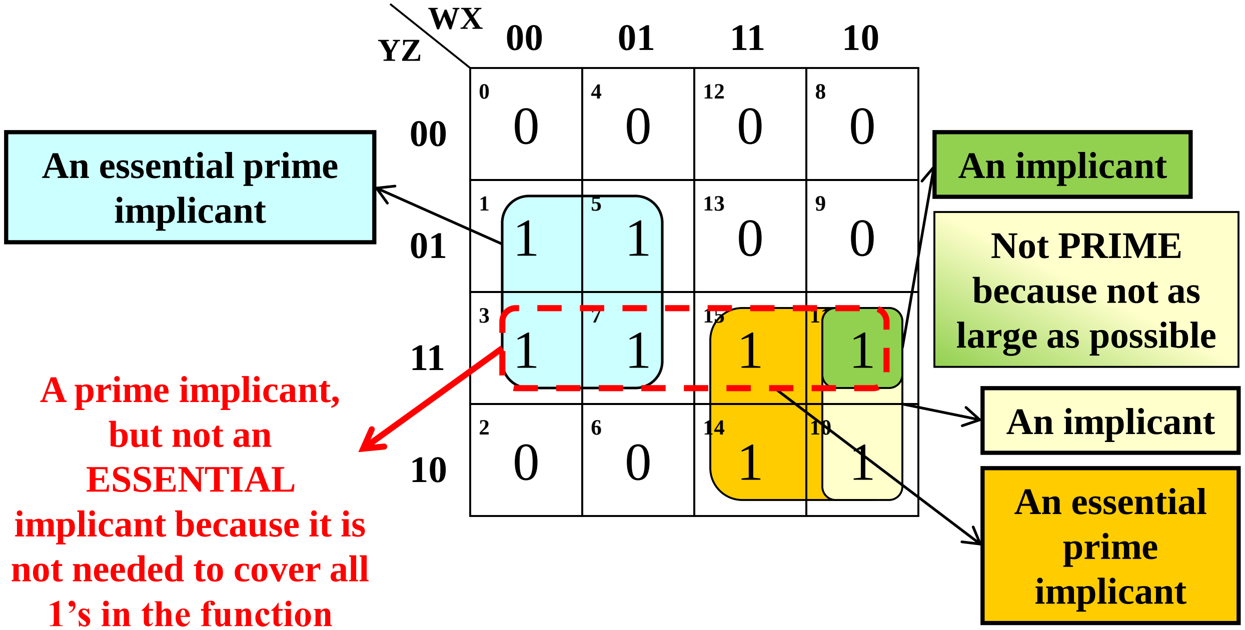
An implicant

An essential prime implicant

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



WX YZ	**00**	**01**	**11**	**10**
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 0
11	3 1	7 1	15 1	11 1
10	2 0	6 0	14 1	10 1

An essential prime implicant

A prime implicant, but not an ESSENTIAL implicant because it is not needed to cover all 1's in the function

An implicant

Not PRIME because not as large as possible

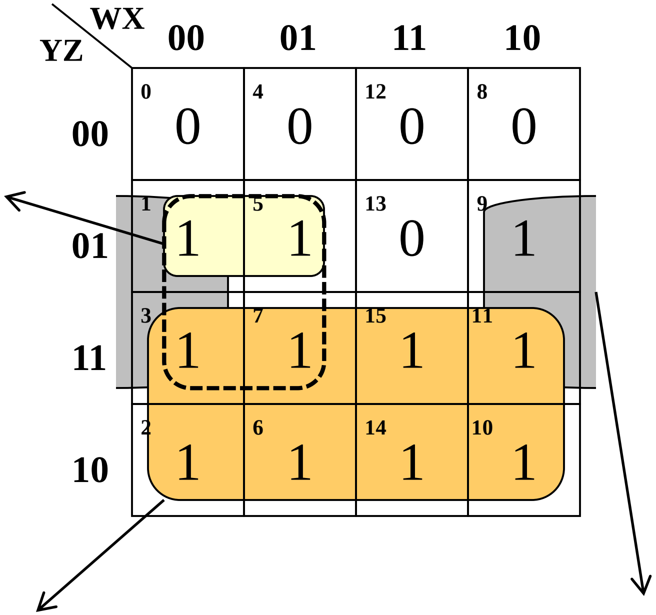
An implicant

An essential prime implicant

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



YZ \ WX	00	01	11	10
00	$_{0}$ 0	$_{4}$ 0	$_{12}$ 0	$_{8}$ 0
01	$_{1}$ 1	$_{5}$ 1	$_{13}$ 0	$_{9}$ 1
11	$_{3}$ 1	$_{7}$ 1	$_{15}$ 1	$_{11}$ 1
10	$_{2}$ 1	$_{6}$ 1	$_{14}$ 1	$_{10}$ 1

An implicant, but not a PRIME implicant because it is not as large as possible (should expand to combo's 3 and 7)

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

An essential prime implicant

K-Map Grouping Rules

- Make groups (implicants) of 1, 2, 4, 8, ... and they must be rectangular or square in shape.
- Include the minimum number of essential prime implicants
  - Use only *essential* prime implicants (i.e. as few groups as possible to cover all 1's)
  - Ensure that you are using ***prime*** implicants (i.e. Always make groups as large as possible reusing squares if necessary)
- Wraparounds are legal

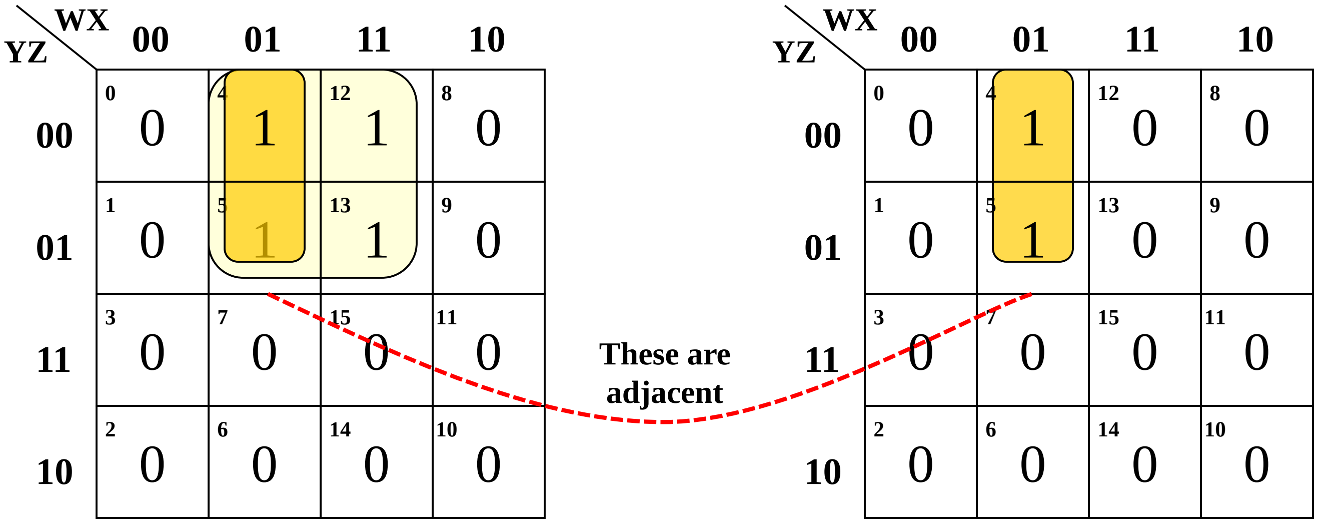
5-Variable K-Map

- If we have a 5-variable function we need a 32-square KMap.
- Will an 8x4 matrix work?
  - Recall K-maps work because adjacent squares differ by 1-bit
- How many adjacencies should we have for a given square?
- 5!! But drawn in 2 dimensions we can't have 5 adjacencies.

YZ \ VWX	000	001	011	010	110	111	101	100
00			▨					
01		▨	🟥	▨		▨		
11			▨					
10								

5-Variable Karnaugh Maps

- To represent the 5 adjacencies of a 5-variable function [e.g. f(v,w,x,y,z)], imagine two 4x4 K-Maps stacked on top of each other
  - Adjacency across the two maps



WX YZ	00	01	11	10
00	0 0	4 1	12 1	8 0
01	1 0	5 1	13 1	9 0
11	3 0	7 0	15 0	11 0
10	2 0	6 0	14 0	10 0

V=0

WX YZ	00	01	11	10
00	0 0	4 1	12 0	8 0
01	1 0	5 1	13 0	9 0
11	3 0	7 0	15 0	11 0
10	2 0	6 0	14 0	10 0

V=1

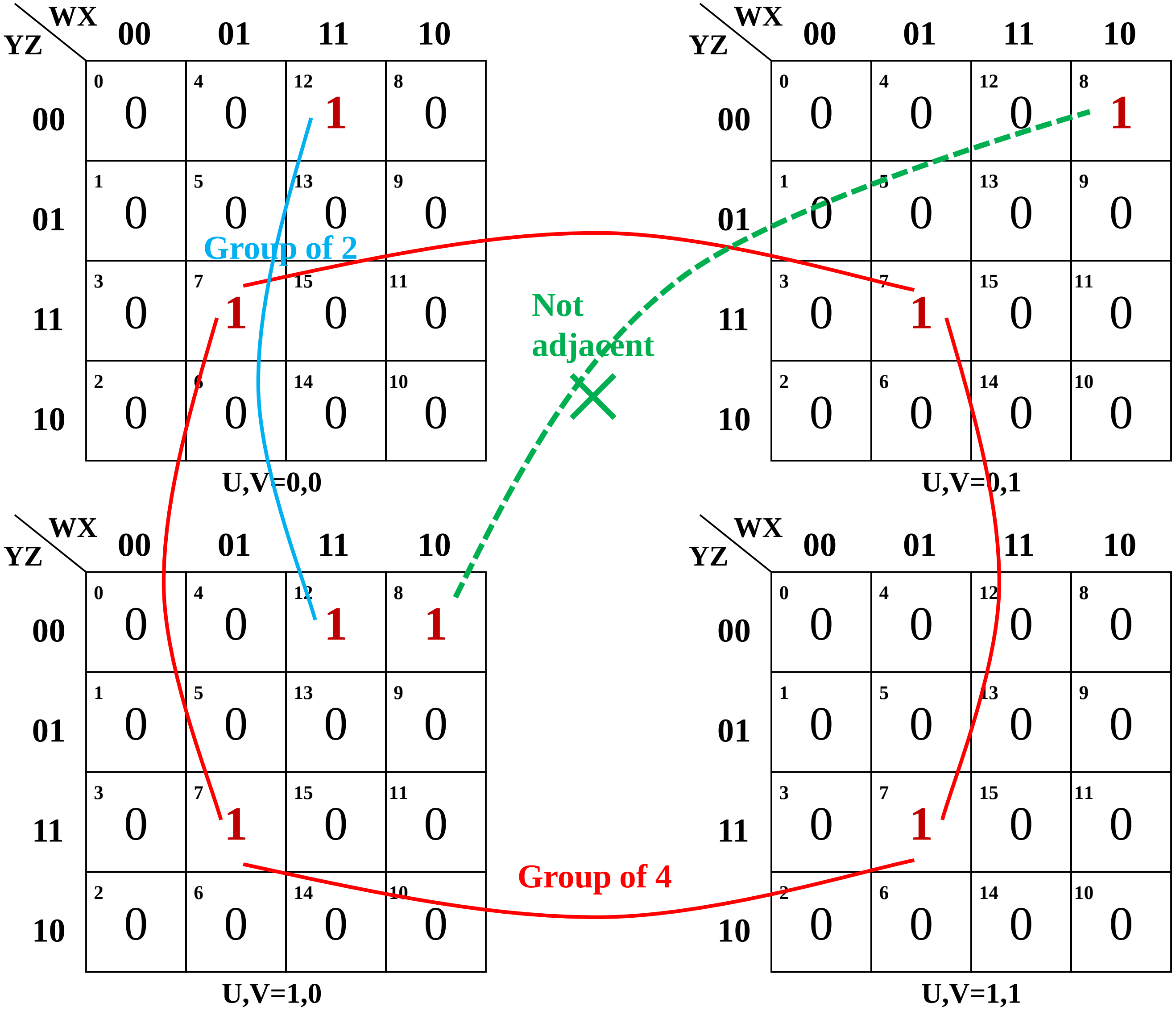
Traditional adjacencies still apply
(Note: v is constant for that group and should be included)
=> v'xy'

Adjacencies across the two maps apply
(Now v is not constant)
=> w'xy'

F = v'xy' + w'xy'

6-Variable Karnaugh Maps

- 6 adjacencies for 6-variables (Stack of four 4x4 maps)



YZ \ WX	00	01	11	10
00	0 0	4 0	12 1	8 0
01	1 0	5 0	13 0	9 0
11	3 0	7 1	15 0	11 0
10	2 0	6 0	14 0	10 0

U,V=0,0

YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 1
01	1 0	5 0	13 0	9 0
11	3 0	7 1	15 0	11 0
10	2 0	6 0	14 0	10 0

U,V=0,1

YZ \ WX	00	01	11	10
00	0 0	4 0	12 1	8 1
01	1 0	5 0	13 0	9 0
11	3 0	7 1	15 0	11 0
10	2 0	6 0	14 0	10 0

U,V=1,0

YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 0	5 0	13 0	9 0
11	3 0	7 1	15 0	11 0
10	2 0	6 0	14 0	10 0

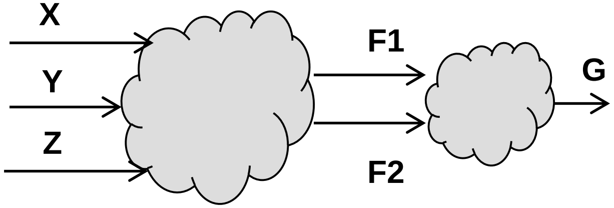
U,V=1,1

Don’t-Cares

- Sometimes there are certain input combinations that are illegal (i.e. in BCD, 1010 – 1111 can never occur)
- The outputs for the illegal inputs are “don’t-cares”
  - The output can either be 0 or 1 since the inputs can never occur
  - Don’t-cares can be included in groups of 1 or groups of 0 when grouping in K-Maps
  - Use them to make as big of groups as possible

Combining Functions

- Given intermediate functions F1 and F2, how could you use AND, OR, NOT to make G
- Notice certain F1,F2 combinations never occur in G(x,y,z)…what should we make their output in the T.T.



X	Y	Z	F1	F2	G
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	1	0

F1	F2	G
0	0	
0	1	
1	0	
1	1	

Don’t Care Example

D8	D4	D2	D1	GT6
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**

D8D4 D2D1	00	01	11	10
00	0 0	4 0	12 d	8 1
01	1 0	5 0	13 d	9 1
11	3 0	7 1	15 d	11 d
10	2 0	6 0	14 d	10 d

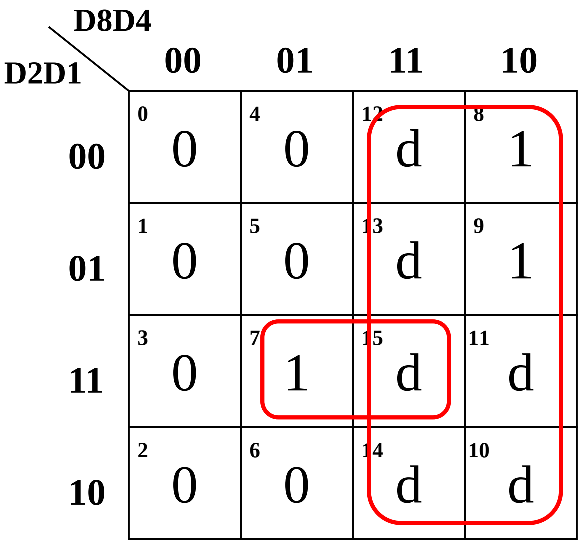
$GT6_{SOP}$=

D8D4 D2D1	00	01	11	10
00	0 0	4 0	12 d	8 1
01	1 0	5 0	13 d	9 1
11	3 0	7 1	15 d	11 d
10	2 0	6 0	14 d	10 d

$GT6_{POS}$=

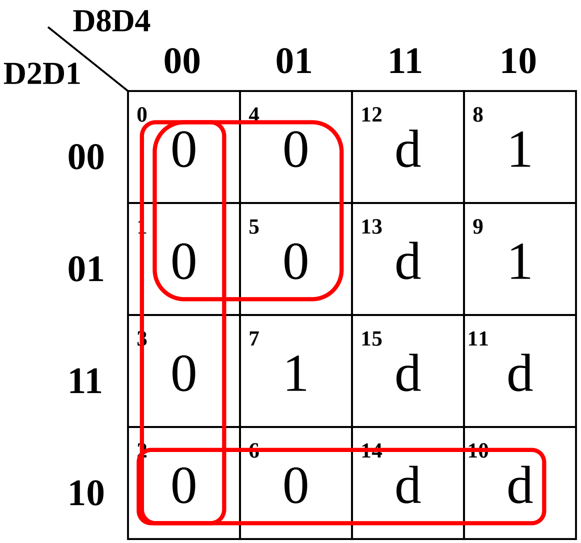
Don’t Care Example

D8	D4	D2	D1	GT6
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**



D2D1 \ D8D4	00	01	11	10
00	0 0	4 0	12 d	8 1
01	1 0	5 0	13 d	9 1
11	3 0	7 1	15 d	11 d
10	2 0	6 0	14 d	10 d

GT6$_{SOP}$=

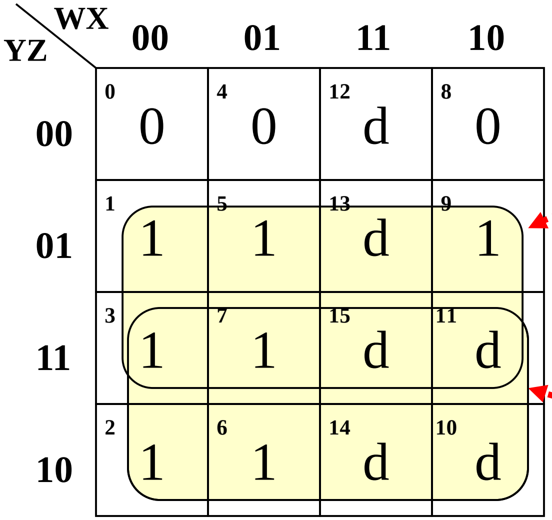


D2D1 \ D8D4	00	01	11	10
00	0 0	4 0	12 d	8 1
01	1 0	5 0	13 d	9 1
11	3 0	7 1	15 d	11 d
10	2 0	6 0	14 d	10 d

GT6$_{POS}$=

Don’t Cares

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**



WX YZ	00	01	11	10
00	0 0	4 0	12 d	8 0
01	1 1	5 1	13 d	9 1
11	3 1	7 1	15 d	11 d
10	2 1	6 1	14 d	10 d

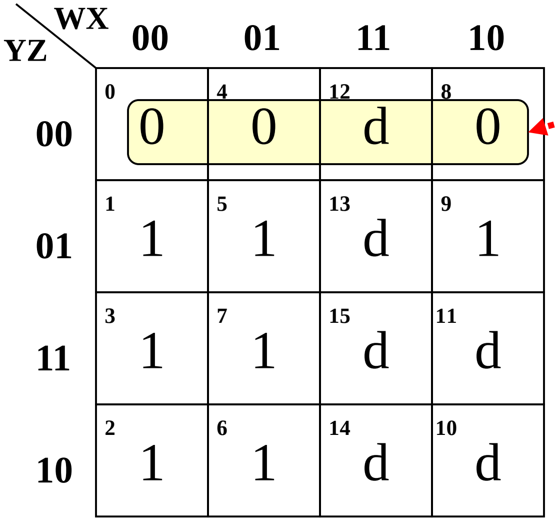
F = Y’Z + Y

Reuse “d’s” to make as large a group as possible to cover 1,5, & 9

Use these 4 “d’s” to make a group of 8

Don’t Cares

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**



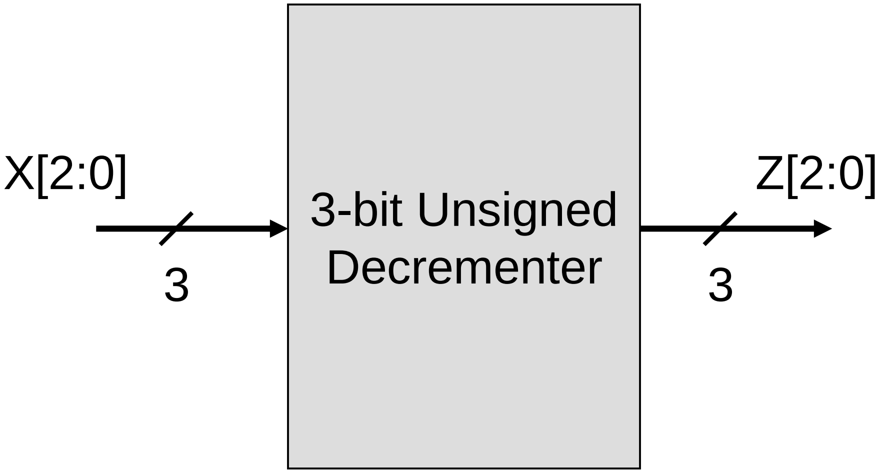
YZ \ WX	00	01	11	10
00	0 0	4 0	12 d	8 0
01	1 1	5 1	13 d	9 1
11	3 1	7 1	15 d	11 d
10	2 1	6 1	14 d	10 d

F = Y+Z

You can use “d’s” when grouping 0’s and converting to POS

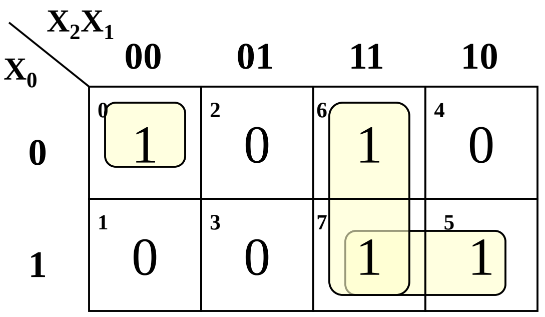
Designing Circuits w/ K-Maps

- Given a description…
  - Block Diagram
  - Truth Table
  - K-Map for each output bit (each output bit is a separate function of the inputs)
- 3-bit unsigned decrementer (Z = X-1)
  - If X[2:0] = 000 then Z[2:0] = 111, etc.



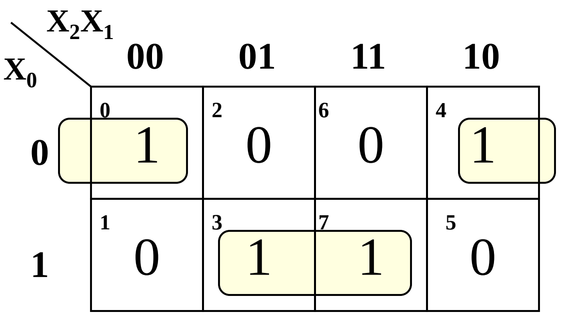
3-bit Number Decrementer

X_2	X_1	X_0	Z_2	Z_1	Z_0
0	0	0	**1**	**1**	**1**
0	0	1	**0**	**0**	**0**
0	1	0	**0**	**0**	**1**
0	1	1	**0**	**1**	**0**
1	0	0	**0**	**1**	**1**
1	0	1	**1**	**0**	**0**
1	1	0	**1**	**0**	**1**
1	1	1	**1**	**1**	**0**



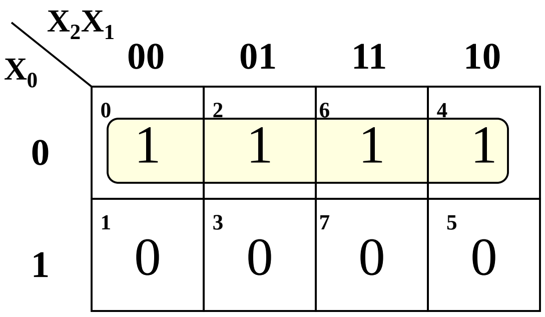
X_0 \ X_2X_1	00	01	11	10
0	^{0} 1	^{2} 0	^{6} 1	^{4} 0
1	^{1} 0	^{3} 0	^{7} 1	^{5} 1

$Z_2 = X_2X_0 + X_2X_1 + X_2'X_1'X_0'$



X_0 \ X_2X_1	00	01	11	10
0	^{0} 1	^{2} 0	^{6} 0	^{4} 1
1	^{1} 0	^{3} 1	^{7} 1	^{5} 0

$Z_1 = X_1'X_0' + X_1X_0$



X_0 \ X_2X_1	00	01	11	10
0	^{0} 1	^{2} 1	^{6} 1	^{4} 1
1	^{1} 0	^{3} 0	^{7} 0	^{5} 0

$Z_0 = X_0'$

Squaring Circuit

- Design a combinational circuit that accepts a 3-bit number and generates an output binary number equal to the square of the input number. (B = A^2)
- Using 3 bits we can represent the numbers from _______ to ______ .
- The possible squared values range from _______ to _______ .
- Thus to represent the possible outputs we need how many bits? ________

3-bit Squaring Circuit

	Inputs			Outputs						
A	A_2	A_1	A_0	B_5	B_4	B_3	B_2	B_1	B_0	$B=A^2$

	A2A1			
A0	00	01	11	10
0	0	2	6	4
1	1	3	7	5

B5 =

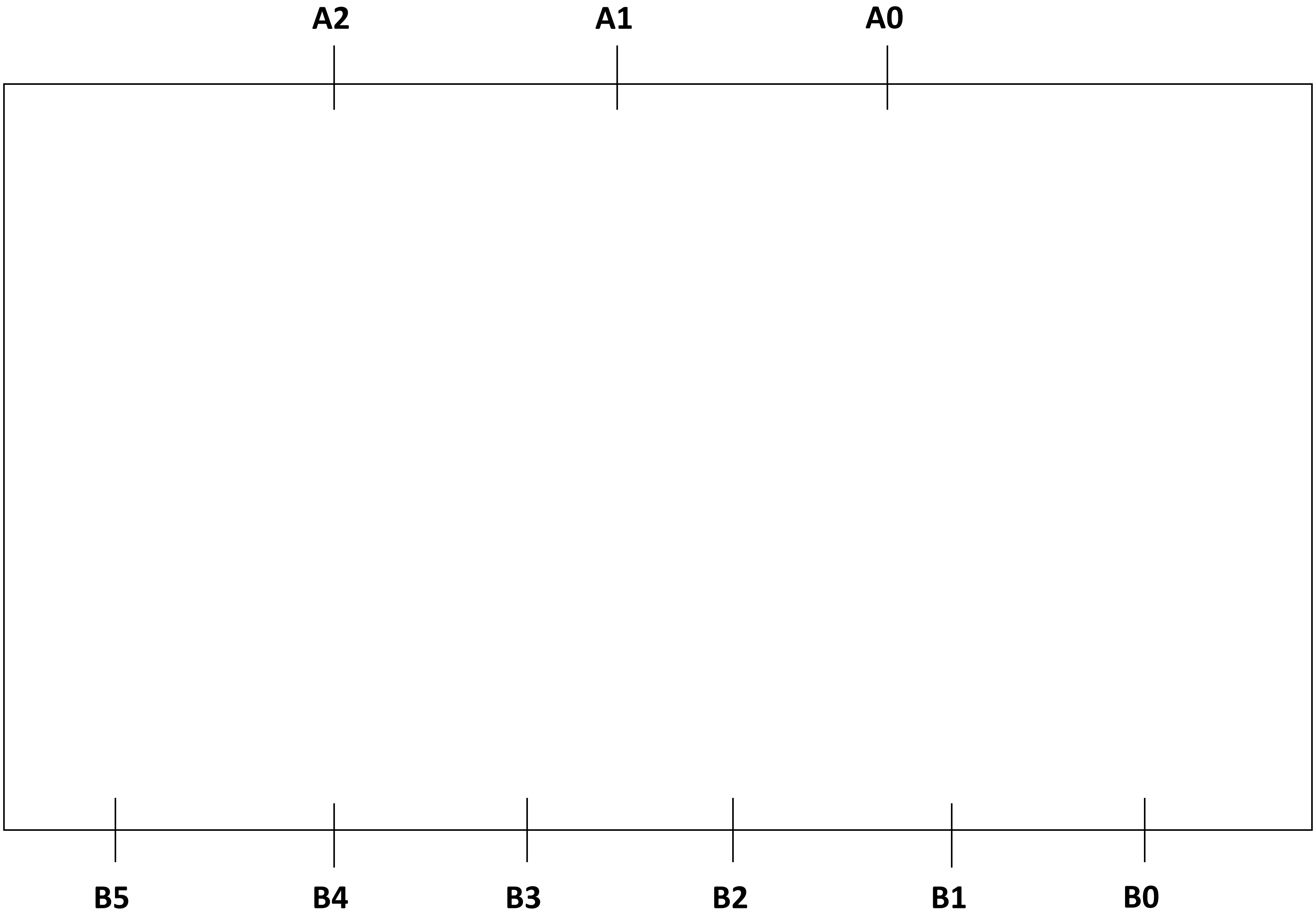
	A2A1			
A0	00	01	11	10
0	0	2	6	4
1	1	3	7	5

B4 =

	A2A1			
A0	00	01	11	10
0	0	2	6	4
1	1	3	7	5

B0 =

3-bit Squaring Circuit



3-bit Squaring Circuit

	Inputs			Outputs						
A	A_2	A_1	A_0	B_5	B_4	B_3	B_2	B_1	B_0	$B=A^2$
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	1	1
2	0	1	0	0	0	0	1	0	0	4
3	0	1	1	0	0	1	0	0	1	9
4	1	0	0	0	1	0	0	0	0	16
5	1	0	1	0	1	1	0	0	1	25
6	1	1	0	1	0	0	1	0	0	36
7	1	1	1	1	1	0	0	0	1	49

A0 \ A2A1	00	01	11	10
0	0 0	2 0	6 1	4 0
1	1 0	3 0	7 1	5 0

B5 = A2A1

A0 \ A2A1	00	01	11	10
0	0 0	2 0	6 1	4 1
1	1 0	3 0	7 1	5 1

B4 = A2A0 + A2A1'

A0 \ A2A1	00	01	11	10
0	0 0	2 0	6 0	4 0
1	1 1	3 1	7 1	5 1

B0 = A0